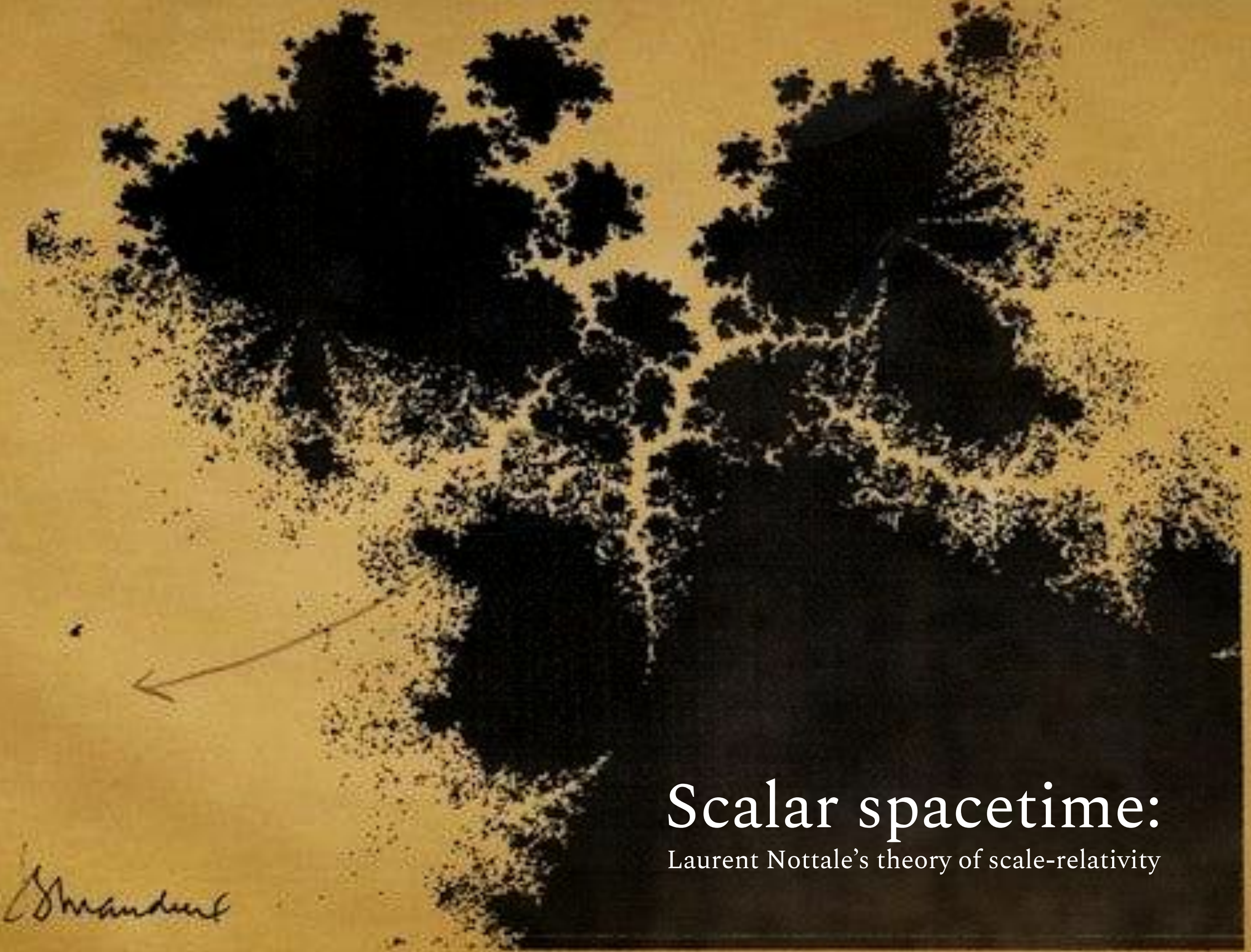


Scalar spacetime:

Laurent Nottale's theory of scale-relativity

1. The history of relativity
2. The principle of relativity as a method
3. The crisis of physics and the hypothesis of differentiability
4. Fractal geometry and scale space
5. Useful tools and insights

Scalar spacetime:

Laurent Nottale’s theory of scale-relativity

Questions and ideas from previous meetings

What are scales and resolutions?

- **language approach:** resolutions as operational concepts to move from a domain of predication to one of another scale of analysis (ex: "pieces of wood" → "molecules")

- **categorical approach:** resolution as endofunctors with certain commutative properties

- **lattice theory approach:** resolution as filters on infinite posets with certain unclear properties

- **type theoretical approach:** resolution as sets of fibers whose elements are made indistinguishable from the perspective of a certain functional process.

- **computational approach:** resolution as iterational limit on a recursive fractal function

- **fractal geometry approach:** resolution as nondifferentiable paths in fractal space - paths that converge to incompatible limits

Attribute: set of possible predicates on parts of a world

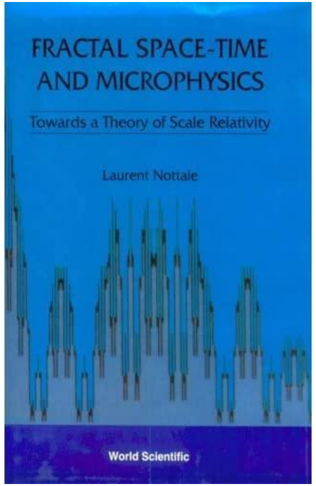
Resolution: a determination of the base unit that establishes a form of measuring the world (attribute + synthetic characteristics)

Resolution-range: the set of possible resolutions that are specifically accessible given the objects of some world (attribute + synthetic+instrumental condition)

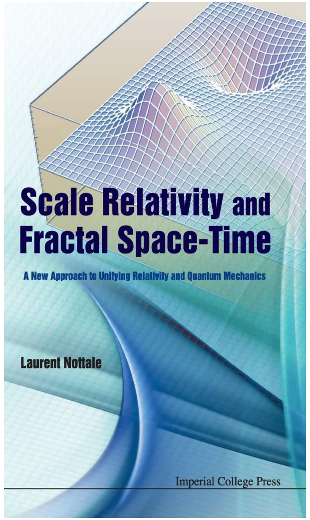
Scale-change: the case where the change in atomic resolution leads to a material synthesis of the world that does not preserve the minimal and maximal of the origin-world.

Layer-change: the case where the change in atomic resolution leads to a material synthesis of the world that adds information that is not accessible through mere predication, but which does not contradict the origin-world.

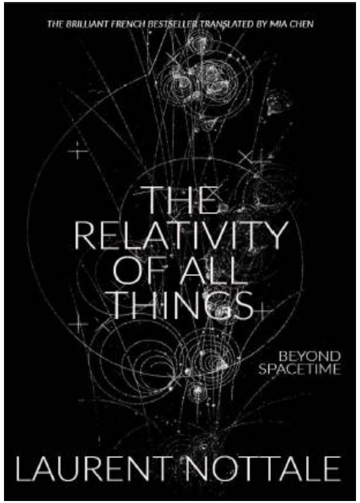
Laurent Nottale



1993



2011



2019

Laurent Nottale (born 29 July 1952) is an astrophysicist, a retired director of research at CNRS, and a researcher at the Paris Observatory. He is the author and inventor of the theory of **scale relativity**, which aims to unify quantum physics and relativity theory.

Nottale began his professional work in the domain of general relativity. He defended his PhD Thesis in June 1980, entitled "Perturbation of the Hubble relation by clusters of galaxies", in which he showed that clusters of galaxies as a whole may act as gravitational lenses on distant sources.[1] Some of these results were reported in *Nature*.[2][3]He also published a popular book *L'Univers et la Lumière*, Flammarion, Nouvelle Bibliothèque Scientifique 1994, Champs 1998) for which he received a prize in 1995 (Prix du livre d'Astronomie Haute-Maurienne-Vanoise).According to Vincent Bontems and Yves Gingras [fr] there are two distinct phases in Nottale's scientific career.[4] From 1975 to 1991 this included conventional topics, such as gravitational lenses, while from 1984 onwards he focused on developing his theory of scale relativity, a proposal for a theory of physics based on fractal space-time.

Scale relativity claims to extend the concept of relativity to physical scales (of time, length, energy, or momentum).[5] Proponents have made wide-ranging claims on its behalf, including applications to the existence of dark matter and the formation of planetary systems, as well as to biology, geology, and the technological singularity.[6][7] Nottale, himself, did not study technological singularities. The proposal has not attracted wide acceptance by the scientific community.[8]

1. The history of relativity
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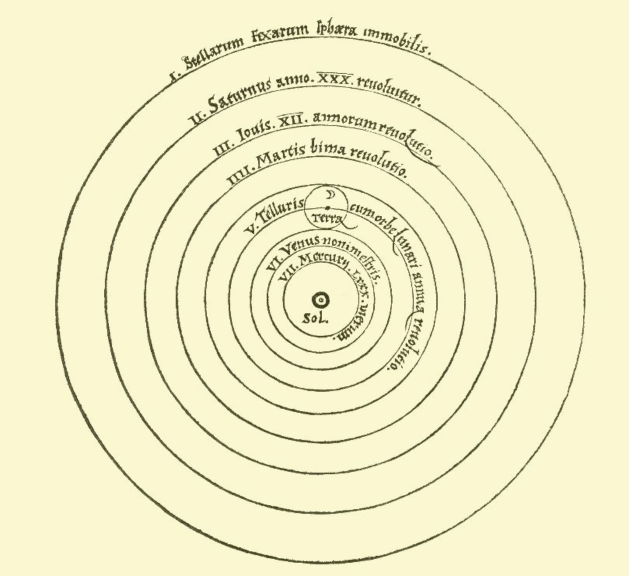
A Quick History of Relativity: Copernicus to Galileo

I. The Copernican move of decentering the earth in favor of a heliocentric model is the start of the process that birthed relativity
   A. Kepler then noticed that the planets do not orbit the sun in circles but rather in ellipses along with the sun around a focal point not located at the suns center
   B. The consequence is that no absolute center of the universe exists

II. Galileo took this observation further stating that the laws of physics must be invariant under translations of reference frames, what is called Galilean Relativity
   A. Importantly, reference frames must be inertial (i.e. not accelerating) in this model

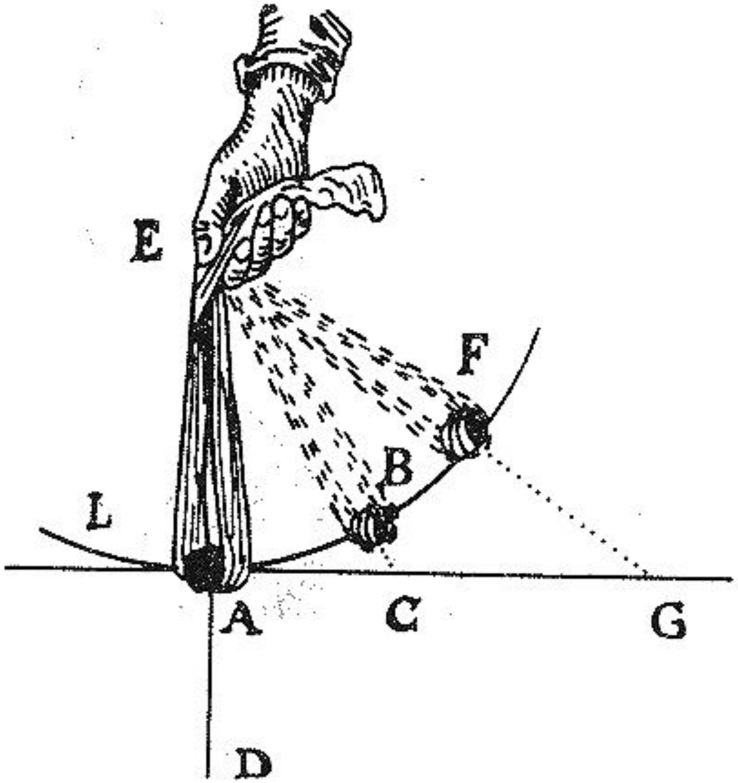


A Quick History of Relativity: Descartes

I. Studying collisions, Descartes formalized the principles of Galilean relativity
   A. By identifying reference frames with choices of cartesian coordinates, Galilean relativity amounts to translating between coordinate systems
   B. This is the declaration of a *symmetry* of the laws of physics under translations

II. This perspective led him to be the first to propose something like the conservation of momentum:

“It is obvious that when God first created the world, He not only moved its parts in various ways, but also simultaneously caused some of the parts to push others and to transfer their motion to these others. So in now maintaining the world by the same action and with the same laws with which He created it, He conserves motion; not always contained in the same parts of matter, but transferred from some parts to others depending on the ways in which they come in contact.”

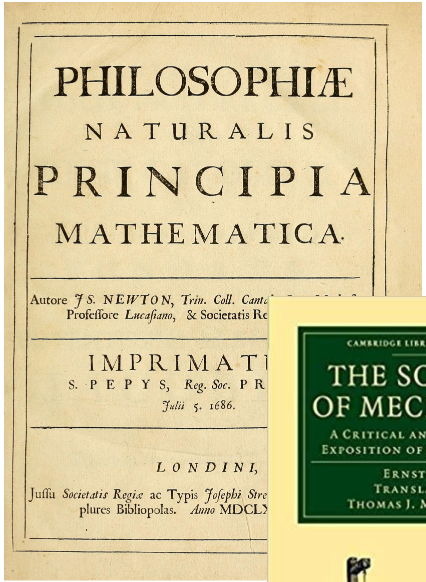


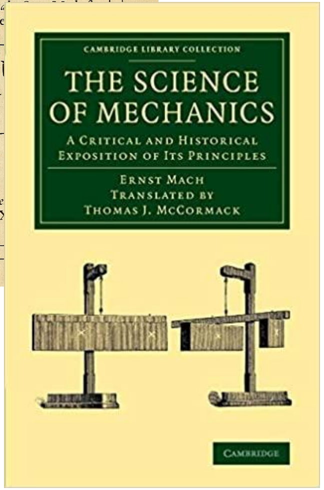
A Quick History of Relativity: From Newton to Mach

I. Universal gravitation: between any two bodies there exists a force of attraction that varies directly with their masses with the inverse square of the distance between them.
   A. Introduces differential calculus to the description and prediction of the physical world
   B. Upheld the existence of an absolute space due to the problem of inertial forces (ex: rotational forces)

II. Mach proposed a new interpretation to the problem of accelerated motion's apparent absoluteness, in favour of a relatitivistic account of reference frames:

"The turning body, within which there appear inertial forces, must turn not with respect to a certain absolute space, but with respect to other material bodies (...) it is within the same frame of reference that the arms are raised and the sky turns, and this will be true for two points of the Earth separated by thousands of km. Mach suggested, then, that the common frame of reference is determined by the entirety of distant matter, of bodies at infinity, of which the cumulative gravitational influence would be at the origin of inertial forces. In other words, the body would turn with respect to a frame of reference, not absolute, but universal""

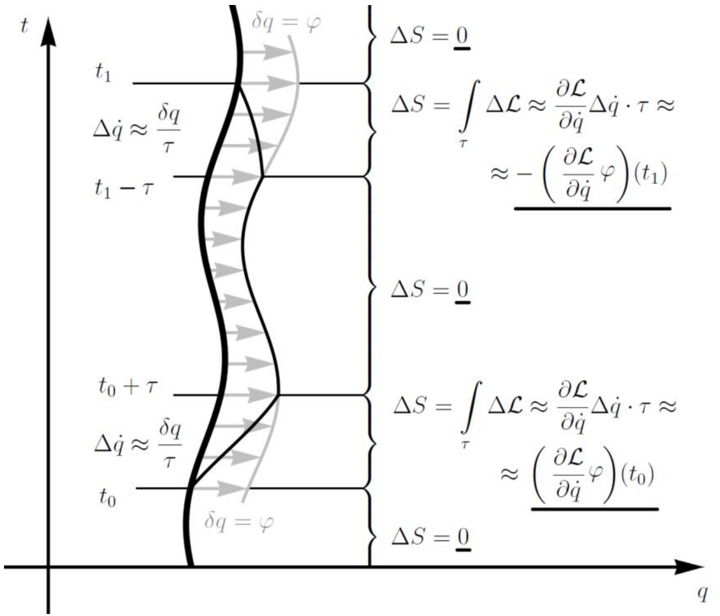




A Quick History of Relativity: Noether

I. Emmy Noether generalized this association of symmetries and conservation laws in the following way:
   A. "To every differentiable symmetry generated by local actions there corresponds a conserved current." (Wikipedia, Noether's Theorem)
   B. The key idea is that symmetries of the actions on the system give constraints on the system in the form of conservation laws

II. Some applications of the theorem:
   A. Translational symmetry implies conservation of momentum
   B. Time symmetry implies conservation of energy
   C. Rotational symmetry implies conservation of rotational momentum
   D. In Quantum Field Theory the analogue called the Ward-Takahashi identity has similar derivations of conservation laws such as the conservation of electric charge from the phase symmetry of the complex field of the particle

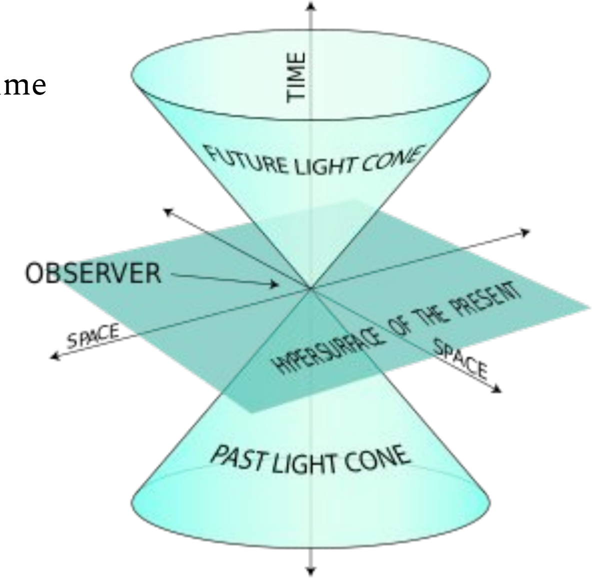


A Quick History of Relativity: Einstein's Special Relativity

I. Special Relativity explains how the relativity of velocity and the speed of light being absolute for all reference frames are compatible
   A. It states that the laws of physics are invariant under Lorentz transformations of coordinate systems rather than Galilean (linear) ones
   B. The new invariant of the system is the spacetime interval

II. Both space and time become relative quantities
   A. Different observers will disagree on measurements of space and time depending on their reference frames
   B. Simultaneity is not agreed upon by all observers
   C. The Lorentz transformations between reference frames give the relative shift of these measurements between one another

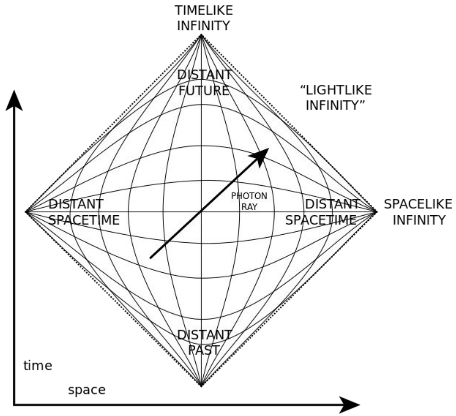
$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$



Interactive spacetime diagram: http://www.trell.org/div/minkvindu.html

Quick Detour: Points of Infinity in Physics

E.g. The compactification of the hyperbolic plane tessellated by M.C. Escher



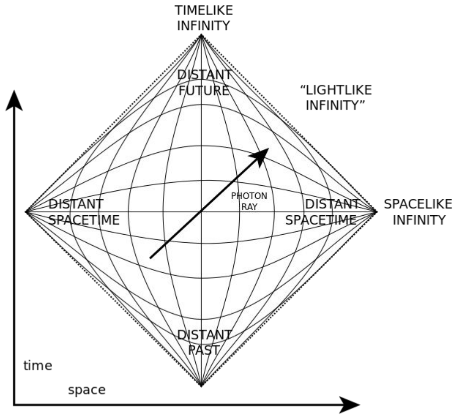
I. The speed of light takes the position of an infinite quantity
   A. For anything with mass, accelerating to the speed of light would require an "infinite" amount of energy
   B. Massless particles on the other hand stand in an absolute position travelling only at this infinite limit speed

II. Compactification
   A. The process of changing perspective such that an infinite space is compacted into a finite volume generally by attaching some "horizon of infinity"
   B. Penrose famously came up with a conformal (angle preserving) compactification of space-time diagrams that helps understand the causal structure when black holes are included
   C. String theorists use compactifications of extra dimensions as a way to explain how it's high dimensional models are compatible with the experience of a 4D space-time

Quick Detour: Points of Infinity in Physics

Ultimately, special relativity demonstrates the existence of spacetime. The complexity of the relativistic formulae compared to their Galilean version is only a consequence of a "bastardized" representation, in which we continue to use three-dimensional concepts such as the usual velocity *v*, while the transformations are four-dimensional in nature (try to imagine what the representations of movements in ordinary three-dimensional space would be like if one limited oneself to using only two variables). Thus, the finitude of the speed of light is only an effect of *perspective*! It is simply the equivalent of a vanishing point. Here again, one will easily understand it by starting from equivalent effects encountered in our vision of space. Two tracks of a railroad which continue infinitely are seen by us, by reason of perspective, to converge at a finite distance on the "plane of the sky." This comes from the projection of a three-dimensional infinity on the celestial sphere, our vision only being two-dimensional. The maximal speed is similar: to be convinced, it is enough to observe that once expressed in four-dimensional representation, the form of Galilean relations are recovered, in which the speed of light now corresponds to a "four-dimensional speed," which is infinite.

E.g. The compactification of the hyperbolic plane tessellated by M.C. Escher



A Quick History of Relativity: Einstein's General Relativity

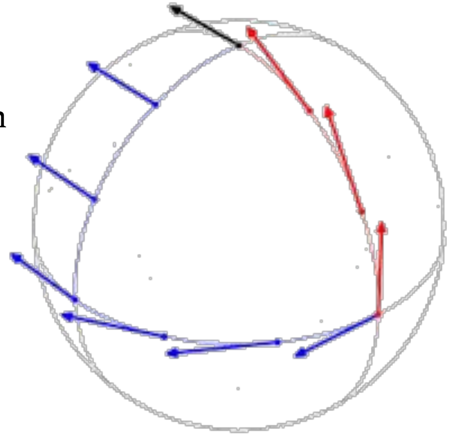
I. The Equivalence Principle
   A. General Relativity begins with Einstein formulating the equivalence principle:

      Locally the effect of the force of gravity is indistinguishable from an accelerating reference frame

   B. This is the impetus for generalizing the model of spacetime from a flat euclidean one to a curved riemannian one

II. Differential forms (the metric tensor in particular) play the role of local coordinates
   A. At each point there is a way to measure lengths and angles, and a way to smoothly move between the points (the covariant derivative)
   B. Space-time under General Relativity is locally Minkowskian (looks like Special Relativity)

III. Geodesics are the "paths of least action" that follow the curvature of space-time
   A. In a curved spacetime there is no global notion of "straight line," geodesics generalizes this concept to curved spaces
   B. Geodesics appear as paths that locally follow the curvature at each point



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5. **Useful tools and insights**

The general principle of relativity

Jean-Marc Lévy-Leblond: distinction between principle of relativity and theories constructed on the basis of it.

It's general formulation can be stated as:

"the fundamental laws of nature are valid in any system of reference"

(easy for philosophy, very crazy for physics!)

The general principle of relativity

*“the **fundamental laws of nature** are valid in any system of reference”*

- presupposition of scientific endeavor

- uniqueness of laws

- connected to Nottale’s defense of the continuum hypothesis in physics: there are no absolutely disconnected patches of physical cohesion in the physical universe

The general principle of relativity

"the <u>fundamental laws of nature</u> are valid in any ***<u>system of reference</u>****"*

- clear and systematic definition of a "coordinate system"

- scientific statements refer to results of measurement

- "it is what allows us to envision the universality of the principle of relativity and the possibility of its application to domains of knowledge other than physics alone"

Frames of Reference Depend on Experimental Setup

Ultimately, a frame of reference can be defined as an abstract system which synthesizes the universal properties of the mechanism of measurement, that is to say, those which are independent of the specific characteristics of the instruments effectively used, but which are common to all these instruments. It is in this way that the result of measuring a point's position along one direction is given uniquely by a number, expressed using a certain unit, associated with an error bar, for example (12.35 ± 0.02) m. This result requires only the definition of an origin, an axis, and a unit which act as references, and a resolution for the uncertainty of the measurement (the resolution is determined by the minimal unit accessible by the given instrument, for example the interval between the two tick marks closest together on a ruler). However, to obtain results in practice, much more has been required than the quasi-abstract characteristics which are a perfect point of origin, an infinitely thin axis and markings traced on this axis. In reality, one must use a wooden, plastic, or other kind of ruler, and trace the markings with such and such a method and such and such a color. All these characteristics which are specific to the ruler actually used disappear as non-significant in the definition of the ruler seen as an archetype of an element of a coordinate system. It is similar with the details of the constitution of a clock: only the numerical value of the time that it gives is of importance, not its form or its inner composition.

Frames of Reference Depend on Experimental Setup

“a frame of reference can be defined as an abstract system which synthesizes the universal properties of the mechanism of measurement”

- centralizes experiments as the determination of reference frames in a materialist way

- resolution is thus an additional degree of freedom determined by the minimum scale the experiment can differentiate

Relativity and invariants

"physical quantities are not defined in an absolute way, but are instead relative to the state of the reference system (...) this principle, which is philosophical in nature, is translated into physics through three constraints: that of covariance, of equivalence and of geodesics" (Scale relativity and Fractal Space-Time)

- **covariance**: construction of equations such that physical quantities transform together as a group as the equation is applied to different reference systems

- **equivalence**: statement that shows that the very existence of some crucial property is dependent on the reference frame - thus determining the existence of a reference frame in which such property disappears.

- **geodesics:** definition of dynamical properties through the very geometry of the reference frame - such that "free trajectories" are identified with the geodesics of space time.

Relativity and perspectives

Here again, a particular point of view plays an essential role: that which consists of putting oneself "in the place" of the system under consideration, to envision it from the inside and not only from the outside. It is a matter of placing oneself in the frame of reference best adapted to an understanding of the problem, in a sort of "empathy" with nature. Such mental exertion is required for one to speak of true understanding: one knows because one sees and directly senses, even though virtually, the nature of the system to be understood. Such was the privileged method adopted by Einstein, when he anticipated special relativity in trying to imagine what an observer travelling with a light wave would see or, later, when he laid the foundations which would lead to general relativity by mentally visualizing the experience of free fall in a gravitational field.

The movement between synthesis and analysis

More generally, equations in physics describe simple relations between physical quantities, defined locally by the tendency of their differentiation. Most often, these take the form of differential equations.

The Cartesian method has sometimes been charged with reductionism. This is, in my opinion, a false charge which stems precisely from a reduction of Cartesian thought. Sure enough, a restricted application of this method to a particular problem can be reductive. But it is remarkable that when a blockage of theory arises (as periodically happens in physics), Cartesian reasoning so often leads to a solution to the problem, simply because it is a general method of *analyzing* the laws of nature. For example, the idea that the whole must be the simple and direct sum of its parts (an example of a "reductionist" idea) is not explicitly part of the Cartesian perspective. One must analyze the system in question to find parts that are easier to describe, but the identification of these parts may well be more complicated than a simple "cutting apart," and the reconstruction of the whole more elaborate than a simple "gluing together."

The movement between synthesis and analysis

- *Cartesian thinking is not essentially reductive, but compositive: the statement that the analysis of parts *always composes in the same way* to become the analysis of the whole is not part of the method. Hence, there can be analytic principles that divide a problem into parts that compose together in different ways - the essential part of the method is the movement between analysis and synthesis, not the form of this movement itself.*

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Crisis of physics today

Has this story been finished? Is physics currently balanced and harmonious, ruled by a single system of laws founded on first principles? It is not, quite the contrary. The crisis of modern physics is at least as profound as that of Aristotelian science. Depending on whether one considers small or large scales (microscopic and macroscopic domains), the phenomena concerned, the experiments, or the observations that one makes, must be explained respectively by quantum or classical laws. Between quantum mechanics, which regulates molecular, atomic, nuclear systems as well as the physics of high-energy particles, and classical mechanics, adapted to ordinary scales, to those of planets, stars, our galaxy, and extragalactic systems, it is not only the laws which differ. It is a whole way of thinking, a manner of treating problems, the choice of mathematical tools and the rules which govern them, which differ at the most fundamental level. One could say, in a provocative manner, that physics currently is in a quasi-"schizophrenic" state, being not one, but two: two almost contradictory physics coexist in an anything-but-peaceful manner.

Thus the relativist "diagnosis" applies especially well with the contemporary physics. One will see that the principle of relativity allows us, once again, to propose remedies for this crisis.

Crisis of physics today

Let us compare quantum mechanics and relativity. General relativity, as we have seen, is a theory that, even if it can become complicated in its application, is based on simple and comprehensible principles. On the contrary, the foundations of quantum theory are purely axiomatic in nature. Relativity is an essentially geometric theory, based on the primacy of the four-dimensional spacetime continuum (which even becomes its primary instrument). Quantum theory is, on the other hand, an algebraic theory for which the principal framework is constituted by spaces of abstract states: there often follows a loss of intuitive comprehension of phenomena. Finally, one can consider that general relativity contains conceptual levels deeper than quantum theory. In effect, the fundamental equations of Einstein's theory are those of the geometric structure of spacetime; the equation of trajectories (that is, of geodesics) is *deduced* from it; ultimately the equation of a *set* of geodesics is itself constructed starting from this latter equation. One might compare this last type of equation to Schrödinger's equation. Currently, there is no equation of an eventual "quantum spacetime." Here again, one can in this respect compare quantum theory to Newtonian theory in its time: an extremely precise theory, which "sticks to the facts," but remains unsatisfactory from the point of view of fundamental concepts.

Crisis of physics today

This dichotomy in contemporary physics is extremely deep and troubling. It goes further than a simple change of laws in the classical sense of the term, which could have been attributed to the existence of new fields at small scale in relation to those known classically (gravitation and electromagnetism). Such new fields indeed exist (the strong and weak nuclear interactions), but the problem is not that. Everything changes between quantum and classical physics: the concepts, the mathematical tools, the manner of posing problems, even of thinking about them, the manner in which theory is formed, the manner in which it is understood.

It seems more and more clear, after a century of concerted effort concerning the relations between classical and quantum physics, that one cannot pass directly from one domain to the other: neither of the two is deduced from the other; they must coexist and be given independently. But is there no other way of unifying them? Cannot each of these two representations of the world be deduced from a third, more general representation, which encompasses them, and is reduced, according to the scale of the observation or the description, respectively to quantum and classical representations? It is such an approach which will be in question in the rest of the present work.

Search for a geometrical basis for the quantum world

What Einstein could not accept as definitive in the theory of microphysics was that quantum mechanics was *by nature* statistical, that probabilities should be placed at the level of the theory's principles. This would abandon an explanation of individual elementary phenomena. Einstein, on the other hand, thought that any physical theory worthy of the name could not be probabilistic in its fundamentals, and that the necessity of a statistical description must, to be fully comprehended, be *derived* from more fundamental principles applicable to the description of "the state of individual systems." As we shall see in what follows, the theory of scale relativity comes under such a view: its fundamental foundation is in terms of nondifferentiable and fractal spacetime, and the statistical description is a consequence of the fact that geodesics are themselves fractal and infinite in number in such a spacetime.

The “independent axiom” of differentiability

We will see an example of the power of the Cartesian mode of analysis with the theory of scale relativity, where we will relinquish the hypothesis of differentiability. The motivation for this relinquishment is the observation that, in actual experiments, the separation into finer and finer sections of space and time does not necessarily yield parts simpler than the initial system (for example, an electron is simple at a large scale, but at a small scale its internal structure becomes enormously complex, involving the whole spectrum of different elementary particles). At first glance, this point of view would seem to be in disagreement with Descartes: the parts are not more simple than the whole. Nevertheless, a solution can be found to this problem by applying Cartesian analysis at a deeper level. Spacetime itself possesses an internal sub-structure (in the “space” of *scales*) to which differential calculus can be applied, even in the case where it is not differentiable.

What is the hypothesis of differentiability?

All contemporary physics carries the assumption that space-time is a differentiable manifold (i.e. every point has a neighborhood that looks “locally flat”)

This has the consequence that spacetime is infinitely divisible meaning smaller scale subsets are simpler and have less structure

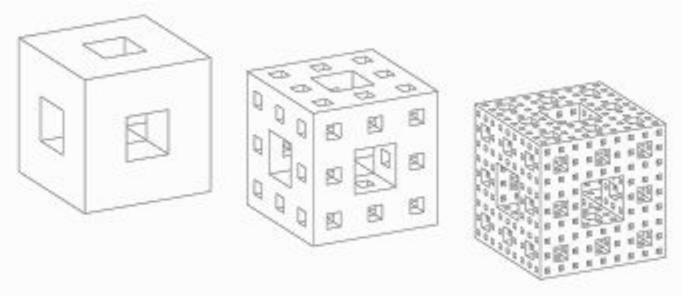
Thus with the hypothesis of differentiability, finer measurements lead to results “closer” to reality

What is the hypothesis of differentiability?

Scale relativity relaxes the constraint of differentiability in order to derive the laws of quantum mechanics as an extension of General Relativity

Giving up the differentiability of space-time implies scale relativity

A. If space-time is non-differentiable then the derivative in its classical sense has no meaning, smaller scales can lead to more complexity potentially ad-infinitum
B. Since finer measurements don't necessarily get you closer to reality, the truth of the measurement is a function that depends on the resolution
    1. Non-differentiable spaces have sets of tangents at a point that depend on the scale rather than one tangent: https://www.geogebra.org/m/q3vqpsw2
    2. The differential equations familiar to physicists can thus be recovered, but with the caveat they are in both space-time and scale-space
C. This implies the principle of scale relativity: structure arises at all scales



From infinitesimals to scale-dependence

Consider a differentiable function $f(x)$. Its derivative is defined since the work of Newton and Leibniz during the 17th century and its formalization during the 19th century as

$$f'(x) = \lim_{dx \to 0} \frac{f(x+dx) - f(x)}{dx}. \tag{3.1}$$

For example, for $f(x) = x^2$, it is easy to compute this limit and to find that $f'(x) = 2x$. Critical analyses of the differential calculus goes back to its origin. Hence Berkeley wrote in 1734 ([49], Sect. XXXV):

> "It must, indeed, be acknowledged that he [Newton] used Fluxions, like the Scaffold of a building, as things to be laid aside or got rid of, as soon as finite Lines were found proportional to them. But then these finite Exponents are found by the help of Fluxions. Whatever therefore is got by such Exponents and Proportions is to be ascribed to Fluxions, which must, therefore, be previously understood. And what are these Fluxions? The Velocities of evanescent Increments? And what are these same evanescent Increments? They are neither finite Quantities nor Quantities infinitely small, nor yet nothing. May we not call them the Ghosts of departed Quantities?"

From infinitesimals to scale-dependence

From the mathematical viewpoint, the modern definition in terms of limit could be considered as an answer to Berkeley's criticism. But then another question arises: does the mathematical definition of derivative agree with its physical meaning?

It is easy to show that this is not the case and that the mathematical derivative is an ideal concept, which is very far from what is actually done in physics. Consider for example the motion of a point-like body. Its velocity is theoretically defined as

$$v(t) = \lim_{dt \to 0} \frac{x(t+dt) - x(t)}{dt}. \tag{3.2}$$

In order to evaluate the validity of such a definition, let us attempt to perform a thought experiment (Gedanken experiment) in the Galileo–Einstein way that would implement it. If really the physical velocity was to be defined in this way, this would involve our ability to make measurements of the position of the body on time intervals, say 1 s, then 10^{-1} s, 10^{-2} s, ..., 10^{-10} s, 10^{-20} s, ..., 10^{-100} s, etc., without any limit. We recover here one of Kant's antinomies, according to which the zero point is actually unreachable and is, therefore, fundamentally an infinite.

From infinitesimals to scale-dependence

Such an infinite series of measurements is indeed impossible for two main reasons.

(i) We are unable to make measurements of position and of time below some minimal resolution scale, however, this minimal resolution scale depends on the quality of our measurement apparatus and it is indeed one of the main progress of experimental physics since the discovery of the microscope to continuously improve their resolution, so this is not a fundamental limitation in the framework of a thought experiment.

(ii) Their number is infinite. In other words, even if one may hope that whatever small a resolution length-scale δx or time-scale δt may be, one will be able in the future to make measurements at these scales, anyway a measurement at $\delta x = 0$ or at $\delta t = 0$ can be considered as definitively impossible.

Now the view point of standard physics is that one does not need to go effectively to the zero limit, since, assuming that the position function $x(t)$ is differentiable, the value obtained for smaller and smaller dt's differs in a smaller and smaller way from the theoretical limit (see Fig. 3.1). Namely, performing a Taylor expansion yields $\{x(t + dt) - x(t)\}/dt = x'(t) + (1/2)x''(t)\, dt + \ldots$, which indeed tends to $x'(t)$ when $dt \to 0$.

From infinitesimals to scale-dependence

However the 20th century experimental physics has brought a fundamental denial to this view. If one really performs the experiment of measuring positions and instants with smaller and smaller space and time resolution intervals, one rather rapidly reaches scales where the quantum laws manifest themselves. Instead of the expected vanishing difference $\delta v \approx (x''/2)\delta t \approx (x''/2x')\delta x$, the velocity becomes more and more badly defined with decreasing scale according to Heisenberg's relation, $\delta v \approx (\hbar/m)\delta x^{-1}$.

This fundamental contradiction has led Heisenberg to construct the quantum theory on the basis of the giving up of the classical concepts of position and velocity. However, from the above analysis, the key hypothesis, which seems to be falsified, is the assumption of differentiability of the position variables, i.e. of space (and more generally of space-time in a motion-relativistic framework), not their very existence.

Let us finally give another equivalent way to exhibit the fundamental contradiction that lies at the heart of today's physics. The Heisenberg's relations also tell us that, if one wants to perform an experiment, e.g. a measurement of position, at decreasing time resolution δt, one needs an increasing energy $E \approx \hbar/\delta t$. The construction of experimental devices aiming at scanning the microscopic world have definitely confirmed this fundamental fact, with magnifying glasses and microscopes using visible light energy, then electronic microscopes, then field effects microscopes, then particle accelerators which now reach several TeV energy scales.

1. The history of relativity
2. The principle of relativity as a method
3. The crisis of physics and the hypothesis of differentiability
4. Fractal geometry and scale space
5. Useful tools and insights

The Scales of Spacetime

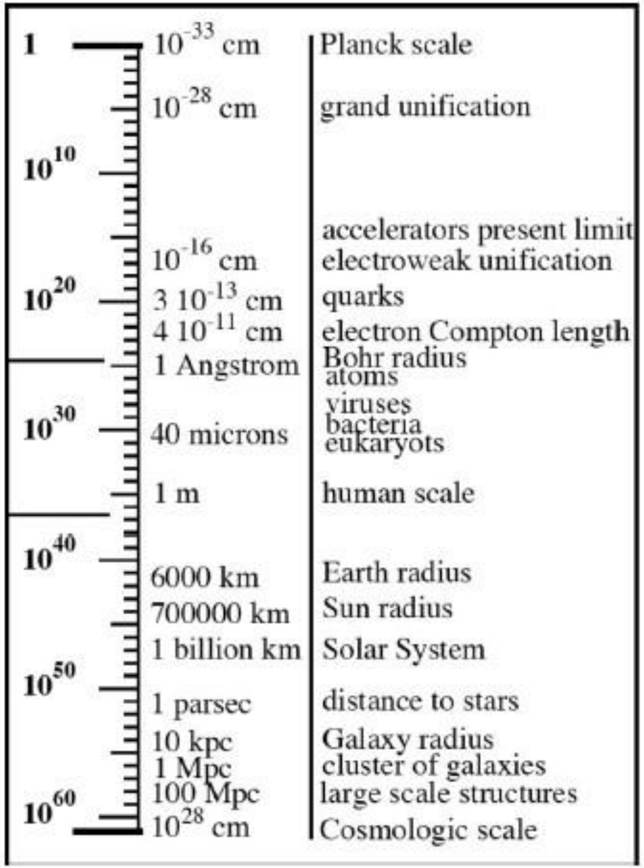
Since antiquity, since the age of Plato, Euclid, and Aristotle, numerous philosophers and writers (such as Voltaire with *Micromegas* or Swift with *Gulliver's Travels*), in addition to mathematicians and physicists (among others, Leibniz, Boscovich, Laplace, and Poincaré), have considered the question of scales of length in Nature. If the philosopher or novelist has often imagined that there might exist men of lilliputian size or giants as large as the Earth, we know that in reality, one sees nothing of the sort. The height of an adult human can hardly vary by more than a factor of two, no one has a height of a millimeter or a kilometer. The size of atoms is of the order of a few angstroms, and there do not exist atoms with a radius of one meter, nor of one fermi: it is their nuclei which one encounters at the latter scale.[97] Inversely, the radius of a star is of the order of millions of kilometers and can reach one hundred times this size, but not one million times. At the scale of 10 kpc (ten thousand parsecs, the parsec being the astronomic base unit), the structures encountered are galaxies, but no galaxy is a thousand times smaller or bigger.[98]

Thus, any given structure is often characterized by a scale or a limited range of scales and, inversely, at any given scale there generally corresponds a certain type of structure. Why is it thus? What determines these scales? This is one of the essential problems, in large part unresolved, of fundamental physics.

The Scales of Spacetime

physical phenomena

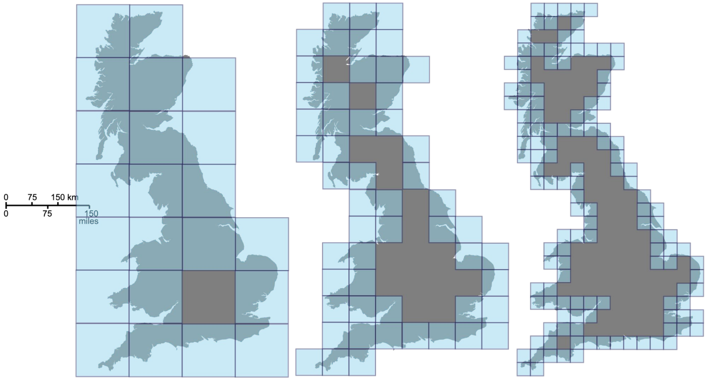
continuous
change of magnitudes



discontinuous types
of instruments of measure

The example of the coast of Britain

Let us return to the question of scale. One does not only see the existence of fundamental scales in nature, but also the existence of laws that are explicitly dependent on scale. In such a case, the physical quantities under consideration take on values which change according to the scale with which one measures them. To give a familiar example, it is so with the length of the coast of Britain, which clearly depends on the scale of the map which one uses to determine it. The more detail this map allows one to see, the more there appear new gulfs, inlets, and recesses which increase this length. Yet this variation of the length as dependent on the resolution (which is the size of the smallest observable detail on the map) is not made in any which way, but following a well-determined law. Such phenomena have led to concepts of scale laws and scale invariance, and to geometric description in terms of fractal objects.[109]

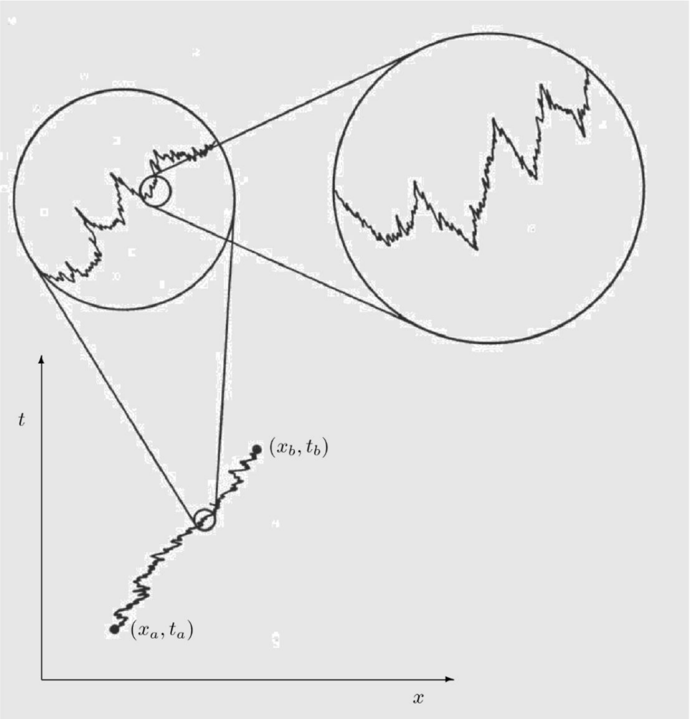


Feynman’s path integrals

“Typical paths of a quantum-mechanical particle are highly irregular on a fine scale. Thus, although a mean velocity can be defined, no mean-square velocity exists at any point. In other words, the paths are nondifferentiable.” (Richard Feynman, Quantum Mechanics and Path Integrals)

I. Feynman’s path integral generalizes the Principle of Least Action to Quantum Field Theory
   A. He did this by integrating over the infinity of all possible quantum paths phase-scaled by the action of each path
   B. Already in this framework, interference between these weird paths leads most paths to cancel out at a large scale--hinting at how classical physics could be emergent

II. The success of this approach is one of the key reasons to believe Scale Relativity might have something interesting to say about the connection between the quantum and classical worlds



Generalization beyond differentiability

The idea serving as Ariadne's thread for this point of view is that there does exist a fundamental principle upon which to base the new theory: this is the *principle of relativity* itself. However, one must go beyond the principle of relativity of motion, as it has been developed by Galileo, Poincaré, and Einstein. It is necessary to introduce an extension of this principle which also encompasses the transformations between scales.

What can we expect from such an extension of our frame of thought?

As we will see, one first consequence is that it implies a profound change to the nature of spacetime. *The general relativity of Einstein had led to a generalization from the flat and absolute space of Newtonian theory to a curved spacetime, dependent on its material and energetic content. Similarly, the idea of scale relativity introduces a new spatiotemporal geometry, more complicated yet: spacetime becomes fractal.* The concept of fractals was expressly constructed by the mathematician Benoit Mandelbrot to designate objects, sets, or functions whose form is extremely irregular and fragmented at every scale, and for which the geometry thus explicitly depends on the resolution used in considering them (see figure 4 and the following). A new usage should nevertheless be made here. One no longer attempts to describe fractal *objects, a priori* included as contents of a space defined beforehand, but fractal *spaces* (more generally, fractal spacetimes), which should therefore be defined intrinsically, from the inside, since it is the container instead of the content. Moreover, as it is already the case in Einstein's theory of general relativity, it is not a question here of an absolute spacetime, but on the contrary, of the construction of a fractal spacetime that must be relative to its material and energetic content.

A second consequence is *a new possibility of understanding quantum mechanics*. We will see how elementary particles can be reinterpreted as families of "geodesics" (being the shortest trajectories from the viewpoint of proper time) of a fractal spacetime showing structures at all scales toward microscopic scales. More generally, the main axioms of quantum mechanics (meaning its mathematical principles, currently arbitrarily posed, and not understood) can be reconstructed and derived in the new approach starting from the principle of relativity itself (applied to motion *and scales*). Similar to how, in Einstein's general relativity, gravitation is nothing other than the set of manifestations of the curvature of spacetime, the theory of scale relativity suggests that quantum effects appear as manifestations of the fractal character of spacetime at small scales.

Generalization beyond differentiability

But we can go further yet. Scale relativity not only allows us to propose a source for quantum mechanics, it also leads to possible generalizations at very small scales. The application of the principle of relativity to scale transformations of coordinate systems leads us to propose possible forms for the laws which govern *the internal structures of spacetime*, structures we postulate to exist in each "point." Original fractal structures, more general than "ordinary" fractals, thus emerge. In this enlarged paradigm, *the problem of the infinite divisibility of space and of time finds a new solution*. It remains possible to divide a spatial or temporal interval in two, and then to divide that new interval in two, and so forth to infinity, but the result of this operation is no longer zero, but a finite interval. Similar to how one can add velocities to each other without end in special relativity, but the result of this sum always remains lower than the speed of light, here there *appears a universal, minimal scale, unsurpassable, upon which any magnification would have no effect*.

As we will see, it seems natural to identify this scale with the Planck scale, whose role as the natural unit of length and of time would find itself to be thus justified. This scale would then possess all the physical properties which were formerly attributed to the point zero and would replace it, all scales between zero and the Planck scale no longer having any existence (similar to how the speed of light possesses, in special relativity of motion, all the physical properties of an infinite speed).

Moreover, the consequences of the theory of scale relativity do not only concern the infinitely small: as I will show in closing, *it also allows us to cast a new light on cosmology (the domain of very large scales of length and of the universe taken in its entirety), as well as the general problem of the formation and development of structures*, in particular gravitational structures.

Mathematics, physics and experimentation

Let us continue with the analysis of the concept of position and instant. If the mathematical point can have a proper definition, is it the same for a physical point? Can one bring into existence a true point? The question can also be asked about other geometrical objects such as lines and surfaces, supposedly without thickness. If one wishes to explain what a point is to a child, one can simply draw a dot. But what has one actually done? The sheet, board, or computer screen upon which the point is drawn, even the receptors that are used to detect this image (our eyes in particular) are in actuality characterized by a certain limiting resolution. The dot on the page or screen may be smaller than this resolution, but it still has nothing to do with the mathematical point, which must be strictly zero in extension. One can simply examine it with a magnifying glass to realize that it is in fact an extended spot. Then, one could take a sharper pencil, a pointier instrument, and make a point recognizable as such under the magnifying glass. But using a microscope would also show us its internal structure, and so on until infinity. The mathematical point or line ultimately cannot be physically realized.[115]

Mathematics, physics and experimentation

Einstein has already insisted on the difference between physics and mathematics:

> It seems to me that when mathematical propositions correspond to reality, they are not certain, and that when they are certain, they do not correspond to reality.[116]

What should we replace the mathematical point, curve, or surface with if they are in actuality inadequate for physical description? The answer is a mathematical tool which includes in its definition itself what the physicist does in practice: one magnifies them more and more with a lens, then an optical microscope, then an electron microscope, then a field-emission microscope, then a particle accelerator. All these instruments change the scale of observation. Toward the larger scales, the use of glasses and telescopes plays a similar role. Yet what experiment and observation has taught us is that never in the course of this type of operation does the strict equivalent of these mathematical objects appear, these objects which we nonetheless use to describe the world. By improving the resolution of an instrument, new internal structures will always appear. But a new geometry now exists, precisely characterized by the existence of structures at every scale: fractal geometry.

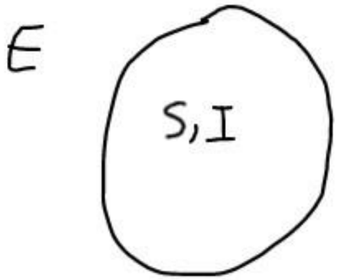
Mathematics, physics and experimentation

experiment

→ environment → system (S)

→ observation → instruments (I)

reference system

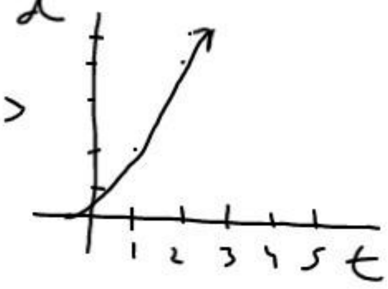


encodes states $s \in S$

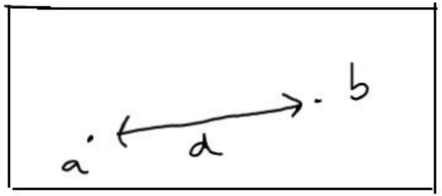
into chronicles of unit $i \in I$

ex:

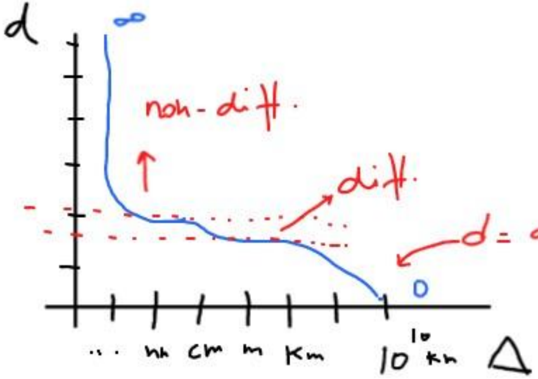
t^i	d^i
1	0
2	2
3	4
4	6



The explicitation of resolutions in coordinate systems

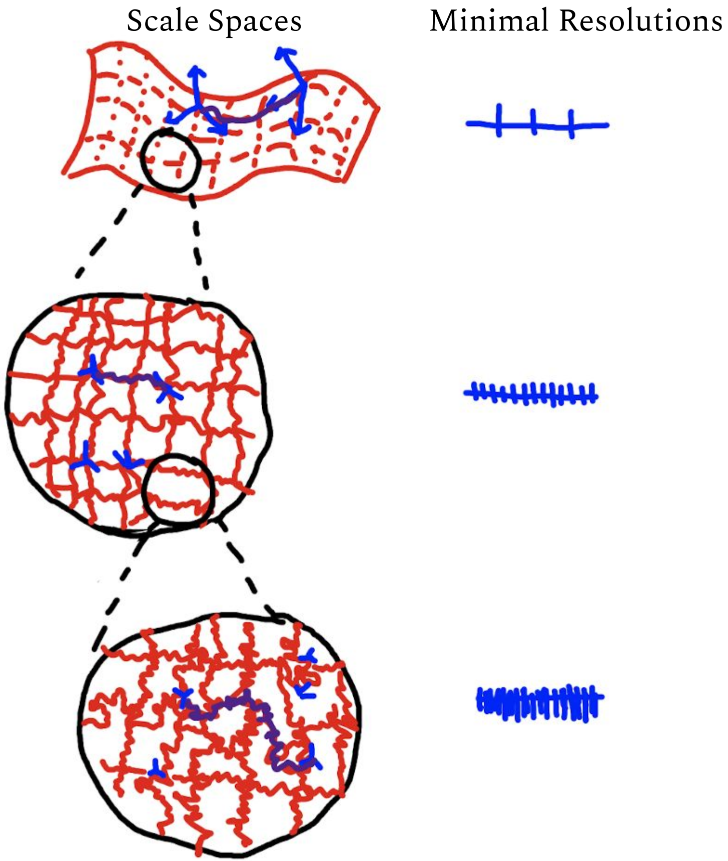


$a : (x, y, z, t, \Delta)$ $b : (x, y, z, t, \Delta)$



Our first proposal for implementing the idea of scale relativity was to extend the notion of reference system by defining "supersystems" of coordinates which contain not only the usual coordinates but also spatio-temporal resolutions, i.e., $(t,x,y,z;\Delta t,\Delta x,\Delta y,\Delta z)$.[33] The axes of such a reference supersystem would be endowed with a thickness: this corresponds, indeed, to *actual* measurements. Then we proposed an extension of the principle of relativity, according to which the laws of nature should apply *to any coordinate supersystem*. In other words, not only general (motion) covariance is needed, but also *scale covariance*.

Non-differentiability and scale-space



I. For each point in scale-space there is a continuum of tangent spaces
   A. The tangent spaces each depend on the minimum resolution of the measurement apparatus at that scale
   B. When restricted to the case of smooth spaces all such tangent spaces would be the same

II. Emergent phenomena arise from the way in which these tangent spaces are mutually incompatible
   A. For each scale, the measurement apparati are compatible with one another
   B. Has the feeling of a sheaf condition
   C. This is the model for the transition between the “two worlds” of physics

1. The history of relativity
2. The principle of relativity as a method
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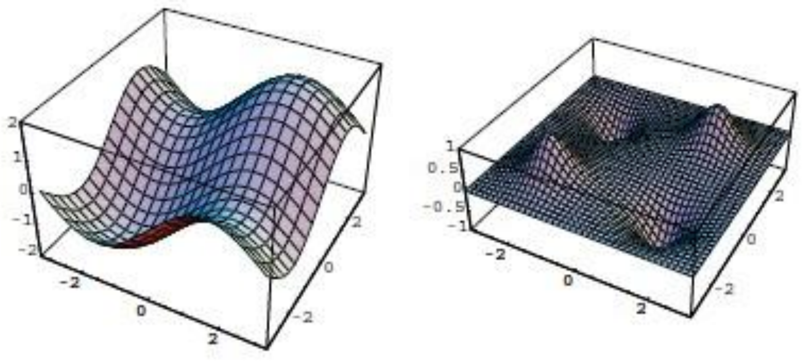
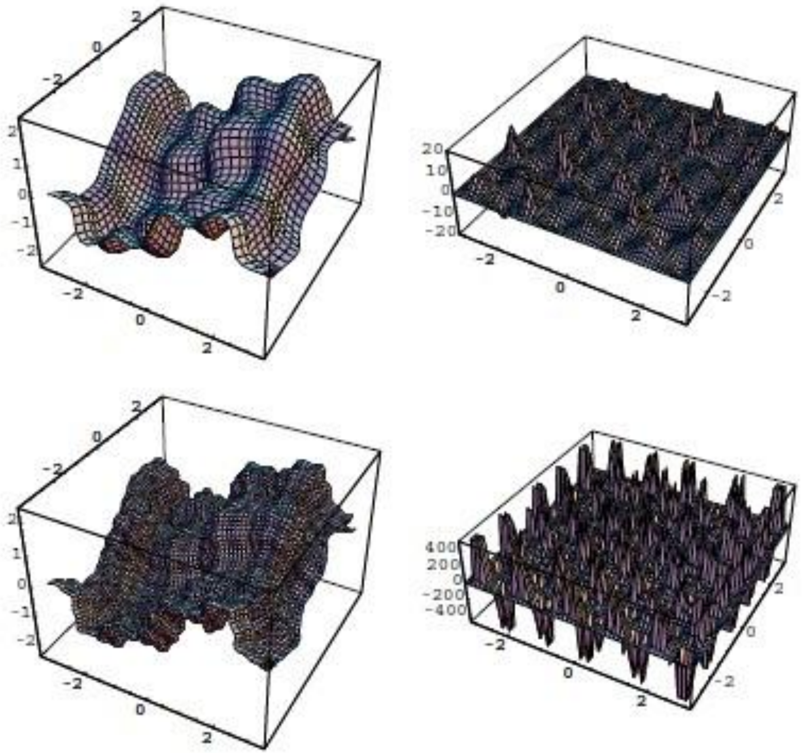
Emergent structures at all scales





Scale-relativity theory gives us a rich example of how we might connect the emergence of complexity and structure through material changes in measurement conditions

Connection between resolutions and experiments

The status of *resolutions* is related to that of units (in particular they are subject to the relativity of scales), but is actually different and of more far reaching physical importance. Changing the resolution of measurement corresponds to an explicit change of the experimental conditions. Measuring a length with a resolution of 1/10th mm implies the use of a magnifying glass; with 10 µm, we need a microscope; with 0.1 µm, an electron microscope; with 1Å, a tunnel microscope. For even smaller resolutions, the measurements of length become indirect, since the atom sizes are reached and exceeded. When we enter the quantum domain, i.e., for resolutions smaller than the de Broglie length and time of a system (as will be specified afterwards), the physical status of resolutions radically changes. While classically it may be interpreted as precision of measurements (measuring with two different resolutions yields the *same* result with different precisions), resolution plays a completely different role in microphysics: the results of measurements explicitly depend on the resolution of the apparatus, as indicated by Heisenberg's relations. This is the reason why we think that the introduction of resolution into the description of coordinate systems (as a *state of scale*) is not trivial, but will instead lead to a genuine theory of scale relativity and the emergence of new physical laws (see Chapter 6).

Proposes a structural connection between the types of structure that are accessible in a given system of reference and the types of measurement that are experimentally possible in that context

Connection between economic optimization and fractality

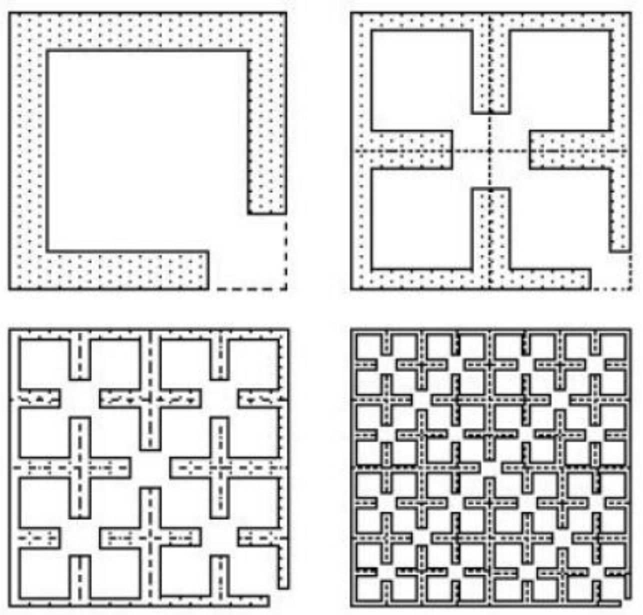
One can obtain fractal structures in terms of a *process of optimization under constraint*, or more generally of the optimization of several quantities, sometimes apparently contradictory. Suppose, for example, that the evolution of a system leads to a maximization of surface area (which is the case in the process of exchange, as in the lung) while minimizing the volume. A solution which optimizes the two constraints is a fractal of dimension greater than two, but smaller than three (it would correspond to a surface tending toward infinity and an infinitesimal volume).

One of the simplest cases of fractal objects (which should not be considered the only case) is that of self-similar fractal objects. These are sets for which one observes the *same* structure after successive magnifications. However, for the mathematical objects which have been most studied thus far, one does not find the *exact* initial configuration unless considered at discrete values of the dilation factor. In other words, when one analyzes them in terms of the variable of resolution (as a logarithm), a self-similar fractal is a *periodic* system in the new dimension of scale.

Let us illustrate these two aspects of fractals, that is, multiple optimization and the return to a preceding state, by a simple example. Suppose that a system needed to increase the energetic intake coming from the environment, which is done with a membrane. This is proportional to the surface of the membrane; thus the system necessitates the maximization of a quantity of topological dimension $D_T = 2$. The simplest solution would be to exchange energy across the external surface limiting the body. Optimization would thus lead to an increase in size. However, the end result of such a process is negative: increasing the size by a factor ρ increases the surface by ρ^2, while the volume increases by ρ^3, so that the energy per unit of volume *decreases* by $1/\rho$. Moreover, the thermal energy (of heat) losses take place across the external surface, so well that it is not possible to increase it by too much (this, with the constraint of gravity, leads to a well-known limit to the size of living organisms on Earth).

Thus the problem is now posed in a different manner: is it possible to increase the surface of exchange without increasing the volume of the body (or equivalently, its external surface)? The only solution is clearly to enlarge the *interior* surface of the body by “invagination” (see Figure 15).

Connection between economic optimization and fractality



- *Correlation between scale and informational thresholds in social formations*
- *Theory of the genesis of different modes of exchange from common process: how an “added dimension” can emerge as a solution to an economic problem posed at lower dimensions*
    - *Reciprocity (0,1,2)*
    - *Contract (0,1)*
    - *Commodity ([0,1])*

Categorical translation of the generalization from differentiability to fractal geometry

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\textit{differential geometry} \arrow[r, "\textit{categorial treatment of analysis}"] \arrow[d, "\textit{suspension of diff hypothesis}"] & \textit{compositional restriction} \arrow[r, leftrightarrow, "\textit{topo-logical interpretation}"] \arrow[d, "\textit{new mereology}"'] & \textit{“convergent” logics} \arrow[d, "\textit{conceptual distinction}"'] \\
\textit{fractal geometry} \arrow[r, "\textit{categorial interpretation of resolutions and scale spaces}"'] & \textit{compositional generalization} \arrow[r, leftrightarrow] & \textit{“scalar” logics}
\end{tikzcd}
$$

Categorical translation of the generalization from differentiability to fractal geometry

Fractals and nonstandard analysis

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We describe and analyze a parametrization of fractal "curves" (i.e., fractal of topological dimension 1). The nondifferentiability of fractals and their infinite length forbid a complete description based on usual real numbers. We show that using nonstandard analysis it is possible to solve this problem: A class of nonstandard curves (whose standard part is the usual fractal) is defined so that a curvilinear coordinate along the fractal can be built, this being the first step towards the possible definition and study of a fractal space. We mention fields of physics to which such a formalism could be applied in the future.

PACS numbers: 02.40. + m

Nottale has already set up interesting connections between fractal geometry and ultrafilters through Robinson's Non-standard analysis

Categorical translation of the generalization from differentiability to fractal geometry

3.4. Non-Standard Analysis and Fractals.

We have proposed[11] to deal with the infinities appearing on fractals, and then to work effectively on the actual fractal F instead as on its approximations F_n by using Non-Standard Analysis (NSA).

It has been shown by Robinson[27] that proper extensions $^*\mathbb{R}$ of the field of real numbers $\mathbb{R}$ could be built, which contain infinitely small and infinitely large numbers. The theory, first evolved by using free ultrafilters and equivalence classes of sequences of reals[28], was later formalized by Nelson[29] as an axiomatic extension of the Zermelo set theory. We do not intend to give here a detailed account of this field which is now developed as a genuine new branch of mathematics; we shall just recall the results which we think to be most relevant for application to fractals.

Let us briefly recall the ultrapower construction of Robinson. Though less direct than the axiomatic approach (which actually has compacted into additional axioms all the essential new properties of Robinson's construction), it allows one to get a more intuitive contact with the origin of the new structure. Indeed the new infinite and infinitesimal numbers are built as equivalence classes of sequences of real numbers, in a way quite similar to the construction of $\mathbb{R}$ from rationals. So, in the end, some of the ideal character of the new numbers is found to be already present in real numbers.

Let $\mathbb{N}$ be the set of natural numbers. A free ultrafilter $\mathcal{U}$ on $\mathbb{N}$ is defined as follows.
$\mathcal{U}$ is a non empty set of subsets of $\mathbb{N}$ [$\mathcal{P}(\mathbb{N}) \supset \mathcal{U} \supset \varnothing$], such that:

(1) $\varnothing \notin \mathcal{U}$
(2) $A \in \mathcal{U}$ and $B \in \mathcal{U} \Rightarrow A \cap B \in \mathcal{U}$.
(3) $A \in \mathcal{U}$ and $B \in \mathcal{P}(\mathbb{N})$ and $B \supset A \Rightarrow B \in \mathcal{U}$.
(4) $B \in \mathcal{P}(\mathbb{N}) \Rightarrow$ either $B \in \mathcal{U}$ or $\{j \in \mathbb{N} : j \notin B\} \in \mathcal{U}$, but not both.
(5) $B \in \mathcal{P}(\mathbb{N})$ and B finite $\Rightarrow B \notin \mathcal{U}$.

Then the set $^*\mathbb{R}$ is defined as the set of the equivalence classes of all sequences of real numbers modulo the equivalence relation:

$$a \equiv b \ , \quad \text{provided} \ \{j : a_j = b_j\} \in \mathcal{U} \ ,$$

a and b being the two sequences $\{a_j\}$ and $\{b_j\}$.

Similarly, a given relation is said to hold between elements of $^*\mathbb{R}$ if it holds termwise for a set of indices which belongs to the ultrafilter. For example:

$$a < b \ \Leftrightarrow \ \{j : a_j < b_j\} \in \mathcal{U} \ .$$

An important result is that any finite number a can be split up in a single way as the sum of a standard real number $r \in \mathbb{R}$ and an infinitesimal number $\varepsilon \in \mathfrak{I}$: $a = r + \varepsilon$. In other words the set of finite hyper-reals contains the ordinary reals plus new numbers (a) clustered infinitesimally closely around each ordinary real r. The set of these additional numbers $\{a\}$ is called the monad of r. More generally, one may demonstrate that any hyper-real number A may be decomposed in a single way as $A = N + r + \varepsilon$, where $N \in {}^*\mathbb{N}$, $r \in \mathbb{R} \cap [0,1[$ and $\varepsilon \in \mathfrak{I}$.

Categorical translation of the generalization from differentiability to fractal geometry

The real r is said to be the "standard part" of the finite hyper-real a, this function being denoted by $r=st(a)$. This new operation, "*take the standard part of*", plays a crucial role in the theory, since it allows one to solve the contradictions which prevented previous attempts, such as Leibniz's, to be developed. Indeed, apart from the usual strict equality "=", one introduces an equivalence relation, "≈", meaning "infinitely close to", defined by $a \approx b \Leftrightarrow st(a-b)=0$. Hence two numbers of the same monad are infinitely close to one another, but not strictly equal.

A practical consequence is that a very large domain of mathematics may be reformulated in terms of NSA, in particular, concerning physics, the integro-differential calculus.[28] The method consists in replacing the Cauchy-Weierstrass limit formulation by effective sums, products and ratios involving infinitesimals and infinite numbers, and then taking the standard part of the result. Hence the derivative of a function will be defined as the ratio

$$df/dx = st\left\{[f(x+\varepsilon) - f(x)] / \varepsilon\right\} \;,$$

with $\varepsilon \in \mathfrak{I}$, provided this expression is finite and independent of ε. The integral of a function is defined from an infinitesimal partition of the interval $[a,b]$ in an infinite number ω of bins, as a sum

$$\int_a^b f(x)\,dx = st\left(\sum_{i=1}^{\omega} f(x_i)\,\delta x_i\right) \;,$$

provided it is finite and independent of the partition. The number of bins, infinite from the standard point of view, is assumed to be a given integer belonging to $^*\mathbb{N}_\infty$. It is said to be *-finite. Such a method allows, more generally, a new treatment of the problem of infinite sums. A summation from 0 to ∞ may be replaced by a summation over an *-finite number of terms ranging from 0 to $\omega \in {}^*\mathbb{N}_\infty$. The sum will be said to converge if for different ω's, its standard part remains equal to the same finite number.

Thanks to its ability to deal properly with infinite and infinitesimals, Non-Standard Analysis is particularly well adapted to the description of fractals. To this purpose we have proposed[11] to continue the fractalization process $(F_0, F_1, \ldots, F_n, \ldots F)$ up to an *-finite number of stages ω. This yields a curve F_ω, from which the fractal F is now defined as

$$F = st\,(F_\omega)\,. \tag{3.4.1}$$

This means that we define an *-curvilinear coordinate from its expansion in the base p:

$$^*s = 0.s_1 \ldots s_k \ldots s_\omega = \sum_{k=1}^{\omega} s_k\, p^{-k}$$

and that the equation of F_ω is given by Eq. (3.3.3) now summed from 1 to ω, $Z(s)$ being its standard part. One of the main interests of the introduction of the curve F_ω is that it contains and sums up all the properties of each of the approximations F_n, and also of their limit F. Moreover a meaning may now be given to the length of the fractal curve. The length of F_ω is a number of $^*\mathbb{R}_\infty$:

$$L_\omega = L_0\,(p/q)^\omega = L_0\, q^{\omega(D-1)},$$

The convergence between Logics of Worlds and Scale Relativity

In our own approach, this logical identity of a world is the transcendental indexing of a multiplicity—an object—as well as the deployment of its relations to other multiplicities which appear in that world. There is no reason to suppose that we are dealing with a fixed universe of objects and relations, from which we would have to separate out modifications. Rather, we are dealing with modifications themselves, situating the object as a multiplicity—including a temporal one—in the world and setting out the relations of this object with all the others. In particular, we accept the great relativist lessons of physics, from Galileo to Einstein and Laurent Nottale, as self-evident: the phenomenon integrates into its phenomenality the variations that constitute it over time, as well as the differences of scale that stratify its space.

Logics of Worlds

TT: And of course through the work of Quentin Meillassoux contingency has become a central question now. His way of approaching the question is very different from yours in terms of how the questions are posed. What do you think of this way of treating the concept of contingency? Is there finally a theoretical relation between the '*hor-lieu*' and the absolutization of contingency developed under the concept of the 'archi-fossil'?

AB: Contingency, for Meillassoux, is first of all with respect to the laws of nature. But for me, there is no 'nature' and thus no laws relating to it. There is an infinite multiplicity of worlds that the transcendental (and thus their respective laws) does not cover. This is the same reason why the infra-molecular universe is not made up of the same laws as the supra-molecular universe. The only interesting path here is the integration, in physics, of these differences in scope. We should admit that the scope of a phenomenon, thus the transcendental of a world where it appears, is an immanent given of its scientific rationalization. Fractal geometry permits the formalization of this point. After which we can let go of a uniform concept of a universe such as that of nature and of the laws of nature. This project is being brought to fruition by Laurent Nottale. In this context, the question of knowing if the 'laws of nature' are necessary or contingent loses its meaning.

Interview with Tzuchien Tho

The convergence between Logics of Worlds and Scale Relativity

The being that underlies every world is composed of multiple-beings whose being-there is realized by their indexing (or function of appearing) on the transcendental of the world in question. The question of knowing in what sense these beings are ontologically 'of the same world' comes down to examining for which constructions of multiplicities a world is closed. It is clear that the mathematical examination of an operational closure concerns the 'dimension' of the set within which one operates. If you apply to a multiple a very powerful operation and if the world is very small, it is quite likely that the result of the operation will overstep the world. For example, if you place yourself in the Quebec-world, and the operation is 'representing the layout of the city of Montreal in 200 million years', there is a good chance that you will exceed the resources of the world, since everything suggests that on this time-scale, there will be no city of Montreal, no Quebec, no Canada and not even a human species, at least in the way that we know it. The properly ontological examination of the question of the limits of a world presumes that it is possible to put forward hypotheses on the number of multiples contained in a world, and that this may be done, for the moment, in a manner entirely independent from the actual appearing of these multiples and thus from the identity-function which articulates them onto the transcendental of the world.

Of course, we saw, and we will confirm, that these hypotheses cannot strictly speaking be formulated *from the interior* of a world. This is what accounts for the fact that the closure in question remains inaccessible.

Aspects of Nottale's theory are already built into Badiou's theory of worlds - such as the use of inaccessible infinities to define the ontological closure/logical completeness of a world. On the other hand, while "atomic logic" is compatible with a scale-dependent interpretation, this is not an explicit feature of the theory, nor are its relative concepts, such as layers and resolution-ranges.