## New Economic Windows

Emmanuel D. Farjoun Moshé Machover
David Zachariah

## How Labor <br> the Global

# Economy 

A Labor Theory of Capitalism
Springer

# How Labor Powers the Global Economy 

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Emmanuel D. Farjoun • Moshé Machover • David Zachariah

# How Labor Powers the Global Economy 

A Labor Theory of Capitalism

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## Preface

In 1983, Emmanuel Farjoun and I published our book Laws of Chaos-A probabilistic approach to political economy (Briefly: LOC). Our aim was to address a long-standing difficulty in Marxian political economy, known as the transformation problem. Rather than taking sides in the dispute between 'orthodox' Marxists and their opponents, we maintained that the dispute itself was misconceived, as both sides shared a traditional, entrenched but fallacious basic assumption. They assumed that in a state of equilibrium the rates of profit across all firms in an entire capitalist economy converge to uniformity. The same, supposedly, applies to prices: the price of a given type of item at a given time is the same across the entire market.

We argued that, on the contrary, in a state of equilibrium these and other economic quantities are not definite numbers but statistical variables (aka 'random variables'), with definite statistical distributions. This is in fact well-established as far as some economic quantities are concerned. For example, it is nonsense to talk about 'the' income of a British family in a given year as if it is a single number of pounds: incomes vary, so that different numbers of families fall within different income brackets. Of course, we can talk about the average family income, but averages do not tell the whole story-far from it. (In fact, most families earn less than the average income!) However, this insight had largely been ignored in macro-economic modeling of prices and profits, including the models used by both sides in the transformation problem controversy.

In retrospect, the probabilistic conceptual apparatus developed in LOC has turned out to be of much wider significance than our treatment of that old transformation chestnut. Indeed, it provides a tool for analyzing central facets of global capitalism: wages, productivity, growth and accumulation.

For years LOC elicited little response, apart from a few mainly positive reviews. This was perhaps due to the novelty of its conceptual framework, and to its being regarded as too unorthodox by most Marxist economists. But eventually it gained a significant circle of followers-most of whom are broadly in the unorthodox wing of the Marxian tradition-who have applied its methodology to various macro aspects
of the capitalist economy. Some regard LOC as the seminal publication of probabilistic macro-economics, sometimes referred to as 'econophysics' (due to an analogy between it and statistical mechanics, a branch of theoretical physics).

> If "econophysics" is taken to denote the principle of applying statistical mechanics to economic analysis, as opposed to a particular literature or network, priority of innovation is probably due to Emmanuel Farjoun and Moshé Machover (1983). Their book Laws of Chaos: A Probabilistic Approach to Political Economy proposes dissolving (their words) the transformation problem in Marx's political economy by re-conceptualising the relevant quantities as random variables. ${ }^{1}$

The great global economic crisis that broke out in 2007 undermined the prevalent faith in conventional economic theories promoted as incontrovertible truths by the establishment and taught in most academic departments of economics. Doubts were especially widespread among young and left-leaning people interested in economics. This no doubt helped to increase interest in probabilistic economics as advocated in LOC.

In the following year, Julian Wells organized a conference to mark the 25th anniversary of the publication of LOC. It was held at Kingston University on 14 17 July, 2008. About 25 scholars from several parts of the world participated and presented research work related to the ideas promoted by LOC. The participants included (apart from Julian Wells and the two authors): Masanao Aoki, Paul Cockshott, Allin Cottrell, Jürgen Essletzbichler, Alan Freeman, Mauro Gallegati, Steve Keen, Andrew Kliman, Paul Plummer, David Rigby, Anna Maria Variato, Michael Webber, Victor Yakovenko, and David Zachariah.

Interest in probabilistic macro-economics, and with it demand for copies of LOC, continued to grow. As the book was out of print, readers used PDF versions that were prepared by Julian Wells, to whom the authors and many readers are indebted.

In 2017 we were told by Verso, the publishers of LOC, that they were ready to publish a reprint. At first Emmanuel and I wished to add a new chapter to the reprinted edition. The original book contained a development and justification of a methodology, but no application of this methodology to the realities of the global economy, and to the central role of labor as its power source. The proposed added chapter would remedy this. But we soon realized that a single chapter could not do justice to the ideas we have developed and the empirical material collected over the years. The second edition of LOC eventually came out in 2020 as a simple reprint, while we set about working on what has become the present book.

During the course of this work I felt unable to complete my share of the task within a reasonable period of time. At this point we were fortunately able to persuade David Zachariah—with whom we were in regular contact since the 2008 conference-to

[^0]join us as a third author. He has made a major contribution to both the content and the organization of the material in the book we are presenting to the reader.

London, UK
Moshé Machover
August 2021

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We are deeply grateful to Rachel Wachs, who volunteered to create many of the figures and diagrams.

We owe an inestimable gratitude to our respective life partners, Aliza Dror, Ilana Machover, and Preethi Thomas for their patience and support.

Finally, it is our pleasant duty to thank our publisher, Springer, and particularly Johannes Glaeser, Gowtham Chakravarthy and Yogesh Padmanaban for their welcome, encouragement, and technical support.

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## Chapter 1 <br> Introduction and Major Propositions


#### Abstract

... The entire globe now operates according to the same economic principles - production organized for profit using legally free wage labor and mostly privately owned capital, with decentralized coordination... ${ }^{1}$


The expansion of capitalism from its local confinements to its global scale has led to profound changes in the lives of countless millions. Through all of its chaotic development, dramatic transformations and local variations, spanning more than two centuries, several patterns are discernible. Profit levels, measured as returns on capital, have varied dramatically across individual assets and steer the direction of development. Economic output has grown exponentially and material wealth has accumulated to historically unprecedented levels. At the same time the total wage share of this output has varied within a fairly narrow range. But the resulting rapid improvements of material living standards have also been coupled with poverty, misery, and degradation in vast areas.

The persistence of many visible economic patterns raise a number of puzzles. For instance, what are the bounds to growth and accumulation in capitalism? Within what bounds are wages confined globally? What explains the regional variation of wages? What basic economic factors contribute to reproducing deep poverty in large parts of the Global South as well as in regions of the Global North? ${ }^{2}$ Why have real wages stagnated or even declined for large sections of the workforce in the older capitalist regions? Why is productivity and growth not accelerating, despite all apparent rapid technical changes? Why is the level of yearly return on financial assets essentially the same as in Ancient Rome (apart from the occasional Ponzi schemes)? How does one account for the significant economic differences, in most indicators, between large regions such as China, the US, and Europe?

[^1]Puzzles such as these concern the livelihoods and well-being of millions of people across the globe. Yet, in our view, standard economic theories cannot address them, let alone resolve them, in a theoretically consistent and empirically testable manner. As one prominent economist put it:
[In economics] it is often impossible to bring one's ideas to a conclusive test either formal or experimental. ${ }^{3}$

These words were penned in the context of a devastating economic depression in the 1930s. Little progress appears to have been made since then. Writing in the aftermath of another severe depression many decades later, a former Chief Economist of the World Bank and one-time Nobel Memorial Prize winner stated that the
> methods and conclusions of macroeconomics have deteriorated to the point that much of the work in this area no longer qualifies as scientific research. [...] On simple questions of fact, such as whether the Fed can influence the real fed funds rate, the answers verge on the absurd. The evolution of macroeconomics [is an example of] fields of science that rely on mathematical theory in which facts can end up being subordinated to the theoretical preferences of revered leaders. The larger concern is that macroeconomic pseudoscience is undermining the norms of science throughout economics. If so, all of the policy domains that economics touches could lose the accumulation of useful knowledge that characteristic of true science, the greatest human invention. ${ }^{4}$

In our view, the failure of standard economic theory to address these puzzles is a major lacuna and stems from its capital-centric perspective on phenomena that are readily observable in the sphere of market exchange. In this book, we take a laborcentric perspective, in which the properties of labor expended in the structured sphere of production constrains key variables such as wages, prices, and accumulation in the chaotic sphere of exchange. We show that most of these questions can be taken up theoretically and answered within this labor theory framework, at least up to a good approximation.

### 1.1 Central Propositions

The fundamental thesis of this book is that labor is the source that powers the capitalist economy. Let us turn to a few central propositions that follow from this and address the puzzles above. All but one are stated as theoretical quantitative bounds on important global economic variables. A substantial part of the book is devoted to deriving them from a few basic postulates and corroborating them with empirical data.

- A mature capitalist economy cannot grow over a long period of, say, 10-15 years by more than $2-3 \%$ a year-as measured, e.g., by the total monetary outputabove the growth rate of the working population employed by capitalist firms.

[^2]Hence growth is bounded by a ceiling of about $4 \%$ a year for any extended period of time. ${ }^{5}$

- Wage incomes, as a share of aggregate output, are confined to vary within a rather narrow range. Any persistent real-term increases are restricted to some $2-3 \%$ a year at most. Moreover, these increases are a minimal condition for maintaining the well-being of the working classes as well as keeping their relative position in a capitalist society.
- Overall productivity in a mature capitalist region cannot grow by a rate of more than $3-4 \%$ a year for an extended period; this rate normally decreases to $1-2.5 \%$ with the technical development of agriculture and industry. ${ }^{6}$
- Total accumulated material wealth in a sizable region is bounded from above in terms of the number of persons employed in that region. Thus, it is rarely higher than a low multiple, about 4-5 times, of the annual output of that region. ${ }^{7}$
- To preserve their relative share in total output, owners of financial capital, private banks, etc., need to charge interest of at least $2-3 \%$ above expected inflation. This rate is determined by the efficiency of living labor in producing goods and services in a specific era. ${ }^{8}$
- Total expenditure on wages of all workers in any sizable sector of the economy cannot fall for an extended period below $10-15 \%$ of the value of its working capital. ${ }^{9}$
- Near total automation of all production-a currently fashionable projection-is a process that is incompatible with the institutions of capitalist economies.

Let us now turn to the basic ideas that this book builds upon in order to arrive at testable propositions such as these.

### 1.2 Context and Methodological Message of the Book

Wages, prices, and profits are three basic categories through which market economies organize and allocate resources. Attempts to explain their levels and movements are scattered throughout the history of economic thought. Take, for instance, the attempts to model market prices which have given rise to several increasingly complicated and unwieldy competing definitions of so-called 'value', 'utility' and 'equilibrium prices.' None of which seem fit to tackle the puzzles posed by the reality of presentday capitalism.

[^3]The main aim of Laws of Chaos (LOC), ${ }^{10}$ the precursor to this book, was to assert the centrality of labor in political economy as a basis for analysis. This was a line of thought clearly developed in the classical and Marxian traditions of political economy, which related market prices in exchange to quantities of labor expended in production-an idea that came to be known as the labor theory of value. Several modern accounts formalized the theory and tried to square it with theories of market economies in a hypothesized state of competitive equilibrium. ${ }^{11}$ A substantial part of LOC was devoted to reconstructing the labor theory of value on a more robust foundation that eschews the idealized state of equilibrium. It developed a 'probabilistic' framework for understanding a multitude of wages, prices, and profits that takes into account their fundamental indeterminacy and unpredictability ${ }^{12}$ from a macroeconomic viewpoint. The chaotic nature of ceaselessly changing wages, prices, and profits are taken to be an irreducible feature of capitalist market economies.

The theory developed in LOC opened a wider vista of methods and questions, which will be employed and considered in this book. ${ }^{13}$ The probabilistic approach-which we will explicate in the first part of the book-abandons altogether any attempt to directly and rigidly relate prices of hundreds of millions individual goods and services exchanged in the market to the individual labor time expended in their production. Total labor expended on a given class of goods or services, which we term here their labor content, is considered as a measure with which one can derive constraints on movements of key macroeconomic variables over large regions and long periods of time. ${ }^{14}$ In doing so, the book aims to expose the role of living labor in the dynamics of global capitalist development.

We have termed this approach a labor theory of capitalism to reflect the intellectual heritage associated with the analysis developed in the labor movement from the 1820s onwards. ${ }^{15}$ The term is also taken to reflect the concern with broader questions of principle characteristics and modes of development of the system, rather than the technicalities of prices and profits.

Building on the probabilistic framework, the theory captures the irreducibly chaotic aspects of capitalism-as witnessed by unpredictable markets swings, variations in prices, surges and collapse of profits-while still capable of drawing some far-reaching conclusions in the spirit of the propositions outlined above. Methods for studying chaotic but structured processes originated historically in human sciences, notably in epidemiology and demography. They are of course widely used in the natural sciences, such as the theory of heat and turbulence, where global measures

[^4]such as temperature and pressure allow us to capture important collective aspects of random motions of individual particles. Some mathematical formulations in this approach are unavoidable, since we are looking for rather narrow, well defined, quantitative bounds that constrain capitalist development. Mathematical formulations are used rather sparingly and the key propositions can be understood, we hope, without closely following the formal steps, which are largely relegated to a technical appendix.

The basic methodological message of the labor theory of capitalism is rather general and, indeed, an old idea in the history of economic thought: labor and labor time are regarded as a foundation of political economy. Apart from raw natural resources, such as sunlight, air, virgin land, and water, the ultimate and principal source of economic wealth and social well-being is human labor. Under capitalism, economic development and wealth are based on a specific mode of employment of human labor capacity in all its forms. The creation and movement of wealth, even in its symbolic form of money, cannot be adequately understood by merely following the paths and transformations of money or other non-labor resources.

In addition to its crucial social and economic role, which is the center of our interest here, the workforce and its working time has the remarkable dual properties of being both a strictly limited and renewable resource: It is naturally limited by the size and growth of the human population; and it is constantly renewed by the most natural of biological renewal process. Unlike currency-money and capital in all its forms, working time is incorruptible, it is the one and only hard currency. Time is money, nay, working time is the only real cost! The central obstacle for the owners and managers of capital is the periodic reassertion of autonomy by the labor forces-the working women and men-who alone can turn abstract time into material products for consumption and accumulation.

Contrary to conventional economic wisdom, the labor theory of capitalism asserts that capital, whether financial or physical, in spite of being an essential part of enabling production, makes no independent contribution whatsoever to economic growth. Just like conservative gravitational forces, in spite of being essential to the movement of planets, make no contribution to the total energy of the planetary system. This proposition follows an existing, if mostly unused, path of analysis and is developed and empirically substantiated in the chapters to come.

### 1.3 The Importance of Labor Time

We can now sum up the theoretical approach of the book by two basic propositions, developed later on, which can be taken as systemic properties that emerge from the local interaction of people in the chaotic sphere of market exchange and the organized sphere of production:

1. The labor content of the aggregate output during a given period governs and bounds from above and below the movement of standard monetary measures of commodity-products and financial assets. Capital as such, whether in financial or material form, makes no independent contribution to the value added as expressed either in money terms or in labor content.
2. The principal agents of the capitalist economy-workers, firms and their ownersmake decisions on the basis of monetary quantities, but operate so as to maintain or expand their relative share of the aggregate output, quantifiable in hours of work.

The underlying balancing principles of capitalism, the integration of its many-faceted chaos, side-by-side with canny profit-motivated calculations, are captured by the accounting of working hours, past, present, and future. The true calculus of capitalism is that of labor-time allocation.

The prime insight of the labor theory of capitalism from its very early formulations is that the most basic resource of human survival, activity and development is time, pure linear time, particularly working time. This resource, associated with individual life itself, is provided to all living humans, to each and every individual. In the current era of global capitalism, the drive of rulers and elites to control and appropriate the basic daily work efforts of large masses of people can be best expressed not as a territorial imperative, striving to acquire or dominate territories that contain natural and human resources; but rather as a working-time imperative, aiming to extract benefits from working time. In a nutshell: the movement of modern evolving capitalism is away from conquest of space that humans occupy towards the conquest of time they use and inhabit.

Working time is by far the central source of material improvement but also of misery and deterioration in people's daily lives. Time span-as opposed to a piece of land or a collection of tools-is nowadays the only resource available to the vast majority of humans. By hiring themselves out to capitalist firms, small and large, the vast majority of the world's population earn access to their own autonomous time, a freer use of the portion of their time that is not hired out.

The choice of labor time as the basic measure and analytic tool is motivated by the view of the political-economic system as a social organism driven by human labor and metabolizing it-hence the title of the book. This choice addresses the key puzzles mentioned above in a theoretically coherent and empirically verifiable manner, with a series of propositions about the dynamics and bounds on developments. Like all such choices, ours is part of a wider social-political perspective and agenda: We view capitalism from the standpoint of the interests of those who provide its fundamental source of power-the working classes-in all their complexity and divisions.

### 1.4 Plan of the Book

The book is divided into three parts: Foundations, Results, and Futures.
The first part lays the foundations of our analysis of capitalism. Chapter 2 introduces a few basic economic concepts and defines the central measure of the book: labor content quantified in worker-hours for a given class of products. Chapter 3 restates the probabilistic framework, first developed in LOC, and contrasts the methodology with that of standard economic theory. Further discussion and elaborations on the concept of labor content as a measure is given in Chap. 4. Distinctions between this measure and the more conventional concept of 'labor value' are explained.

The second part of the book encompasses the central results of the book that deal with key variables of capitalist development: productivity, wages, economic growth, and accumulation of both of material goods and monetary wealth. Chapter 5 addresses the limits of overall productivity gains as well as limits to 'full' automation. The question of the existing range of wages and its potential variation within and across regions is considered in Chap. 6. It also discusses factors that determine both the distribution of wages among workers as well as between them and the upper classes. Chapter 7 deals with strong limitations on growth and accumulation implied by the theory. All chapters in the second part of the book end with empirical data that corroborate the theoretical propositions.

The third part is devoted to the limits of capitalist development in terms of human well-being and the possibilities to overcome them. Chapter 8 shows why the limits tend to narrow as a result of the dynamic patterns discussed in the second part of the book. The types of measures required to overcome them, we argue, are incompatible with private, exponential accumulation of wealth. The tradition that the labor theory of capitalism follows has always looked to future developments within the system that can help surpass the limitations of this mode of organizing societies. Chapter 9 considers four such potentials and the open problems in bringing about transitional changes of the existing mode towards one in which human well-being takes priority.

Such a socialized political economy has a material base in present-day capitalism. Its emergence would enable socioeconomic development that is no longer subjected to the sanctity and mediation of private firms; but rather deliberate development that allows much greater autonomy and participation to workers, while preserving common domains, both social and natural.

The book ends with two appendices. Appendix A states some theoretical problems touched upon, but not completely resolved, in the main part of the book. Appendix B provides details and proofs of some mathematical assertions that are used or alluded to in the main text.

## Part I <br> Foundations

## Chapter 2 <br> Production and Labor

The production of goods and services is imperative to all human societies. In this chapter we begin by introducing a few related economic categories, some of which belong specifically to economies based on market exchange. We then consider the process by which societies with a social division of labor reproduce themselves economically. This leads to the concept of labor content, a measurable quantity that will form the basis of our analysis of how capitalism is powered by labor.

### 2.1 Basic Economic Categories

Any object or service of use to human beings can be called a usable. It may be an abstract capacity or object, such as a skill or an algorithm. The totality of usables available to an individual, group, or society is the material wealth of that individual, group or society. Some usables are pure gifts of nature, in the sense that no input of human labor is used, directly or indirectly, to make them available. Examples of such usables are sunlight, wind, untapped minerals, the vast and deep oceans and their life forms, wild herbs or fruit.

A usable that does require direct or indirect input of labor for its creation is referred to as a product. Note that a product may be a concrete or abstract object, or a service. The kinds of direct inputs that go into a product are all, or some, of the following three: gifts of nature, labor, and previously produced products. But previously created products require themselves the same three kinds of direct input. By repeating this decomposition several times over, we are left with only two kinds of ultimate inputs: gifts of nature and labor. ${ }^{1}$

[^5]Commodities are a special class of usables that presuppose highly abstract notions such as ownership and enforceable entitlement. Whereas production, the making of products, is inherent in social human nature, commodities appeared relatively late in human evolution-first sporadically, when trading was barely distinguishable from exchange of gifts ${ }^{2}$ —and only gradually becoming widespread and much more impersonal. A commodity is realized in a transaction of exchange or a sale, between seller and buyer, which may be real persons, groups, or fictitious 'legal persons' such as a corporation. For a usable to become a commodity, therefore, it must be owned by the seller: a possession enforced by law (which is one of the main functions of the state), by custom, or by brute force ('the law of the jungle', 'might is right').

While both products and commodities are usables, they are by no means identical categories; they merely overlap. On the one hand, there are many kinds of commodities that are not products as they have no labor input. Example of this are virgin land or untapped mineral resources held as property; and a great variety of abstract commodities such as patents, copyrights, and other transferable legal rights. Note the distinction between say an image or text created by work and the copyright to its use over many years. On the other hand, many kinds of products are not commodities. Examples of this are material and abstract objects and services provided free as public goods by state institutions or charities; many roads and bridges, theoretical discoveries and natural laws, and those produced privately for the producer's domestic use or as gifts. But also some things produced by capitalist firms are not commodities. These include tools and services produced solely for in-house use, as means of obtaining the end-product that is sold as a commodity. An interesting example of an abstract product of this kind is algorithms that are the principal product of information technology firms, such as Google. These valuable products are not sold but held as jealously guarded secrets, used for obtaining the commodities that Google sells: information and advertising facilities. Another example of a product that is not a commodity but a means of obtaining a salable commodity is freebie newspapers: these are products that are not sold by publishers, nor are they guarded as secrets, but given away; what the publishers do sell is advertising space, which is a semi-abstract commodity but hardly a product. The overlap between the two categories is very large: many products are also commodities. For the sake of precision, we shall refer to such a product as a commodity-product.

Money is a special kind of commodity: it is universal in the sense of being acceptable in exchange for any commodity. The state's ability to tax its subjects in monetary units determines the dominant unit of account. While the state monopolizes, regulates, and protects the net creation of money, its magnitude is easily dwarfed by the pairs of monetary credit and debt created by private creditors and banks. Modern money is a universal commodity, which, as opposed to precious-metal money, requires virtually zero inputs to create.

In the following, we shall assume that all sales are money transactions: the buyer of a commodity pays its seller a sum of money, or (in the case of credit transaction) makes an enforceable promise to pay a sum of money, in exchange for the transfer

[^6]of ownership of the commodity. This sum of money is the (actual) price of the commodity. There is no real loss of generality in this assumption. Barter transactions, in which no money changes hands, do of course exist. But in modern times and modern economies money is normally used as a notional means of accounting, to make sure that the exchanged commodities are equal in price.

The price of certain kinds of commodity is usually called rent. Strictly speaking, rent is the price of an entitlement to the use-often for a definite period of timeof some usable which may itself be a material or abstract commodity. But noncommodities, such as mortmain property, may also be rented. Rent in this strict sense is measured by a unit of money divided by a unit of time; for example, US dollars per annum. Loan of money is a special case of renting; and the rent paid in this case is interest, whose rate is measured in percentage (of the total sum paid) divided by a unit of time. In practice the distinction between rent and other prices is not always adhered to: 'rent' is sometimes used in a broader sense, and some kinds of rent are not normally called by this name.

Labor power, that is, the capacity to perform concrete labor tasks, is a rather unique product. It requires both gifts of nature and labor as inputs. The latter includes provision of education, training, shelter as well as consumable products used to sustain a worker. But unlike any other product, labor power also produces an ultimate input: labor. In addition, labor power, being a component of a whole personality, unlike almost all major products, is 'produced' outside the direct control of private firms, usually in the intimate family fold. This is an important asymmetry which allocates to labor power a central role in the process of economic reproduction.

### 2.2 Economic Reproduction

Let us now consider a region in a given period of time, say, a month or a year. For the population of this region to survive from one month to the next, a collection of products must continually be reproduced-including food, clothes, shelter and child rearing. This process of reproduction is a minimal stability condition for any economic system. Since our object of inquiry is capitalism, we will focus on how this process takes place through socially coordinated units of production-farms, mines, factories, warehouses, docks, offices-which, unlike self-sufficient households, cannot reproduce themselves individually.

During the given time period, these units of production yield jointly a collection of reproducible products. From the standpoint of human usability, certain individual products are equivalent to others. Different packets of 1 kg caster sugar are practically interchangeable and individually performed buzz haircuts share the same practically usability. We therefore consider all reproducible products in a given period to be divisible into distinct product-types. These will of course vary across regions and periods, e.g., what was offered in a typical department store in London 1920 may have greatly differed from that in a mall in Cape Town 2020.

The specific number of product-types is therefore mainly a matter of deciding on the level of resolution of analysis. To provide a bit of formalism for our analysis, we let $\mathcal{B}$ denote a collection or 'basket' of products. In the simplest case, $\mathcal{B}$ may just be a single unit of some product-type, say, one kilogram of sugar or one haircut. In a more complex case, $\mathcal{B}$ is the basket of all goods and services produced during, say, a year.

Let us now consider all products produced during the year. A certain quantity of these products are used $u p$ within the process of production itself. For instance, some of the steel output enters the production of cars as inputs that are used up. After deducting the basket of products used up in production, we let $\mathcal{B}_{\text {out }}$ denote the basket of (net) output products of the entire economy considered as a joint process. ${ }^{3}$ This aggregate output basket includes products consumed by people, workers or non-workers, during that year. If $\mathcal{B}_{\text {out }}$ were empty, economic reproduction could not take place and the survival of the population in the region would be at stake. At the very least, $\mathcal{B}_{\text {out }}$ must be able to contain a basket of products consumed by the workforce that is sufficient to ensure the survival and reproduction of labor power. The basket $\mathcal{B}_{\text {out }}$ is not the only collection of things coming out from the process of economic reproduction. As with all irreversible work processes-human or otherwise-disorganized energy dissipates into the surroundings in the form of heat. Since production requires material inputs, it will also output different forms of unwanted by-products, that is, waste, that are absorbed by the surroundings.

### 2.3 Labor Content as a Measure

Given the centrality of product baskets in the process of economic reproduction, and their vast diversity, we seek a numerical yardstick for measuring and comparing various baskets of products.

Of course, money is the standard measure of commodities and commodity baskets. We are however interested in understanding the material conditions and constraints to production and growth. To do so, we trace the genesis of products to their ultimate inputs. That is, we may choose one type of input-say, water, energy or crude oil-as yardstick and assign to any given basket $\mathcal{B}$ a number equal to total quantity of this input used up directly or indirectly in the production of $\mathcal{B}$. (This is similar to the approach used in calculating the carbon footprint of a given basket.) This measure renders heterogeneous basket, for instance, one laptop, five liters of cow milk, or a haircut, into commensurable units of, say, litres of water used.

In principle, any quantifiable input could be used as a yardstick. The use of ultimate inputs is informative of the material constraints of production: there is only so much water or crude oil available at any given period of time. But one choice of yardstick suggests itself as being of unique interest and importance to human beings:

[^7]labor, quantified in units of time. As we will demonstrate in coming chapters, the working times of humans play a pivotal role by constraining fundamental properties in capitalist market economies, such as prices, wages, growth, and accumulation. This yardstick has been considered by many authors since antiquity, with some variations on its precise definition. Here, we provide a fairly simple and straightforward way to measure the labor content of products. ${ }^{4}$

## Basic Properties of the Labor Content Measure

We consider the average quantity of labor-time used to produce a basket $\mathcal{B}$, which we will call labor content (or simply L-content). We denote this quantity by $L(\mathcal{B})$, measured units of worker-hours. The labor content of a packet of granulated sugar may be 0.02 hours, while that of a passenger airplane may be 6 million worker-hours. The following concept of L-content will be at the foundation of the labor theory of capitalism that is presented and developed in this book. The theory does not depend on the specific way in which the measure is defined or constructed ${ }^{5}$ as long as it satisfies the following basic properties:

- (L1) The L-content of any basket of products $\mathcal{B}$ equals the total hours of direct labor expended its production plus the proportional labor content of the products used up in the production process.
- (L2) The L-content of gifts of nature, such as sunlight, wind, in situ minerals, is zero. The labor content of an empty basket $\mathcal{B}$ is naturally also zero, whereas the labor content of the basket of all (net) output products of the entire economy during a given period, $\mathcal{B}_{\text {out }}$, equals the total number of hours worked during this period.
- (L3) The L-content measure is defined over a specified economic region of interest and its resolution is specified by the given division of products into various types.
- (L4) The L-content of an existing product, from a past production process, is no greater than that of newer products of the same type.

Further elaborations about these properties are provided in Chap. 4, but a few important remarks are in place here.

First, the L-content of a product-type is obtained by simply averaging over all labor time expended across units of production. Individual factories or offices will by definition expend labor that is above or below the averaged L-content of a product. Note that this measure counts all concrete labor tasks equally, whether or not they involve different skills or intensities. An hour's work required by a programmer or a farm worker amount to the same expenditure of an ultimate input: universal

[^8]labor time of persons capable of being trained and deployed to perform a range of concrete tasks. It includes the L-content of raw material, energy, and the proportional L-content of the machinery used.

Second, our macroeconomic analysis is largely insensitive to the chosen resolution of the measure. Two different shirts would, for instance, have the same L-content at one resolution when specified to belong to the same type. But when analyzing their usability at a finer resolution, they may be considered to belong to two different quality types, in which case their L-content could differ. Our conclusions are, however, insensitive to this choice.

Third, the L-content of a product basket can be altered by changes in the methods and/or organization of production. For instance, by increasing the intensity of work, more potatoes can be harvested and car repairs performed per hour, which thereby reduces the labor content of a kilogram of potatoes or a car repair. Large reductions in L-content is often achieved by the introduction of more labor-efficient production methods.

## L-Content as a Measure of Productivity and Economic Distribution

Decreasing labor content will play a central role in our analysis. Suppose, for instance, that the rate of decrease of $L(\mathcal{B})$ is $1 \%$ per year, then the amount of labor required to reproduce basket $\mathcal{B}$ is halved every 70 years. By contrast, if the rate of decrease is $10 \%$ per year, the amount is halved every 7 years. ${ }^{6}$ This is a measure of the growth of material productivity. When we consider baskets of consumption goods, different rates translate into dramatic differences in material living standards over time. Similarly, durable products, such as a random-access memory cell produced in 2015, might have lower L-content in 2020 than when it was produced even before wear and tear, simply due to technical changes in production. Advances in copying and information processing technologies have brought the L-content of certain product-types down to virtually zero, such as copies of computer software, digital music recordings, or generic pharmaceutical drugs, even though the initial creation of the product required substantial amounts of labor.

The basket that is allotted to the workforce is a part of all products in $\mathcal{B}_{\text {out }}$ and is central to maintaining material living standards. We denote this basket by $\mathcal{B}_{\text {workers }}$ and consider its L-content in relation to the output:

$$
\begin{equation*}
\frac{L\left(\mathcal{B}_{\text {workers }}\right)}{L\left(\mathcal{B}_{\text {out }}\right)} \tag{2.1}
\end{equation*}
$$

[^9]This labor share is naturally bounded between $0 \%$ and $100 \%$ since products for consumption are always a part of the (net) output of production. The remainder of $\mathcal{B}_{\text {out }}$ is the share of surplus labor-content created in economy, which includes new investments in production and infrastructure, weapons acquired by the state and luxury goods for the upper classes. This distribution of output is the outcome of a social process of negotiation, contestation or struggle. In a capitalist market economy, as we shall see, the labor share is determined via the distribution of wages.

The specific processes through which the real distribution of labor content comes about demarcate different historical societies and the dynamical laws that govern them. In this we are following Marx, who wrote:

The specific economic form, in which unpaid surplus labor is pumped out of direct producers, determines the relationship of rulers and ruled, as it grows directly out of production itself and, in turn, reacts upon it as a determining element. Upon this, however, is founded the entire formation of the economic community which grows up out of the production relations themselves, thereby simultaneously its specific political form. It is always the direct relationship of the owners of the conditions of production to the direct producers-a relation always naturally corresponding to a definite stage in the development of the methods of labour and thereby its social productivity-which reveals the innermost secret, the hidden basis of the entire social structure and with it the political form of the relation of sovereignty and dependence, in short, the corresponding specific form of the state. ${ }^{7}$

With the above remarks about L-content in place, we shall proceed to derive in the coming chapters its role in constraining certain fundamental properties of capitalist market economies.

### 2.4 Remarks on History of the Concept

The centrality of labor in economic analysis can be traced throughout the history of thought, at least back to antiquity. The idea that labor is the ultimate source of economic value was clearly articulated already by the Arab historian and protosociologist Ibn Khaldun in the fourteenth century. ${ }^{8}$ Several centuries later, in the context of an emerging capitalist economy, Adam Smith wrote that the
> real price of everything, what everything really costs to the man who wants to acquire it, is the toil and trouble of acquiring it. [...] What is bought with money or with goods is purchased by labour as much as what we acquire by the toil of our own body. That money or those goods indeed save us this toil. They contain the value of a certain quantity of labour which we exchange for what is supposed at the time to contain the value of an equal quantity. Labour was the first price, the original purchase-money that was paid for all things. It was not by gold or by silver, but by labour, that all the wealth of the world was originally purchased; and its value, to those who possess it, and who want to exchange it for some new productions, is precisely equal to the quantity of labour which it can enable them to purchase or command. ${ }^{9}$

[^10]It seems likely that what led and attracted many thinkers to this labor theory of value is the observation that there is a positive association between the labor content of a commodity and its price in exchange: commodities whose production requires (under normal conditions) a greater amount of labor tend to fetch a higher price. While it is tempting to posit a deterministic, causal relation between the two categories, we will argue in the next chapter that this is untenable.

David Ricardo made several amendments and clarifications to Smith's idea. Using the example of stockings, he considered the total quantity of labor necessary to manufacture them and bring them to the point of market exchange:

First, there is the labour necessary to cultivate the land on which the raw cotton is grown; secondly, the labour of conveying the cotton to the country where the stockings are to be manufactured, which includes a portion of the labour bestowed in building the ship in which it is conveyed, and which is charged in the freight of the goods; thirdly, the labour of the spinner and weaver; fourthly, a portion of the labour of the engineer, smith, and carpenter, who erected the buildings and machinery, by the help of which they are made; fifthly, the labour of the retail dealer, and of many others, whom it is unnecessary further to particularize. The aggregate sum of these various kinds of labour, determines the quantity of other things for which these stockings will exchange, while the same consideration of the various quantities of labour which have been bestowed on those other things, will equally govern the portion of them which will be given for the stockings. ${ }^{10}$

## More generally, he claimed that the

value of a commodity, or the quantity of any other commodity for which it will exchange, depends on the relative quantity of labour which is necessary for its production, and not on the greater or less compensation which is paid for that labour. ${ }^{11}$

## This applied to

those goods which are the objects of desire, are procured by labour, and they may be multiplied, not in one country alone, but in many, almost without any assignable limit, if we are disposed to bestow the labour necessary to obtain them. In speaking then of commodities, of their exchangeable value, and of the laws which regulate their relative prices, we mean always such commodities only as can be increased in quantity by the exertion of human industry, and on the production of which competition operates without restraint. ${ }^{12}$

Thus labor requirements in the production process was thought to regulate the relative variations of individual prices of commodity-products. As Karl Marx put it, they oscillated around a purported 'real value':

At the moment when supply and demand equilibrate each other, and therefore cease to act, the market price of a commodity coincides with its real value, with the standard price round which its market prices oscillate. ${ }^{13}$

[^11]However, the principle by which labor time regulates prices needed to be 'considerably modified by the employment of machinery and other fixed and durable capital', ${ }^{14}$ that is to say, by industrial capitalist production. This modification turned out to be a theoretical challenge for economists following the tradition of classical political economy. We shall return to this point in our remarks in Sect. 3.6.

For a popular account of theories of economic value, see Mazzucato (2018). A more technical discussion is provided in Kurz and Salvadori (1997).

[^12]
## Chapter 3 <br> Probabilistic Framework

In capitalism, the core of economic reproduction takes place through workers who produce commodity-products in firms, and households that consume them. Thus wages, prices, and profits are microeconomic variables that jointly affect millions of people and shape the global development of inequality, growth, and capital accumulation, which are the focus of this book.

In any large region at any given point in time, microeconomic variables exhibit a range of different values. For instance, workers earn many different wage rates for a wide variety of local reasons: employment opportunities, skills, bargaining powers, and so on. With such detailed information at our disposal, we could narrow down the range of wages that an individual worker is likely to receive. From a macroeconomic vantage point, however, individual wages, prices and profits are largely indeterminate and unpredictable. Rather than attempting to eliminate this fundamental fact of unpredictability in large-scale economies, by invoking idealized variables or ignoring their irreducible heterogeneity, we take it to be foundational and build on a framework that was first developed in LOC. This 'probabilistic' framework, presented here, considers distributions of wages, prices and profits in the entire economy. That is, it eschews modeling, for instance, the wage rate of an individual worker and considers instead the proportion of all employed labor that earns wage rates in a given range, say, between $\$ 10$ and $\$ 15$ per hour. Such proportions are not arbitrary but are subject to important macroeconomic constraints.

The analysis in the subsequent part of this book builds on certain basic properties of the wage, price, and profit distributions discussed here. A central point is that the framework establishes a link between two spheres of capitalist market economies: the visible sphere of exchange and market prices and the opaque but fundamental sphere of production and labor content. This probabilistic connection will play a central role in the pages that follow. We will end the chapter by contrasting some important methodological differences between our framework and that of standard economic theory.

### 3.1 Capitalist Market Economies

In capitalist market economies, economic life is coordinated, managed and mediated through money and market exchange. It is a seemingly trivial fact that a large collection of separate firms employ workers to produce and sell commodities. However, in reality it presupposes several historical novelties.

First, firms and households are subjected to market dependence. Firms organize a wide array of units of production-farms, factories, offices, repair shops, department stores, and so on. But they cannot reproduce themselves without non-labor inputs from other firms and labor inputs from workers, on the one hand, and monetary revenues from buyers of their outputs, on the other hand. While the productive assets are concentrated within firms owned by a tiny minority, most households in turn lack sufficient assets to meet an adequate material living standard by themselves. Their members must therefore obtain necessary commodities in exchange for money, and to do so they must seek to sell entitlements to utilize their capacity to work-labor power-to firms in need of it. ${ }^{1}$ This precludes the widespread existence of self-sufficient households or collective units of production. ${ }^{2}$ Second, economic processes coordinated by market exchange require contract enforcement. That is, buyers and sellers of commodities must uphold each side of a contract of exchange. Without a third party to enforce this, market exchange is brittle and can break down under systematic violations. A capitalist market economy is thus thoroughly dependent on state institutions and other powerful third parties that can enforce contracts. However, states themselves typically lack vast productive assets and are thus also market dependent. Their reproduction depends on purchasing commodities offered on the market and on taxing wage and property incomes in the economy. Third, the operation of capitalist firms is predicated on the private appropriation of nature. The totality of interacting firms are dependent on the entitlement to use the natural environment, both to extract gifts of nature and as an outlet for waste. We take market dependence, contract enforcement and the private appropriation of nature to be three necessary preconditions for capitalism to emerge.

Large and heterogeneous collections of workers, commodities and firms have aggregate properties that no individual unit can significantly alter. For instance, no individual worker can produce a noticeable change to the total number of hours worked or the overall share of wages in all incomes. On the one hand, there are numerous microeconomic configurations of workers, commodities, and firms that are compatible with a given set of global macroeconomic values. For instance, the

[^13]average wage rate is virtually unchanged from one day to the next. But a worker earning $\$ 40$ per hour could be made redundant by one firm, only to be employed in another at $\$ 10$ per hour the next day. This indeterminacy is not merely theoretical, but reflects the systemic disorder and the relentless turbulence that emerges from weakly coordinated buyers and sellers, workers and firms, constantly trying to improve their positions in heterogeneous markets. On the other hand, a given set of macroeconomic constraints also rules out a great number of local microeconomic configurations. Thus amid the apparent disorder of capitalism these constraints give rise to persistent patterns. The focus on stable global constraints leads to a probabilistic framework for macroeconomic analysis that is robust against unpredictable microeconomic variations.

To lay the foundational concepts of the framework, we will initially consider a capitalist market economy in any specified region for a given period of time, say, a year. For convenience, we consider a region with a common currency, say, US dollars. Then we let $M(\mathcal{B})$ denote the monetary price in dollars of a particular basket of commodity-products $\mathcal{B}$ that is bought-and-sold on the commodity market.

### 3.2 Workers and the Labor Market

Workers can earn a wide range of wages and salaries in an economy. But what determines the limits of this variation and how are wages distributed? We shall begin addressing such questions by linking wages earned to labor time expended by workers.

Over the given year, there is a large number of labor powers available on the labor market, a large portion of which is hired by firms. Conceptually, we may list each labor contract as

$$
\{1,2,3, \ldots\}
$$

where the order is of no importance. This is a list of many millions of labor contracts in which workers perform a range of concrete tasks under widely varying conditions. They may be located in air-conditioned offices or under smokestacks; the contract may require scarce skills or no qualifications; the tasks may be creative and purposeful or mind-numbingly monotonous; and the work may be carried out over regulated workdays or sporadic hours of causal work (not counting travel hours).

Now, consider a specific labor contract $i$ in the list above. Suppose it spans over a total period of $T_{i}$, say, 100 hours. This contract yields a wage rate $W_{i}$-before taxes and transfers. If $W_{i}=\$ 20$ per hour, then the labor contract yields a wage income of

$$
W_{i} T_{i}=20 \cdot 100=\$ 2000
$$

The wage rate and labor time are essential factors to meet the worker's costs of living, and therefore play a central role for the long-term survival of the worker and her dependents. But the arithmetical product of both factors is also a cost of production

Fig. 3.1 Distribution of wage rates (described by its density function). The shaded area gives the proportion of total labor hours that earns wage rates between $\$ 0$ and $\$ 20$ per hour, denoted $\mathrm{P}(0 \leq W \leq 20)$. The proportion P in any other interval of wage rates can be obtained using the area under the curve

that impinges on the firm's profitability. This feature constitutes an irremediable source of contestation and conflict over wage rates $W_{i}$ and over the utilization and duration of labor time $T_{i}$. The firm purchases the entitlement to use a worker's labor power at its disposal during a stipulated period of time. This constrains the workers' autonomy and enforces an unequal distribution of risks at the workplace between those who perform the concrete work and those who supervise it. An individual worker's bargaining power over $W$ and $T$ is determined by her position on the labor market, relative to the availability of labor contracts and equally qualified labor powers.

Given the importance of wages in capitalist market economy, we are interested in characterizing them across all labor contracts above. Since the duration of contracts differ, we shall consider how the total labor time expended in the market sector over the given year is distributed over different wage rates. Let P denote the proportion of total labor time that earns wage rates $W$ in any specified range. Then, for instance,

$$
\begin{equation*}
\mathrm{P}(0 \leq W \leq 20) \tag{3.1}
\end{equation*}
$$

denotes the proportion of employed labor time in production that earns wage rates between zero and twenty dollars per hour. It may equal, say, $50 \%$ of all workerhours. How the proportion of labor time P varies across different intervals of wage rates characterizes the distribution of all wage rates in the economy. An illustration of a distribution of $W$ is given in Fig. 3.1. At the macroeconomic level, the wage rate of any labor contract is largely indeterminate and unpredictable without further specifications. ${ }^{3}$ Individual wage rates can change, new contracts appear and old disappear, but the proportion of labor in any given interval of wage rates cannot be arbitrary. This is because workers and firms are subject to certain global constraints, two of which we will discuss next.

[^14]
## Constraints on the Distribution of Wage Rates

To sustain the reproduction of labor power, resources must be allocated for shelter, consumable goods, child rearing, education, and so on. A significant part of these resources is organized outside the capitalist market economy by domestic households and public institutions. The remaining part must be met through market exchange against wage incomes, which is a critical precondition for the market dependence of workers. This imposes a lower barrier such that the proportion of labor that earns wages rates lower than a certain rate, say, $\$ 5$ per hour, is small. That is, a constraint on the distribution of wage rates such that

$$
\begin{equation*}
\mathrm{P}(W \leq 5) \quad \text { small } . \tag{3.2}
\end{equation*}
$$

The specific bottom floor varies historically and is in some regions set as a legal minimum wage. It can be lowered if the commodities that the workforce pays for are cheapened or if a greater share of necessities is met outside of market exchange, by domestic households and public institutions.

Let us turn to another constraint that follows by considering the wage rate averaged across all employed labor time. This average rate is often referred to as the 'expected' wage rate and denoted $\mathrm{E} W$. By averaging $W$ across all labor time, it can be shown that the expected wage rate equals

$$
\begin{equation*}
\mathrm{E} W=\frac{\text { total wage income }}{\text { total labor time }}, \tag{3.3}
\end{equation*}
$$

which is the ratio of two macroeconomic quantities that no individual worker can affect much. The total wage income is limited by the overall output $\mathcal{B}_{\text {out }}$ during the year, quantified as $M\left(\mathcal{B}_{\text {out }}\right)$ and often referred to as the 'value added' in the whole economy. Similarly, the total labor time depends on the size of the workforce and the structure of employment. For the skewed distribution of $W$ illustrated in Fig. 3.1, the average rate is $E W=\$ 25$ per hour. Note that more than half of all labor must earn wage rates below $\$ 25$ per hour.

More generally, the type of macroeconomic constraints considered above is based on some property of the wage rates, described by a function $f(W)$. We may consider a microeconomic configuration of workers producing a particular allocation of labor time across all wage rates. Suppose that due to the churning of the market economy, each possible allocation of labor could occur but that the average of $f(W)$ across all wage rates is fixed or limited by some macroeconomic variables-such as the total wage income. Then some microeconomic configurations of workers are vastly more probable to occur than others. In this way persistent patterns, observable as a stable distribution of $W$, can emerge amid chaotic change. ${ }^{4}$

[^15]
## Stable Features of Distribution

One problem with measuring wage rates in units of dollars as above is that the purchasing power of a dollar also changes over time. Thus the above constraints are expressed in nominal terms that vary across time. In Chap. 6, we shall therefore consider measuring $W$ in terms of purchasing power instead and, in doing so, derive certain structural features of the wage distribution that are comparable across time and geographical regions.

Two key structural features will emerge from our analysis: the level of inequality between workers and the proportion of labor earning rents that a majority of workers cannot access. These two features are determined by the structure of employment, workers' cohesion and organizational capacities, and political forces. They shape the overall distribution of wage rates and thus constrain incomes that affects the lives of many millions of people. They therefore also determine the workers' collective share of the overall value added in the economy and affect the general level of inequality. We shall denote this wage share by

$$
\begin{equation*}
\omega=\frac{\text { total wage income }}{M\left(\mathcal{B}_{\text {out }}\right)} \tag{3.4}
\end{equation*}
$$

Our analysis in Chap. 6 will show that while the level of inequality in the distribution of $W$ can change significantly over longer periods of time, the wage share $\omega$ is constrained to a fairly narrow range around $50-60 \%$ !

### 3.3 Commodity-Products and Their Labor Contents

Workers produce commodity-products that are bought-and-sold at a wide range of prices on the market. But what determines the limits of this variation and how are prices distributed? To address such questions we will be linking prices to labor time, similar to our treatment of wage rates above. This establishes a connection between market price and labor content, introduced in the previous chapter, that will play a central role in the pages to come.

Over a given year, there is a large number of products available on the market, a large portion of which are bought-and-sold. Conceptually, we may symbolically list all transactions in a given period of time and a given large region as

$$
\{1,2,3, \ldots,\}
$$

This list covers many millions of transactions involving a wide range of individual products: steel rolls, cases of peaches, haircuts, and so on. These are produced under a multitude of different working conditions in different firms. They are also bought-and-sold in a variety of different circumstances such that, say, two peaches produced and delivered by the same firm to two different grocery stores may fetch two different

Fig. 3.2 Distribution of specific prices (described by its density function). The shaded area gives the proportion of total labor content bought-and-sold at specific prices between $\$ 25$ and $\$ 75$ per hour, denoted $\mathrm{P}(25 \leq \Psi \leq 75)$. The proportion P in any other interval of specific prices can be obtained using the area
 under the curve
unit prices. Now consider a given transaction $i$ in the list above. Suppose it involves a product with labor content $L_{i}$, say, 5 hours and fetches a certain amount of money $M_{i}$ dollars. In analogy with wage rates, we shall define the price rate of this individual product as the ratio $\Psi_{i}=M_{i} / L_{i}$ measured in dollars per hour. We shall call this rate the specific price.

Suppose, for instance, the specific price was $\Psi_{i}=\$ 42$ per hour. Then transaction $i$ involves a sum of

$$
\Psi_{i} L_{i}=42 \cdot 5=\$ 210 .
$$

Given the importance of prices for firms purchasing and producing commodities, as well as for households consuming the products, we are interested in characterizing prices across all transactions above. Since the production of each item-say, a cup of tea, a haircut, or a passenger airplane-involves a given quantity of ultimate input of labor, each transaction is in effect a purchase and sale of a definite amount of labor content. We shall consider how the total labor content bought-and-sold over the given year is distributed over different prices. Let P now denote the proportion of bought-and-sold labor content that fetches specific prices $\Psi$ in a given specified range. Then, for instance, the expression

$$
\begin{equation*}
\mathrm{P}(25 \leq \Psi \leq 75) \tag{3.5}
\end{equation*}
$$

denotes the proportion of labor content (materialized as bought-and-sold products) that fetches specific prices between twenty-five and seventy-five dollars per hour of labor. This proportion may equal, say, $70 \%$ of all labor content embodied in the items bought-and-sold in the period and region under consideration. As in the case of wage rates, the variation of this proportion P across different intervals of specific prices characterizes the distribution of all specific prices in the economy. An illustration of a distribution of $\Psi$ is given in Fig. 3.2.

Specific prices in each transaction are indeterminate and unpredictable at the macroeconomic level, but the proportion of labor content in any given interval of specific price cannot be arbitrary. We shall consider some important global constraints next.

## Constraints on the Distribution of Specific Prices

To derive certain constraints on the distribution of specific prices $\Psi$, we begin by relating this rate to production costs. Consider a product in any given transaction, say, a steel roll or case of peaches. The direct labor expended to produce it is paid a certain amount of wages. These are the direct wage costs. Next, the non-labor inputs used up in production have their direct wage costs too. By proceeding backwards iteratively through the chain of production, we trace the direct wage costs of all firms involved in the production of the given product. Summing them we obtain the total wage costs. ${ }^{5}$ In order to study the distribution of $\Psi$, it will be helpful to decompose it into a product of two factors:

$$
\begin{equation*}
\Psi=\frac{\text { price }}{\text { labor content }}=\frac{\text { total wage costs }}{\text { labor content }} \cdot \frac{\text { price }}{\text { total wage costs }} \tag{3.6}
\end{equation*}
$$

where the first factor is the total wage costs divided by average number of workerhours expended in the production of products of the same type. This first factor in (3.6) has therefore the form of an averaged wage rate, paid directly and indirectly in the production of products of this type, and is measured in dollars per hour. This fact, to be used below, relates specific prices $\Psi$ to costs of production via the distribution of wage rates $W$.

We now turn to the second factor in (3.6), the price of an item exchanged in a given randomly chosen transaction divided by the total wage costs paid in the course of producing that item. We can immediately see that across many transactions this factor will tend to be greater than 1 ; if it were systematically lower than that, the price that a product fetches in exchange would not cover all wage costs so that one or more firms involved in its production and sale would soon be out of business. To make this point a bit more precise, we decompose the price of a product into the costs of its used up non-labor inputs plus a residual: the firm's value added, from which wages are paid and profits are derived. ${ }^{6}$ Similar to the way in which we added up all direct and indirect wage costs, we may now trace the value added for the prices of each non-labor input. By proceeding backwards through the chain of production, we end up decomposing the price of a given product into the total value added by all firms involved in its production and sale. In other words: the price of a product equals the accumulated total of the value added in the entire 'history' of its production. This enables us to rewrite the second factor in (3.6) in terms of total value added, and we end up with a final expression for specific price:

$$
\begin{equation*}
\Psi=\frac{\text { total wage costs }}{\text { labor content }} \cdot \frac{\text { total value added }}{\text { total wage costs }}, \tag{3.7}
\end{equation*}
$$

[^16]where the ratio of total wage costs to total value added is similar to the wage share $\omega$ in (3.4).

With the expression (3.7) in place, we will now derive two general properties that constrains the distribution of $\Psi$. We begin by averaging the specific prices over all exchanged labor content to obtain the 'expected' specific price, denoted E $\Psi$. Using (3.7) it can then be shown that the average specific price is well approximated by the average wage rate $\mathrm{E} W$ divided by the wage share $\omega$ of all the value added in the economy. ${ }^{7}$ That is, we have the following approximation

$$
\begin{equation*}
\mathrm{E} \Psi \simeq \frac{\mathrm{E} W}{\omega} \tag{3.8}
\end{equation*}
$$

Suppose, for instance, that the average wage rate is $\$ 25$ per hour and the wage share in the economy is $50 \%$, then the average specific price is

$$
\mathrm{E} \Psi \simeq \frac{25}{0.50}=\$ 50 \text { per hour. }
$$

This result does not tell us anything about the specific price in any given transaction. However, if the distribution of $\Psi$ is skewed as illustrated in Fig. 3.2, then it follows that more than half of all products, as measured by their labor content, exchange for specific prices below $\$ 50$ per hour. That is to say, the proportion

$$
\mathrm{P}(\Psi \leq 50) \text { is greater than } 50 \% .
$$

We now turn to constraints on the distribution at the lower range of $\Psi$, by considering the viability conditions of firms. As discussed above, prices must be sufficiently high to meet the total costs of labor in production and yield some profit. Therefore only a small proportion P of labor content in exchange can exhibit very low specific prices $\Psi$. In particular, we take the specific prices realized in market exchanges that are insufficient to cover the costs of the average labor time expended, when paid at the average wage rate $\mathrm{E} W$, to be a small portion. That is,

$$
\begin{equation*}
\mathrm{P}(\Psi \leq \mathrm{E} W) \text { small. } \tag{3.9}
\end{equation*}
$$

A guesstimate would be $10 \%$ of all bought-and-sold labor content, which constrains the distribution of $\Psi$. We will take this to be a stable property.

A reasonable candidate form for the distribution of $\Psi$ is then the so-called 'lognormal' distribution, already illustrated in Fig. 3.2. ${ }^{8}$ An important consequence of this particular form is that it is fully determined by the two macroeconomic constraints, (3.8) and (3.9), above. In other words: if, as we conjecture, the distribution of $\Psi$ is log-normal, then the average wage rate and the wage share will determine how all labor content in exchange is distributed over different specific prices.

[^17]
## Connection Between Labor Content and Market Price

From a macroeconomic viewpoint, individual specific prices $\Psi$ are indeterminate and are thus treated as a random variable above. The constraints derived above establish an important connection to L-content that emerges out of interdependent firms that must meet and exceed their wage costs. To better appreciate its practical consequences, let us now consider the specific price of a basket of products $\mathcal{B}$, which we denote:

$$
\begin{equation*}
\bar{\Psi}(\mathcal{B})=\frac{M(\mathcal{B})}{L(\mathcal{B})} \tag{3.10}
\end{equation*}
$$

It can be shown, using simple algebra, that $\bar{\Psi}(\mathcal{B})$ is a weighted sum of the specific prices of all individual products in the basket, weighted by their labor content.

Now suppose $\mathcal{B}$ is a heterogeneous sample of the collection of products in transaction. Then, the specific price of this sample basket, $\bar{\Psi}(\mathcal{B})$, will be very close to the expected specific price $E \Psi .{ }^{9}$ In light of (3.8), this means that we have an excellent approximation

$$
\begin{equation*}
\bar{\Psi}(\mathcal{B}) \simeq \frac{\mathrm{E} W}{\omega} \tag{3.11}
\end{equation*}
$$

By combining (3.10) and (3.11), this establishes a basic connection between labor content and market prices: The monetary price of a large sample basket $\mathcal{B}$ is wellapproximated by

$$
\begin{equation*}
M(\mathcal{B}) \simeq L(\mathcal{B}) \cdot \frac{\mathrm{E} W}{\omega} \tag{3.12}
\end{equation*}
$$

that is, by its L-content multiplied by the average wage rate and and divided by the wage share in the economy.

Returning to our numerical example above, a sample basket $\mathcal{B}$ with $L(\mathcal{B})=10$ hours will fetch a market price of about

$$
10 \cdot \frac{25}{0.50}=\$ 500
$$

Conversely, a sum of $M=\$ 500$ will have a purchasing power of 10 hours of Lcontent in the form heterogeneous products in exchange. This provides a noteworthy perspective on the fundamental categories of money and prices. Money is a symbolic means by which labor content can be appropriated and distributed in market economies. The total monetary income in the market sector is the value added $M\left(\mathcal{B}_{\text {out }}\right)$; but in units of $L$-content purchasing power it equals the annual number of hours expended in production, which we denote $\mathbb{L}$ and call total labor added for

[^18]brevity. Quantifying incomes-such as wages, profits and rents-in this unit will be a cardinal point in our analysis of capitalism. ${ }^{10}$

### 3.4 Firms and the Profit Imperative

Firms employ workers and productive assets to produce and sell commodityproducts. In this process, they earn a wide range of profits rates. What determines the limits of this variation and how are profits distributed? To begin addressing these questions, we link profits earned to the capital invested and the total economic output.

Over the given year, there is a large number of operating firms and we list them as

$$
\{1,2,3, \ldots\} .
$$

This list includes hundreds of thousands, or millions, of farms, factories, offices, restaurants, repair shops, stores, warehouses, and so on. We take here a 'firm' to be each unit of production with its separate accounting of investments, costs and profits. Consider a firm $i$ in the list, with productive capital assets worth $K_{i}$, say, $\$ 1,000,000$, including plant, machinery and equipment. The assets yield a rate of profit $R_{i}$ which plays a key role in the evolution of the firm. Suppose $R_{i}=20 \%$ per year; then the capital assets yield a profit income of

$$
R_{i} K_{i}=0.20 \cdot 1000000=\$ 20000 \text { per year. }
$$

Here we use a broad measure of profits at the firm level that corresponds to its 'operating surplus'-before deducting payments for interest on loans, dividends, and taxes. Part of this surplus is retained by the firm, and the remainder is paid out as interest, dividends, and taxes.

Profits are clearly central to the operation and viability of firms. Given their importance to firms and their capitalist owners, we are interested in characterizing profits across all the firms. Since a tea shop and an airplane factory have different amounts of capital investments, we shall consider how the total capital is distributed over different profit rates. Let P denote the proportion of capital that yields profit rates $R$ in a specified range. Then, for instance,

$$
\begin{equation*}
\mathrm{P}(0 \leq R \leq 0.10) \tag{3.13}
\end{equation*}
$$

equals the proportion of capital that receives rates between 0 and 10 percent per year. It may equal, say, $55 \%$ of the total capital. The variation of the proportion $P$ across different profit rate intervals characterizes the distribution of the profit rate

[^19]Fig. 3.3 Distribution of profit rate random variable $R$ (described by its density function). The shaded area gives the proportion of capital earning profit rates between $0 \%$ and $10 \%$ per year, denoted by $\mathrm{P}(0 \leq R \leq 0.10)$. The proportion P in any other interval of profit rates can be obtained similarly, using the
 area under the curve
random variable $R$ in the economy. Figure 3.3 illustrates the kind of distribution that $R$ may have. From a macroeconomic viewpoint, the profit rate of any individual firm is indeterminate and unpredictable without further specifications. Individual profit rates change as firms clash and compete on new and old commodity markets; but the proportion of capital in any given interval of profit rates is far less volatile and, as we shall soon see, cannot be arbitrary. Here we are only interested in certain features of the distribution of $R$ rather than its precise form. Two global constraints on the distribution are particularly relevant.

Let us first consider the expected profit rate $\mathrm{E} R$, obtained by averaging over all capital. It is readily shown that this rate equals the following ratio

$$
\mathrm{E} R=\frac{\text { total profits per year }}{\text { total price of capital stock }}
$$

To better appreciate the material meaning of this rate, we first look at the numerator. Total profits are a fraction of the overall value added $M\left(\mathcal{B}_{\text {out }}\right)$ in the market sector. Since workers are allotted the wage share $\omega$, the remaining fraction $1-\omega$ yields the operating surplus. In other words,

$$
\text { total profits per year }=M\left(\mathcal{B}_{\text {out }}\right) \cdot(1-\omega) .
$$

Now consider the denominator, which equals the current monetary value of all capital goods in production, denoted by the basket $\mathcal{B}_{\text {capital }}$, so that

$$
\text { total price of capital stock }=M\left(\mathcal{B}_{\text {capital }}\right)
$$

In order to establish constraints on profit rates, it is useful to express these aggregate monetary values in terms of labor content. Since both $\mathcal{B}_{\text {out }}$ and $\mathcal{B}_{\text {capital }}$ form large sample baskets, we can relate their prices to L-content using the central connection (3.12). Recalling that $\mathbb{L}$ is the total labor added in the economy, we arrive at the following approximation of the average profit rate:

$$
\begin{equation*}
\mathrm{E} R \simeq \frac{\mathbb{L}}{L\left(\mathcal{B}_{\text {capital }}\right)}(1-\omega) \tag{3.14}
\end{equation*}
$$

which is positive. This constrains $R$ by the size of the workforce and the quantity of labor required to produce capital goods. Thus, profitability measured in the visible sphere of exchange is materially constrained by labor content measured in the opaque sphere of production. The quantities that appear in (3.14) are subjects to intense social and political struggle, both local and global. The number of working days in a year, the length of a working day, the wage rates, etc., are all subject to continual conflicts and fiercely fought outcomes.

Firms that persistently make losses, that is, $R<0$, accumulate debts and are eventually eliminated, in which case their assets are partly appropriated by others. This lower barrier leads in part to a skewed distribution as illustrated in Fig. 3.3 and this constrains the bulk of all capital to yield profit rates lower than $E R>0$. In the chaotic sphere of exchange, the profit rate of a firm can move erratically, from loss to profit-making and back. At any given point in time, a substantial proportion of capital could therefore earn negative profit rates. ${ }^{11}$ This proportion is determined by structural factors such as the access to credit to run operations. Based on recent data, we would estimate the proportion of capital with a negative operating surplus at any given time to be less than thirty percent, i.e.,

$$
\begin{equation*}
\mathrm{P}(R \leq 0) \quad \text { less than } 30 \text { percent. } \tag{3.15}
\end{equation*}
$$

## Importance of Profitability

Profit rates affect business decisions. The owners of a firm $i$ expect a 'decent' return on the capital assets $K_{i}$ in comparison to rates of return that could be made from other assets. This imposes a profitability criterion on the daily operations of the firm as well as on decisions to invest in its productive assets. The criterion determines which sectors of the economy will expand and steers the direction of capitalist development.

Profit rates limit the pace at which individual firms can expand by their own revenue. For instance, consider a firm $i$ with capital worth $K_{i}$ that reinvests all its profits to expand productive capacity at a rate $R_{i}=10 \%$. Then the capital stock and annual profit income could double every seven years at the fastest. ${ }^{12}$ Thus the profit imperative together with reinvestment of profits gears firms toward exponential growth. Note that the firm earns an annual profit of $R_{i} \cdot K_{i}$. This income flow can be measured in units of L-content purchasing power instead of dollars, using (3.12) and the specific price (3.7):

$$
\frac{R_{i} \cdot K_{i}}{\mathrm{E} \Psi} .
$$

[^20]Suppose the average wage rate in the economy is $\$ 25$ per hour and the wage share is $50 \%$. Then the average specific price $E \Psi$ is $\$ 50$ per hour. If the firm has productive capital assets worth $K_{i}=\$ 1000000$, then its yearly profit income can purchase large sample baskets with labor content of about

$$
\frac{0.10 \cdot 1000000}{50}=2000 \text { worker-hours. }
$$

This purchasing power is partly retained by the firm, while the rest is allocated as unearned property incomes.

Profit incomes measured as such must add up to $(1-\omega) \cdot \mathbb{L}$ worker-hours over a year. From this perspective, the scramble for profit by firms is deep down a clash over newly added labor. Behind their daily operations of firms, managers and owners are struggling to expand or preserve the labor content that is appropriated and accumulated. We shall return to this point in Chap. 6, and in Chap. 7 we will show a corollary: the process of exponential growth is constrained by the source of this labor content, namely, the workforce-by its size and capacities. ${ }^{13}$

### 3.5 Emergent Reduction of Labor Content

It is a well-established fact that capitalist market economies have historically exhibited rapid changes in production methods and the variety of goods produced, in comparison with prior economic systems.

As pointed out in Chap. 2, the effects of such changes can be quantified by the rate at which the labor content of any given product basket decreases over time. For instance, available data indicate that the total labor time needed to produce a wide variety of different products-a bushel of corn, a kilogram of cotton, an electronic memory cell, or passenger transport across the Atlantic-has decreased quite dramatically over decades. We will call this persistent pattern the law of decreasing labor content (LDLC) which will be central to our analysis of capitalist development in the subsequent part of the book.

Using the connection between price and labor content, we now show that LDLC is likely to emerge through a process of cost-cutting by firms. So while L-content constrains prices at any given point in time, decisions based on actual prices reorganize production so as to reduce the L-content of many products across time.

Consider a firm producing a basket $\mathcal{B}$. To stay profitable as well as to undercut its competitors, the firm adopts various strategies to reduce its costs per unit of produced output. The L-content $L(\mathcal{B})$ is however a macroeconomic property and can be decomposed in two parts: the hours of direct labor expended its production plus the proportional labor content of the products used up in this production process. We now turn to the effects on the L-content of $\mathcal{B}$ when many firms try to reduce the

[^21]costs per unit of output over a brief period with negligible structural change in the economy.

## Reduction of Labor Costs

To reduce the unit costs that arise from direct labor inputs, the firm can

- cut wages for workers,
- rationalize labor by changing production methods,
- intensify labor by increasing the pace of the work process.

Wage bargaining leaves the structure of production intact and will therefore not alter the quantities of direct labor. By contrast, labor intensification and rationalization increase the output per hour of work in a firm. As such changes permeate across larger sections of production, they also contribute to decreasing the L-content of product baskets $\mathcal{B}$. But both have their technical and human limitations. Machines and equipment have maximum operating capacities and workers cannot remain productive if pushed beyond mental and physical exhaustion. The scope for intensification and rationalization is thus limited by the workers' ability to resist pressures toward exhaustion.

## Reduction of Non-labor Costs

To reduce the unit costs that arise from direct non-labor inputs, the firm can

- beat down the prices paid to suppliers of inputs,
- replace existing non-labor inputs $\mathcal{B}_{0}$ with cheaper alternatives.

Price bargaining leaves the structure of production intact and will therefore not alter the L-content. By contrast, replacement of inputs does impact the production process. If an alternative input basket $\mathcal{B}_{1}$ is found to be cheaper, it replaces $\mathcal{B}_{0}$. That is, a new input basket $\mathcal{B}_{1}$ adopted, provided that the cheapening factor,

$$
\begin{equation*}
C=\frac{M\left(\mathcal{B}_{0}\right)}{M\left(\mathcal{B}_{1}\right)}, \tag{3.16}
\end{equation*}
$$

is greater than 1. This change has no immediate connection to the labor contents of $\mathcal{B}_{0}$ and $\mathcal{B}_{1}$, but we can express their ratio in terms of specific prices:

$$
\begin{equation*}
\frac{L\left(\mathcal{B}_{0}\right)}{L\left(\mathcal{B}_{1}\right)}=C \cdot \frac{\bar{\Psi}\left(\mathcal{B}_{1}\right)}{\bar{\Psi}\left(\mathcal{B}_{0}\right)} \tag{3.17}
\end{equation*}
$$

Thus the new basket $\mathcal{B}_{1}$ has lower L-content than $\mathcal{B}_{0}$ only when the ratio on the left-hand side in (3.17) is greater than 1 . This depends on both the positive cheapening factor $C$ and the specific prices of non-labor inputs, $\bar{\Psi}\left(\mathcal{B}_{1}\right)$ and $\bar{\Psi}\left(\mathcal{B}_{0}\right)$. For an individual firm, it is therefore not possible to say whether the L-content increases or decreases after replacing the input basket $\mathcal{B}_{0}$ by a cheaper alternative basket $\mathcal{B}_{1}$.

Instead of a single firm, we consider a large collection of cost-cutting firms that independently seek to cheapen their non-labor inputs. From a macroeconomic viewpoint, the cheapening factor $C$ and specific prices in each individual replacement are indeterminate and unpredictable. However, across a large number of independent replacements, the L-content ratio in (3.17) is on average greater than $1 .{ }^{14}$ Now consider an input replacement strategy as a sequence of replacements from one input basket to the next,

$$
\mathcal{B}_{0} \rightarrow \mathcal{B}_{1} \rightarrow \mathcal{B}_{2} \rightarrow \cdots \rightarrow \mathcal{B}_{m}
$$

then after $m$ changes the cumulative effect is an L-content ratio $L\left(\mathcal{B}_{0}\right) / L\left(\mathcal{B}_{m}\right)$ and it can be shown that the ratio is greater than 1 with very high probability. In other words, the proportion P of all replacement strategies in which the L -content of input decreases as a result is very high:

$$
\begin{equation*}
\mathrm{P}\left(L\left(\mathcal{B}_{m}\right)<L\left(\mathcal{B}_{0}\right)\right) \text { high. } \tag{3.18}
\end{equation*}
$$

The implication of this result is that the widespread application of cost-cutting input replacement strategies reduces the L-content of the non-labor inputs for many firms. As this process permeates into larger sectors of production, the L-content of many product baskets $\mathcal{B}$ will decrease as an unintended result.

## Limits and Drivers of the Law

We have seen that as firms locally adopt strategies that cut their total unit costs through labor-saving strategies and input replacement strategies, these give rise to a molecular change with profound structural consequences in capitalism. ${ }^{15}$ We will analyze structural changes that arise from LDLC in the second and third parts of the book.

While our analysis of the types of changes discussed above establishes that Lcontent of sample baskets $\mathcal{B}$ is very likely to decrease, the analysis does not determine the rate of decrease, which varies across product types. Available technologies certainly set upper limits to these rates. The scope for labor-saving is, for instance, very different in manufacturing firms than in firms producing personal services. We will discuss upper limits further in Chap. 5. Within these limits, one factor that can accelerate the rate of L-content decrease is a greater bargaining power of workers. While this increases the resistance against various forms of labor intensification, it also pushes upward the bottom of the distribution of wage rates $W$, illustrated in Fig. 3.1, and therefore raises unit costs of direct inputs in many sectors. This puts

[^22]increased pressure on firms to instead find means of rationalizing labor and using cheaper non-labor inputs per unit of output.

### 3.6 Remarks on Theoretical Framework

To better appreciate some of the methodological principles that underpin the probabilistic framework, it is instructive to contrast them with those of standard economic theory below. We then place the origins of the framework-developed in LOC-in its intellectual context.

## Contrasts with Standard Economic Methodology

Standard economic theory builds on fairly rigid assumptions about the behavior of individual agents-workers, households, firms, buyers, sellers-in the economy. The agents are thought to be adopting optimal actions with respect to various economic response functions, often visualized as 'curves' that shift or intersect. ${ }^{16}$ The collective outcome of optimizing agents is a supposed state of equilibrium where opposing economic forces balance each other, until they are disrupted by 'external' factors. This optimization process determines the allocation of economic resources and key variables such as wages, prices and profits. Often the microeconomic assumptions are transferred to macroeconomic modeling by means of a 'representative' agent that optimizes in lieu of all individual agents.

By contrast, the probabilistic framework has radically different (even indifferent!) microfoundations. Few assumptions are posited on the individual agents, who are taken to be only weakly and locally coordinated in a capitalist market economy. They operate in a variety of ways but are subject to certain fundamental economic constraints.

The variability of wages, prices, and profits is taken to be an irreducible fact representing heterogenous economic conditions which drive agents to endeavor to improve their positions in a multitude of ways. The economic system can never settle in some 'balanced' state in this framework. Rather, if certain structural variables of global production, employment and money supply were constant, then the economy is assumed to conform to a statistical equilibrium in which the distributions of wages, prices, and profits are stable. Within this state, microeconomic configurations of workers, commodity-products, and firms are still subject to ceaseless change. ${ }^{17}$

A central concept of standard economic theory is that of the 'production function'. It allegedly specifies the technological relation between quantities of productive inputs and produced outputs of a given firm. In practice, this relation is quantified in monetary terms such that $Y=M(\mathcal{B})$ is the value of the basket $\mathcal{B}$ produced by the firm and is assumed to be determined by a production function

[^23]$$
Y=f(K, T)
$$
where $K=M\left(\mathcal{B}_{\text {capital }}\right)$ is the value of capital and $T$ is the direct labor time employed. The most widely used production function ${ }^{18}$ has the following form:
\[

$$
\begin{equation*}
Y=A \cdot K^{\beta} \cdot T^{1-\beta}, \tag{3.19}
\end{equation*}
$$

\]

where $A$ is a supposedly 'technical' coefficient and $\beta$ is a parameter between 0 and 1 . Through the prism of the production function, labor and capital become substitutable 'factors of production'. That is, any number of hours of labor input can be replaced by a certain equivalent amount of dollars of capital that leave output $Y$ intact. It is often thought that the output of the entire economy, measured as $Y=M\left(\mathcal{B}_{\text {out }}\right)$, can be described by an aggregate production function of the same form as in (3.19). This function is at the heart of standard macroeconomic analysis and forms the basis of its theories of growth. It also conforms to the principles of income distribution outlined by one of the founding figures of standard economic theory:
[...] the distribution of the income of society is controlled by a natural law, and that this law, if it worked without friction, would give to every agent of production the amount of wealth which that agent creates. ${ }^{19}$

Specifically, this takes the form of theoretical profit and wage rates matching the 'marginal product' of capital and labor, respectively. That is, the rates should equal the extra dollar of produced output $Y$ that is obtained per additional dollar of capital $K$ or additional hour of labor $T$, which we may express as:

$$
\begin{equation*}
\mathrm{E} R=\frac{\partial Y}{\partial K} \quad \text { and } \quad \mathrm{E} W=\frac{\partial Y}{\partial T} \tag{3.20}
\end{equation*}
$$

Capital and labor are here seen as factors that make independent 'contributions' to output. Variations of $R$ and $W$ across firms and labor contracts are not considered here. In the aggregate production function, $1-\beta$ corresponds to the theoretical wage share $\omega$ and $A$ measures the so-called 'total factor productivity' in the economy, despite lacking any meaningful unit. ${ }^{20}$

While it is widely used, the production function is riddled with deep theoretical problems. For instance, the individual production functions of all firms in the economy do not readily aggregate into one. Moreover, there is overwhelming empirical evidence that average wages are above the theoretical 'marginal product of labor' (3.20) in most capitalist economies. ${ }^{21}$ It may be thought that, for all its weaknesses, the aggregate production function still captures underlying characteristics of pro-

[^24]duction due to its overall fit to aggregate output data. But this fit was found to be a mere artifact that arises from the wage share $\omega$ being remarkably stable. ${ }^{22}$ Thus the production function serves more as a curve-fitting exercise than actually modeling underlying laws that would give some predictive power. For instance, prior to fitting (3.19) to empirical data, the range of theoretical wages and profits that is admissible by (3.20) is very unrealistic.

The probabilistic framework, by contrast, does not rely on any assumptions about firms having well-behaved production functions. Starting with a few basic postulates, discussed in Sects. 4.1 and 4.4, it generates theoretically consistent and empirically testable claims about wages, productivity, growth and accumulation, which are central to the development of capitalism.

## Context of Probabilistic Framework

The use of probabilistic concepts for studying incomes was pioneered by Pareto at the end of the nineteenth century and developed by Gibrat in the 1930s. The wider use of such concepts in macroeconomic analysis appears to have been scarce. In 1983, the probabilistic framework, summarized in this chapter, was developed in LOC. Drawing upon methodological ideas in statistical mechanics, developed by the physicists Maxwell and Boltzmann in the mid-nineteenth century, that book proposed that central variables of political economy-wages, prices and profits-should be understood in terms of distributions.

It may appear strange that statistical properties of molecular interactions should have any relation to large-scale properties of human interactions. But in fact, Maxwell motivated this new methodology with reference to the study of demography:

> The modern atomists have therefore adopted a method which is I believe new in the department of mathematical physics, though it has long been in use in the Section of Statistics. When the working members of Section F get hold of a Report of the Census, or any other document containing the numerical data of Economic and Social Science, they begin by distributing the whole population into groups, according to age, income-tax, education, religious belief, or criminal convictions. The number of individuals is far too great to allow of their tracing the history of each separately, so that, in order to reduce their labour within human limits, they concentrate their attention on a small number of artificial groups. The varying number of individuals in each group, and not the varying state of each individual, is the primary datum from which they work. ${ }^{23}$

LOC was written in the context of a raging debate between economists writing in the tradition of classical and Marxian political economy. ${ }^{24}$ A central idea in this tradition was that the competitive dynamics in capitalism make wages, prices and profits fluctuate around underlying 'centers of gravity'. The underlying 'natural' levels of these variables were thought to reflect stable states of competitive equilibrium. Then

[^25]the economic system could be studied in a hypothetical state in which $W$ and $R$ are equal across all workers and firms, respectively. Using formal models of input-output relations between different sectors of global production, it can then be shown that in this state a uniform profit rate is determined directly by the technical conditions of production as well as the level of the 'natural' wage rate. This state also jointly determines the 'natural' prices for each product-type. Labor content is entirely redundant in this framework and the amount of labor plays no role in constraining the average profit rate $E R$ as in (3.14). This was a point of contention for economists who wanted to assert the centrality of labor in political economy.

LOC sidestepped this point of contention by eschewing the premise of the debate: the hypothetical state of competitive equilibrium. It was argued that the very mechanism of equalization-competition and capital reallocation-also creates irreducible motion of firms away from a state in which all profit rates $R$ equal the average $\mathrm{E} R .{ }^{25}$ Subsequent work investigated the profit rate as a random variable-see for instance Wells (2007), Fröhlich (2013), Scharfenaker and Semieniuk (2017). ${ }^{26}$ The probabilistic connection between labor content and prices derived in LOC produces two testable hypotheses that contradict the analysis of competitive equilibrium. First, across sectors of firms, the monetary value of the sector outputs would be correlated with its L-content. Second, sectoral profit rates would be negatively correlated with capital-intensity. These predictions were consistent with the findings of numerous subsequent econometric studies, e.g., Shaikh (1984), Cockshott et al. (1995), Shaikh (1998), Tsoulfidis and Maniatis (2002), Zachariah (2006), Fröhlich (2013) and Shaikh (2016, Chap. 9).

Two notable works that apply and develop the probabilistic framework of analysis are Cottrell et al. (2009) and Cockshott (2020).

[^26]
# Chapter 4 <br> Labor Content—Properties and Postulates 

In the previous chapter, we established a probabilistic connection between market prices and labor content. This forms the basis for the analysis of capitalism that we develop in the subsequent chapters. Given the centrality of the L-content measure in our analysis, we will in this chapter further elaborate on its basic properties and on the peculiarities of labor. We end the chapter with three postulates from which constraints on the dynamics of capitalist economies will be deduced later on.

### 4.1 Elaborations on Properties

In Sect. 2.3, we stated the basic properties (L1-L4) of labor content as a measure of products. Due to the complexity of the production methods, whereby a given firm may produce a multitude of products more or less simultaneously, there may well be several somewhat different measures that satisfy these basic properties. But the analysis in the coming chapters depends only on the existence of at least one measure that satisfies the four properties. Here we assume that such a measure exists. An example of a measure that can be approximated using national accounting data is given in Sect. B.1.

Any measure of labor content that satisfies four basic properties, (L1-L4) stated in Sect. 2.3, will suffice for our analysis. Several measures have been formalized since the heyday of classical political economy and we therefore find it appropriate to elaborate on the required properties here.
(L1) The L-content of any basket of products $\mathcal{B}$ equals the total hours of direct labor expended its production plus the proportional labor content of the products used up in the production process.

This means that we are counting the actual hours expended across all firms producing each given product-type in the basket. For example, suppose one firm produces $80 \%$ of all caster sugar within an economic region, and another firm produces the remaining quantity. If the labor expended in both firms were 10 and 15 worker-hours for a given quantity of sugar, respectively, then its L-content is simply

$$
10 \cdot 0.80+15 \cdot 0.20=11
$$

worker-hours (leaving aside the L-content of the input products used up in the process). The averaging property obviates several conventional qualifiers, including attempts to count labor under 'normal intensity', 'average skill', or counterfactual 'optimal' conditions.

In the production process, products are used up wholly or partially in the production of the given basket. This includes the wear and tear of durable machines and tools. In sum, the production process preserves the L-content measure of everything used up in it. ${ }^{1}$ The measure is also additive: if a basket $\mathcal{B}$ is divisible into several separate baskets, then its L-content is equal to the total L-contents of all the separate baskets summed together. That is, the labor content of a curry dish and a soda is the same as the contents of the dish and the soda added together.
> (L2) The L-content of gifts of nature, such as sunlight, wind, in situ minerals, is zero. The labor content of an empty basket $\mathcal{B}$ is naturally also zero, whereas the labor content of the basket of all (net) output products of the entire economy during a given period, $\mathcal{B}_{\text {out }}$, equals the total number of hours worked during this period.

L-content is a measure of products only and is not assignable to other usables, no matter how important they may be humans or production systems. For example, the natural, unprocessed water used up to produce any product has zero L-content. But the discovery of a new source of crude oil may require substantial amounts of qualified labor, in which case we can assign L-content to the discovered oil.

We count the total number of hours worked in all firms that jointly output a basket $\mathcal{B}_{\text {out }}$ of (net) products. Since our analysis is about capitalist economies, this excludes the crucial time expended in domestic households to reproduce the collective capacity to work. The interaction between the market and non-market sectors of the economy is an avenue for future work.
(L3) The L-content measure is defined over a specified economic region of interest and its resolution is specified by any given division of products types.

The concept of labor content assumes that all products in a given large region are divided into types, so that certain models of cars or pencils are regarded as being equivalent and thus have the same L-content.

The measure counts all working time in a region on an equal footing. Given the spatially interconnected character of modern capitalist production and exchange, we find it reasonable to take the region of interest to be the entire global economy. Then

[^27]L-content is measured by averaging across all firms that produce equivalent producttypes. Naturally, some firms use more labor than the average, while others use less. In so far as farms in Mexico and the United States produce the same type of corn, the L-content of corn is averaged over the labor expended in farms across both countries. The analysis developed below is insensitive to any choice of product-types. Many products that are sold on the world market are considered to belong to same types.

This global measure does not, however, preclude the analysis of sub-regions such as individual countries, sub-national territories, or transnational unions. On the contrary, the constraints set by global L-content play an important role in determining the variability of economic development across sub-regions producing different mixes of product baskets.
(L4) The L-content of an existing product, from a past production process, is no greater than that of newer products of the same type.

This means we are counting labor time as a replacement cost. That is, for any existing durable product, such as a memory cell, a four-door car or a building, its L-content cannot be greater than an equivalent product-type produced by existing firms.

### 4.2 Measuring Heterogeneous Labor in Homogeneous Units

The measure of labor content takes an hour's work performed by a programmer, farm worker, doctor or nurse all equally, irrespective of different skills, work intensities or demands for labor-power. There is no need to assess different types of labor in any other unit than linear time. ${ }^{2}$ It may perhaps appear remarkable that one could derive any meaningful inferences about complex economies by simply counting working times equally across heterogeneous labor tasks.

Reservations to this measure may indeed arise if one follows standard economic theory with its focus on relative 'contributions' of factors of production. We consider no such thing here; contributions from various groups of workers may differ significantly. Our task is neither normative nor prescriptive. In our view, science is about identifying patterns of reality that are compressible into principles and laws. Labor content is a simple measure of what is actually happening in real-world production processes and enables us to derive informative constraints on the dynamics of complex capitalist economies, as we will demonstrate in the coming chapters.

Counting time in production, even minutes of 'work' and 'non-work', has in fact been a feature of firm management for more than a century, from the production of Ford Model T to present-day computerized warehouses. To take a recent example,

[^28]
#### Abstract

Amazon continued to track every minute of most warehouse workers' shifts, from how fast they packed merchandise to how long they paused-the kind of monitoring that spurred a failed unionization drive led by frustrated Black employees at an Alabama warehouse this spring [of 2021]. [...] Two measurements dominated most hourly employees' shifts. Rate gauged how fast they worked, a constantly fluctuating number displayed at their station. Time off task, or T.O.T., tracked every moment they strayed from their assignment-whether trekking to the bathroom, troubleshooting broken machinery, or talking to a co-worker. ${ }^{3}$


### 4.3 Peculiarities of Labor as an Ultimate Input

It is possible of course to conceive of other material measures of products: from energy use to carbon footprint. These various measures can and do have their own uses and advantages.

All such ultimate inputs impose quantitative constraints on the process of economic reproduction. Labor, however, has certain distinguishing features that are of interest to us. We are, of course, mostly interested in labor and labor time as a sociopolitical factor of production. But it also has its own physical, mechanical properties. For instance, labor-like all life processes-tends to increase the order of the immediate physical surroundings. That is to say, it processes various forms of matter and energy into more organized forms within a specific region of space and period of time. It is virtually certain that a vast array of ultimate non-labor inputs will not spontaneously assemble into packets of milk, concrete bridges, or tanks of gasoline. Tilling the land, cleaning warehouse floors, or writing computer code are processes that reduce the number of possible ways in which matter and energy is configured locally. Of course, they are accompanied by increased disorder elsewhere through the release of gases into the air, emission of heat, mixing of waste compounds into water and so on. ${ }^{4}$

In this physical sense human labor is similar to that of bees and ants or birds and even trees that transform their environments into functioning hives, nests and life-supporting environment and structures. We will now consider several economic reasons why labor time is the central quantity that powers capitalism.

## Alternative Production Inputs?

Standard economic theory considers 'capital' to be an essential input that is substitutable for, and at par with, labor. But as Abraham Lincoln remarked long ago

Labor is prior to, and independent of, capital. Capital is only the fruit of labor, and could never have existed if labor had not first existed. ${ }^{5}$

[^29]In other words, capital goods are themselves products that can be resolved into ultimate inputs: labor and gifts of nature. Indeed, quantifying the heterogeneous collection of capital goods employed in production in any other way leads to quite deep theoretical problems. ${ }^{6}$ Capital, however measured, does not therefore impose the same economic constraints as does the quantity of employable labor time.

Certain capital goods do, of course, displace labor over time. Through the application of technical innovations, various machines can replace routinizable human work tasks. Indeed, automation has been a persistent feature of capitalist production, at least since the application of Watt's steam engine, and a source that periodically weakens the bargaining power of workers. But despite all historical projections of a fully automated economy, capital goods are still reducible to and constrained by the ultimate input.

Projections of an automated future have become fashionable after important advances in machine learning and, so-called 'artificial intelligence' (AI) technologies made in the 2010s. Here is a recent example:

> It sounds utopian, but it's something technology can deliver (and in some cases already has). Imagine a world where, for decades, everything-housing, education, food, clothing, etc. became half as expensive every two years... [...]
> If robots can build a house on land you already own from natural resources mined and refined onsite, using solar power, the cost of building that house is close to the cost to rent the robots. And if those robots are made by other robots, the cost to rent them will be much less than it was when humans made them. ${ }^{7}$

Computational and information processing technologies have made it possible for computers to learn to perform certain challenging tasks using large amounts of training data. However, current AI technology has two fundamental limitations. It is either restricted to forms of pattern matching by learning associations in passively observed data: for instance, recognizing typical x-ray scans of lungs with pneumonia. Or, if it is used to perform interventions, it is restricted to a closed model of the world, similar to strategy board games such as chess. This technology can therefore not operate as a general-purpose productive resource in a world with only partially known causal mechanisms and open-ended outcomes. Suppose the game of chess constantly threw up new pieces with novel moves and the dimensions of the board could change. Autonomous machines within the existing paradigm are not capable of adapting to worlds like these. Even in best-case scenarios they would require astronomical amounts of trial-and-error, which is non-feasible in tasks with resource constraints or risks to human lives.

By contrast, labor is an adaptable productive resource. The continual process of training and redeployment of laboring capacity across production renders concrete work tasks into the expenditure of a universal resource quantified in units of person time. ${ }^{8}$ Unlike other direct inputs, the capacity to perform work-labor-power-is not produced within capitalist firms. And unlike other, seemingly crucial material

[^30]resources-coal, gold, or oil-labor has not been an incidental or historical requirement but rather an ultimate input since the dawn of humanity. Measures of other ultimate inputs in production, such as quantities of water or crude oil, are certainly relevant when studying various physical constraints. But unlike labor, they provide very little information about the immediate constraints on a wide variety of economic processes: from design and assembly of machines, to writing lines of computer code and performing heart surgeries.

## Non-Conservation of Labor Content

Any framework of economic analysis must account for the basic fact that human production systems yield a social surplus and can grow. That is to say, economic production is not a conservative process since it yields diverse output products that are 'greater' than their inputs in some quantitative sense. Many ultimate non-labor inputs of products-such as water, iron, crude oil, or energy-are subject to conservation laws such that their total quantities cannot grow through economic processes. This makes them quite impractical as a measure for the analysis of capitalism; using crude oil content is hardly useful when studying the limits of central variables such as wage rates, profit rates and the expansion of production. For similar reasons, we cannot resolve labor input into energy (along with other inputs that perform mechanical work, such as electrical motors or industrial robots).

One important reason for taking labor time as the central quantity that powers a capitalist economy is that it measures a finite universal productive resource, unlike the incommensurable capacities of special-purpose machines. The flow of labor time supplied to firms can both grow or shrink. Its limits are ultimately set by the growth of the total workforce, which is often not under the control of firms that ultimately need it. In this sense it is an external quantity, whose availability and size is not directly influenced by the firms whose products and inputs are to be measured.

The flow of labor time passes the point of production as the labor content of diverse product baskets-steel, electricity, rice, and poultry meat-that enter other processes to produce new baskets-machines, haircuts, chicken fillets, weapons, and luxury goods. These products are either consumed or accumulated as durable material goods. Collectively, workers play a dual role here. They are the source of the labor content but also a major sink through their consumption in individual households. In the socioeconomic process, labor content is created, transmitted, and partly destroyed. By tracing the flow of products we see that a portion of the added labor content is destroyed in the act of consumption. Waste, wars, natural disasters and other material causes are also non-conservative forces.

Another major force mentioned in the postulates below and discussed at length in Chap. 5, is the law of decreasing labor content, LDLC. This force, a direct result of labor itself, brings about the reduction of total labor time needed to produce a typical product, by technological improvements, new products replacing older ones etc. This is a global force usually operating on firms by the collective efforts of
workers in many other firms, in addition to its own workers. The elimination of older chemical photographic equipment, of older airplanes and ships are typical examples. In accounting for L-content of a new electronic camera one does not add to its Lcontent the destroyed content of the older camera-making equipment rendered almost useless by the new ones. ${ }^{9}$

### 4.4 Postulates

We now turn to three basic postulates on L-content. These will form the starting point for our analysis of several central features of capitalist dynamics in the next part of the book.

1. The L-content of the aggregate output constrains the movement of all other key variables of capitalist development (such as wages, productivity, growth, profitability, and capital accumulation). Capital, whether in material or financial form, makes no independent 'contribution' to the output of the economy.
2. We take the global economy as the main economic region of interest, so that the L-content of its output is created by the workforce employed globally.
3. The law of decreasing labor content (LDLC) is a central dynamic force of the capitalist economy. It asserts that, with a high probability, the labor content of any large sample basket decreases continuously. For sake of analysis, we postulate that the L-content of a heterogeneous sample basket of commodity-products is halved every 20-30 years over a long period.

The first postulate stands in direct contrast with standard economic theory, as it denies treating capital on a par with labor. Capital-quantified as accumulated labor content in the form of productive assets-does however enable various forms and scales of productive processes in which the workforce is employed. The second postulate implies also the importance of the regional allocation of production. Firms located in different sub-regions will produce different parts of the global output basket $\mathcal{B}_{\text {out }}$ with varying degrees of relative labor efficiency. As we shall see, this will constrain their local variables of development. The third postulate sets LDLC at the center of capitalist development. The rates of decreasing labor content vary of course significantly between product-types, between services and mass production, and across technical eras of production. But we take the postulated fall of labor content to hold for any basket $\mathcal{B}$ that samples the output of a large industrialized capitalist economy (based on data presented in Chap. 5).

[^31]
## Part II <br> Results

## Chapter 5 <br> Law of Decreasing Labor Content

The previous chapter ended by postulating that the law of decreasing labor content (LDLC) is a central dynamic force of the capitalist economy. It applies to the L-content of any large sample basket, especially those containing product-types that are produced on a mass scale. For example, the labor content of 1 kg of granulated white sugar today is lower than the labor content of an equivalent item was 20 or 30 years ago. The same applies to the replacement of an outdated product by another having similar use and fulfilling a similar function. This is a real material gain in the economy as a whole.

But what are the implications for persistently decreasing L-content of various product baskets? What happens to employment across the economy? Can LDLC proceed to the near elimination of labor in production? What are the limits to the rate of decreasing L-content? How does this rate relate to conventional measures of labor productivity-based on monetary value added per worker-hour? These are the key questions we address in this chapter. Standard economic theory, in contrast, provides very little insight as to why the growth of money value added per worker-hour for a firm, or collection of firms, should be limited.

Based on empirical data to be discussed below, we postulate that the L-content of a heterogeneous sample basket of commodity-products is halved every $20-30$ years over a long period. This has important implications that will be explored further in the chapters to come. For instance, on the one hand, LDLC makes a case for workers to improve their material living standards to keep pace with labor productivity. On the other hand, it lowers the bottom floor for wages sufficient to cover an ever-cheaper basket of minimum consumption goods needed for survival. We shall return to this point in Chap. 6.

### 5.1 Consequences of the Law

For the sake of illustrative simplicity, let us first consider a particular type of product, say, granulated white sugar. Suppose that at a given point in time, the L-content of 1 kilogram of sugar is $v$ worker-hours. In other words, it takes $v$ hours of direct and indirect labor to produce 1 kilogram of sugar. Next, suppose that after several years, the L-content has decreased by a factor of $1 / 2$ so that 1 kilogram of sugar has now an L-content of $v / 2$ worker-hours. This relative rate of decrease is independent of the choice of units in which sugar is measured: if we were to use units of a pound weight (lbs) instead of kilograms, the decrease would still be a factor of $1 / 2$. Put differently, the economy is capable of doubling the quantity of sugar produced with a given amount of labor. It follows that L-content is an (inverse) measure of material labor productivity that counts the total-direct and indirect-labor required to produce sugar in the economy of a given large region, or the world as a whole. ${ }^{1}$ The argument readily applies to any fixed basket $\mathcal{B}$, which may include products ranging from steel rolls and airplanes to haircuts and car washes. Let its labor content in year $t$ be denoted by $L_{t}(\mathcal{B})$, and suppose this quantity decreases at a rate of $\delta$ per year. Then its labor content in the next year $t+1$ can be expressed as

$$
\begin{equation*}
L_{t+1}(\mathcal{B})=(1-\delta) \cdot L_{t}(\mathcal{B}) . \tag{5.1}
\end{equation*}
$$

The effect of LDLC on given basket $\mathcal{B}$ is quantified by $\delta$. This is a measure of the rate of decrease of labor content-or, in other words, the rate of increase of material productivity-in the entire system of production. ${ }^{2}$ What is a plausible range of values for this rate?

Our analysis in Sect. 3.5 showed that, as a consequence of firms' cost-cutting, $\delta$ is very likely to be positive for large baskets $\mathcal{B}$, so that the labor content of $\mathcal{B}$ goes down over time. Specifically, based on empirical data discussed below, we postulated that the L-content of a large sample basket $\mathcal{B}$-representative of all products in exchange-is approximately halved in 25 years. This can be stated, for any given year $t$, as follows:

$$
\begin{equation*}
L_{t+25}(\mathcal{B})=\frac{1}{2} L_{t}(\mathcal{B}) \tag{5.2}
\end{equation*}
$$

Combining the postulate with (5.1) translates into an average rate of decrease of

$$
\delta \simeq 2.7 \% \text { per year, }
$$

when considering heterogeneous sample baskets. ${ }^{3}$

[^32]This is a schematic calculation, which involves averaging over products (yielding a hypothetical 'typical' product) as well as over time (yielding percentage change per year). But it gives us a reasonable idea of the ballpark in which we can expect to find realistic figures for a range of product-types. We shall return to evidence for this estimate in Sects. 5.4 and 5.5. The analysis applies also to the replacement of an outdated product by another having similar use and fulfilling a similar function. Thus, the labor content of a standard ballpoint pen today is lower than the labor content of a basic fountain pen was before it was replaced in general use by the ballpoint pen.

While LDLC emerges from firms driven by the profit imperative, it is important to note that the resulting rising material productivity for a given product-type may, paradoxically, drive down profit rates in this line of production. Extreme cases of this sort are cures of certain medical conditions that used to require prolonged and laborintensive procedures, but were made simpler following an advance in medical or pharmaceutical science. A well-known example is the cure of common peptic ulcers, which in the past involved elaborate and expensive treatment, until it was proved that a major cause of this condition is a bacterial infection that can be eradicated using simple and low-cost antibiotics. ${ }^{4}$ The same goes for basic personal computers and simple timepieces, whose production became standardized and requires a fraction of labor time compared to old methods and techniques. So we have an effect whereby local increase of productivity allows for short-term higher profit and higher wages in the production of a given item, while the longer term and wider effect may well bring down profits derived from the same product type.

However, the loss of profits from one specific product is often compensated by the extension of new techniques to other products. A case in point is that of photography using light-sensitive film: this has become a rarity, largely superseded by electronic photography. In the past, firms such as Kodak made massive profits from developing and printing photos. Only professional and skilled amateur photographers processed their own photos. Now, due to electronic photography, the labor content of an individual photo has gone down to virtually zero. Most people keep electronic albums or, if they wish, can produce their own prints. Printing photos may no longer be a source of large profits; but at the same time, the sale of cameras in various forms, mainly incorporated in cellphones, has emerged as a massive and profitable business.

Thus, LDLC brings about profound structural changes, especially if the average rate of decrease at about $2.7 \%$ per year is maintained. As the L-content of various product-types declines, fewer worker-hours are needed to reproduce a given basket of products. Thus, overall employment drops unless the volume of consumption and investment products increases sufficiently. For the sake of illustration, let's consider granulated white sugar again. Suppose its L-content decreases at a rate of $\delta=2.7 \%$ per year. Then the per-capita consumption of sugar must increase by a sufficient amount or else total employment in sugar production will drop. This principle applies to any basket or product-type $i$. Suppose the per-capita demand for such products increases at an annual rate denoted by $\kappa_{i}$. Then the total labor requirements to meet this demand grow at an annual rate of approximately

[^33]

Fig. 5.1 Growth of labor requirements to meet demand for a given product-type $i$ versus decreasing its labor content at rate $\delta_{i}$; see (5.3) with a population growth of $1 \%$ per year. The dotted line shows the growth of labor requirement when the demand grows at a rate of $\kappa_{i}=2.5 \%$ per year. The shaded region shows the range of labor requirement growth rates that follow when demand grows between $0 \%$ (dashed bottom line) and 5\% (dashed upper line) per year. For rapid rates of decreasing labor content, the total employment that contributes to producing the product $i$ shrinks

$$
\begin{equation*}
g+\kappa_{i}-\delta_{i} \tag{5.3}
\end{equation*}
$$

where $g$ denotes the population growth rate. ${ }^{5}$ Thus, when (5.3) is negative, labor requirements decline as a consequence of productivity growing faster than demand. For the sake of illustration, let us consider a specific type of goods, such as shirts. Suppose realistically that the population grows at a rate of $1 \%$ per year and that the per-capita demand for shirts is saturated. Then total labor required to meet the demand for shirts will decline by

$$
0.01+0-0.027=-1.7 \% \text { per year, }
$$

and workers will have to be laid off or transferred. Of course, productivity growth rates and changes in demand vary across time, and the latter is affected by the distribution of incomes. But the typical pattern is that the rate of increase $\kappa_{i}$ in per-capita demand goes up as the market prices of products decline. Then it approaches zero (and may even be negative in certain cases). More generally, we may illustrate the relationship as in Fig. 5.1 which shows the range of employment growth rates that the demand for product-type $i$ induces.

The unplanned discrepancies between the rates of LDLC and changes in demand for various product-types lead to a persistent restructuring of workers' employment across the capitalist market economy.

[^34]
### 5.2 Limits to the Rate of Decrease of Labor Content

The rate of decrease of L-content for any type of product is of course determined by numerous factors. We will now show that the average rate $\delta$ cannot be arbitrary but rather has a limited range such that it cannot greatly exceed the postulated $2.7 \%$ per year over extended periods of time. ${ }^{6}$

Clearly, technological advances in each production process are central, but are not readily quantifiable by numerical parameters. General discoveries and breakthroughs-such as the development of antibiotics, the synthesis of insulin, determination of the helical molecular structure of DNA, and the construction of silicon-based transistors and memory cells-are enablers for structural economic change. Similarly, a body of foundational knowledge-such as classical mechanics, the laws of electromagnetism, and quantum theory-may have profound economic impact. But despite being an outcome of countless hours of labor of skilled workers and scientists over many generations, the resulting information content lacks any definable labor content. Take, for instance, atomic theory, a foundation of modern technology and science, which was foreshadowed by Greek philosophers and their Roman followers around the first millennium BCE, as well as by their Indian contemporaries, but was rigorously established only in the late nineteenth century. The huge efforts and countless hours of work on these basic abstract products of the human mind and industry have neither price nor labor content attached to them. They are not a particular product of any firm and have not been made directly to manufacture goods. They do, however, have immense impact on the productivity of labor, and are principal enablers of LDLC in almost every product used in production and consumption.

This leaves us with a puzzle: What factors bound the rate of decrease of L-content from above? Why, given all the much-vaunted rapid innovations, would the average rate not be something like $15-20 \%$ per year over an extended period? Popular science writing, often smacking of science fiction, abounds with predictions of the imminent advent of intelligent robots, self-reproducing machines, and fully robotic medical care. Total automation is approaching fast, or so we are told. By definition, this means that the labor content of most products would drop to almost zero, so that the decrease in L-contents of many basic products should be, say, $90 \%$ over the coming $10-20$ years. For this to occur, the labor content of a vast number of products would have to decrease at an average rate of something like $20 \%$ per year. We will argue below that these expectations are not consistent with the reproduction of capitalist market economies.

[^35]
## Persistent Shift Toward Labor-Intensive Sectors

The rates of decreasing L-content naturally vary greatly from one field of economic activity to other. At the same time, the per-capita demand for various products cannot rise indefinitely. There are simply limits to how much sugar can be consumed on average, no matter how cheap sugar becomes. As a rise in demand tapers off, employment will therefore typically decline more sharply in sectors with rapid rates of decreasing L-content than in those with slow rates. This process leads to a relative shift of employment toward sectors with lower material productivity growth $\delta$. The output of these sectors therefore makes up an increasing share of the bought-and-sold labor content, which moderates the aggregate rate of LDLC in the economy. ${ }^{7}$

These sectors are primarily service sectors, as has been observed by many economists in the past:

It is standard to distinguish the following three types of goods and services. For industrial goods, productivity growth has been more rapid than for the economy as a whole, so that prices in this sector have fallen relative to the average of all prices. Foodstuffs is a sector in which productivity has increased continuously and crucially over the very long run (thereby allowing a greatly increased population to be fed by ever fewer hands, liberating a growing portion of the workforce for other tasks), even though the increase in productivity has been less rapid in the agricultural sector than in the industrial sector, so that food prices have evolved at roughly the same rate as the average of all prices. Finally, productivity growth in the service sector has generally been low (or even zero in some cases, which explains why this sector has tended to employ a steadily increasing share of the workforce), so that the price of services has increased more rapidly than the average of all prices.
This general pattern is well known. ${ }^{8}$
But what explains the low rate of material and monetary productivity growth among service sectors? To address this question, consider three categories of services. The first category involves tasks that are repetitive, routinizable, and restricted in scope, such as record keeping, transcribing, computing, and so on. The direct labor inputs into such clerical tasks are decimated, as digital storage, communication and computation technologies become increasingly cheaper and powerful. The second category of services involves more open-ended tasks with a much richer repertoire of possible actions, such as repairs, cooking, house painting, and laundry. Not only do they involve relatively complex mechanical motions and manipulation, but they also demand elements of object recognition and planning that cannot be pre-programmed or learned blindly from a massive number of trials. Existing technical paradigms therefore offer far less scope to shedding direct labor inputs, and the LDLC operates at a much lower rate for these product-types. The final category exhibits certain fundamental barriers, such as haircuts, medical treatments, in-class teaching, and personal services, since their quantities are typically measured in human labor time. Clearly, a one-hour math class or flute-playing lesson cannot readily be substituted

[^36]for a half-hour class without affecting the quality of the product; its usability derives in part from the element of human interaction. The labor content of an hour of person-to-person teaching, or a personal service, cannot fall below one worker-hour. These services are thus persistently labor-intensive with an L-content that remains virtually constant. ${ }^{9}$

In contrast to industrial and agricultural products, the L-content of many important services does not decrease exponentially but remains fairly stable over time. For example, the total labor input of a haircut has not changed significantly over many decades. Moreover, the L-content of certain services may even increase substantially due to their changing nature. This can occur in certain kinds of health care and medical treatments. Due to modern research and deepened knowledge of specific illnesses, new possible treatments emerge that are often labor-intensive and use expensive instruments and medication.

As LDLC proceeds to cheapen products and spur greater demand for them, the process also gives rise to a growing sector of associated services. Take, for instance, the dramatic decrease in the L-content of phone calls, which has enabled a huge service industry associated with phone applications, maintenance, and security. This branch is highly labor-intensive and frequently suffers from labor shortages; it is often outsourced to overseas call centers, where large numbers of workers are penned.

Taken together, we see that the persistent shift toward sectors with low rate $\delta$ increases the proportion of labor devoted to services. This bounds the overall rate of LDLC from above. In a global capitalist economy, this shift of employment is often associated with regional differentials, so that in regions that take up a greater share of service production we should expect to observe declining local productivity growth.

## An Upper Bound on Material Productivity Changes

We now turn to another, stricter limit on LDLC. As we argued in Chap. 3, a precondition for capitalism is the concentration of assets in the hands of few private agents. Now, let a basket $\mathcal{B}$ represent all products that constitute accumulated material wealth, in a given region, up to a given year. Its L-content is not arbitrary, as we will show in Chap. 7, and is empirically approximately five times the labor added over a year in a mature economy. ${ }^{10}$ That is,

$$
\begin{equation*}
L(\mathcal{B}) \simeq 5 \cdot \mathbb{L} \tag{5.4}
\end{equation*}
$$

Consider what would happen if this factor, instead of staying put with the value of 5 , were to progressively diminish toward zero. Then all material wealth $\mathcal{B}$ would eventually be reproduced by a small amount of labor each year. In so far as such assets were accessible on the market, they would quickly cease to be concentrated and the dependence of households on wage incomes paid by capitalist firms would soon vanish. This counterfactual limit provides a means of bounding the rate of

[^37]decrease $\delta$. In the following, we use empirical observations of the factor 5 and the wage share to bound the rate of decrease of labor content $\delta$ from above.

Let us consider how the large basket of material wealth $\mathcal{B}$ changes from a year $t$ to the next year $t+1$. First, some of these durable products are consumed or destroyed through wear-and-tear, and we express the resulting basket as $\mathcal{B}^{\prime}$. The L-content of the initial stock of wealth is $L_{t}(\mathcal{B})$ and after wear-and-tear it is $L_{t+1}\left(\mathcal{B}^{\prime}\right)$. Second, new products are produced and accumulated as wealth each year. That is, during year $t$ a portion of the total labor added, $\mathbb{L}$, is accumulated. This portion is bounded by the fraction of surplus L-content that remains after consumption by the workforce, i.e., by $(1-\omega) \cdot \mathbb{L}$. Here, we make the simplifying assumption that labor added is unchanged from one year to the next. ${ }^{11}$

Now if the L-content of the material wealth is not to fall persistently, we can express the accumulation condition as follows:

$$
\begin{equation*}
L_{t+1}\left(\mathcal{B}^{\prime}\right)+(1-\omega) \mathbb{L} \geq L_{t}(\mathcal{B}) \tag{5.5}
\end{equation*}
$$

where the left-hand side is the L-content of material wealth in year $t+1$. We now relate condition (5.5) to LDLC and the rate $\delta$. The first term can be expressed as

$$
L_{t+1}\left(\mathcal{B}^{\prime}\right)=(1-\delta) \cdot L_{t}\left(\mathcal{B}^{\prime}\right)
$$

due to the decreasing L-content. Next, the wear-and-tear of $\mathcal{B}$ into $\mathcal{B}^{\prime}$ will by itself reduce L-content, which we can express in a similar manner:

$$
L_{t}\left(\mathcal{B}^{\prime}\right)=(1-d) \cdot L_{t}(\mathcal{B})
$$

where we call $d$ the material depreciation rate. We do not consider here the possible ways in which this could be accounted, but we estimate $d$ to be in the range of, say, $5-8 \%$ per year. ${ }^{12}$ By combining the two equations above, we substitute the first term in (5.5) by $(1-\delta) \cdot(1-d) L_{t}(\mathcal{B})$. Using the empirical relation (5.4), we can then rearrange the accumulation condition (5.5) to obtain the following long-term bound on the growth rate of decreasing L-content: ${ }^{13}$

$$
\delta \leq 1-\frac{\omega+4}{5(1-d)}
$$

As we will show in Chap. 6 , the wage share $\omega$ is not only rather stable but tends to be in the order of $50 \%$. This results in an upper bound:

[^38]\[

$$
\begin{equation*}
\delta \leq 1-\frac{0.50+4}{5(1-0.05)} \simeq 5.3 \% \text { per year. } \tag{5.6}
\end{equation*}
$$

\]

In other words, if the rate $\delta$ were substantially greater than $6 \%$ per year, the Lcontent of all material wealth $\mathcal{B}$ would soon be reproducible with very little labor, and the economic system would cease to operate along capitalist principles.

### 5.3 Toward Total Automation?

While the near elimination of labor as an ultimate input is logically possible, we have argued that it is economically implausible and not consistent with capitalist production and accumulation.

For LDLC to displace the bulk of labor, the rate $\delta$ must not only be persistently high but the demand for every product-type must also remain relatively low so that the resulting employment growth rate, given by (5.3), is negative. That is to say, for any product-type $i$-present as well as future -the growth rate of per capita demand remains below a threshold, i.e., $\kappa_{i}<\delta_{i}-g$. No known economic law implies that this would be the case for unknown products of the future.

Total, or near-total, automation of production has nevertheless become a fashionable projection, whether utopian or dystopian. We now show why this is implausible when we consider any larger collection of firms.

## Minimal Aggregate Employment Requirements

Let us consider a large collection $i$ of firms with a basket $\mathcal{B}_{i}$ representing all of its capital goods in use at the beginning of an aggregate production cycle. This includes all non-labor inputs, used fully or partially: machine, energy, and raw materials. If the L-content of $\mathcal{B}_{i}$ is drastically reduced over time, the goods could readily be purchased by households at minimal cost which renders the businesses of the firms nonviable. Capital with virtually zero L-content means that production can become a simple service, as its basic productive machinery or methods can be bought by many customers. This is what has happened to certain printing equipment, to photo duplication and creation, and distance voice and visual exchanges. As we will show, this viability condition implies that the size of the workforce employed by the firms, measured in worker-hours $\mathbb{L}_{i}$, cannot fall to zero.

Let $\mathbb{K}_{i}$ be the L-content of the capital in use, $\mathcal{B}_{i}$, in a given year. For the collection of firms, the ratio $\mathbb{L}_{i} / \mathbb{K}_{i}$ will be very close to their total monetary value added divided by the value of the capital in use. Then $(1-\omega) \cdot \mathbb{L}_{i}$ will roughly correspond to the operating surplus of all firms. In the next year, the L-content of the capital in use is

$$
\begin{equation*}
\mathbb{K}_{i}^{\prime}=(1-\delta) \cdot \mathbb{K}_{i}+(1-\omega) \cdot \mathbb{L}_{i} \tag{5.7}
\end{equation*}
$$

where the first term reflects the decreasing L-content and the second term represents the maximum additional capital accumulated by internal revenue from the previous
year. The viability condition, mentioned above, for this collection of firms is that

$$
\mathbb{K}_{i}^{\prime} \geq \mathbb{K}_{i}
$$

Using (5.7), we can express the condition as a minimum employment requirement:

$$
\begin{equation*}
\mathbb{L}_{i} \geq \mathbb{K}_{i} \cdot \frac{\delta}{1-\omega} \tag{5.8}
\end{equation*}
$$

which connects four basic parameters of capitalist production. In other words, the employed workforce for a large collection of firms must be greater than a certain fraction of the labor content of its capital in use for it to remain viable. If we take the typical wage share of around $50 \%$, then

$$
\begin{equation*}
\mathbb{L}_{i} \geq \mathbb{K}_{i} \cdot \frac{0.027}{1-0.50} \simeq \mathbb{K}_{i} \cdot 0.054 \tag{5.9}
\end{equation*}
$$

or in other words, the employed working hours must correspond to at least 5.4\% of the L-content of $\mathcal{B}_{i}$. This figure provides only a conservative bound, since the fraction of added L-content that is accumulated as additional capital is well below $1-\omega$. But it demonstrates that to survive over a long period, a branch of industry needs some minimal new input of labor per year. ${ }^{14}$

Inequality (5.8) can also be turned around. For example, if we take the labor-tocapital ratio, $\frac{\mathbb{L}_{i}}{\mathbb{K}_{i}}$, for a particular large collection of firms to be 0.1 , and $\delta=2.7 \%$ per year, then (5.8) implies that that the capital share in the newly created labor content should be greater than $27 \%$. This is necessary to keep the L-content of the total capital in use from falling.

[^39]
## Consequences of Total Automation

We have argued above that the idea of total, or near-total, automation has no basis in sound theory or observations. Nevertheless, discussing it as a thought experiment is useful-as in the case of some other science fiction-in throwing light on trends and potentialities in the real world.

Will the secular rise in the productivity of labor lead the capitalist system to dispense for good with a large proportion of the current global input of labor? Predictions to this effect have been made repeatedly with each wave of mechanization, automation, and technical innovation. The pessimistic version warned that permanent unemployment would be the lot of a huge section of the population. The optimists, such as J M Keynes, looked forward to a radically shortened working week and much extended leisure for workers. ${ }^{15}$

None of these forecasts have materialized. They failed to foresee that, along with new improved production techniques, novel needs would also be invented or 'discovered', demanding hitherto unknown types of goods and services, requiring workers with new types of skills. There is no compelling theoretical reason to believe that this time things will be very different and-assuming the continued existence of capitalism-the current spate of automation and robotization will result in a drastic reduction in the global input of labor, leading to persistent high unemployment or increase in leisure time enjoyed by workers. Of course, it is quite possible for some individual capitalist firms to make a profit with little if any direct input of labor. In fact, something approaching this already exists; for example, a property-owning company deriving all its profit from rent. The property may need to be managed, maintained, and serviced, but this (to the extent that it is not automated) can be outsourced, as is indeed normal practice.

As a hypothetical but realistic example involving the production of physical goods, suppose a firm owns or rents a source of natural mineral water. It is quite realistic to envisage that the entire process of extraction and bottling could be automated. Maintenance and marketing (insofar as they are not automated) can be outsourced.
Total, or near-total, automation-substituting robots for all or almost all human workers in the entire economic system-is quite another matter; it is not going to happen any time soon. Nevertheless, it is worth considering such a laborless economic system. Like the best kind of science fiction, it can throw some light on our presentday real world.

The first thing that can be said about a hypothetical socioeconomic system based on quasi-total automation is that it cannot be capitalist. This is so by definition: capitalism is a system in which surplus product is extracted, in the mediating form

[^40]of profit, by the private owners of the means of production from the labor of hired workers. ${ }^{16}$

But what kind of socioeconomic system is compatible with quasi-total automation? To begin to answer this question, observe that the L-content of all products in such a system is zero or as near-zero as makes no difference. ${ }^{17}$ The ultimate inputs of these products therefore consist entirely of natural resources, 'gifts of nature'. Note also that automata used as means of production are themselves produced by automata, with natural resources as their only ultimate inputs.

It follows that whoever has access to the requisite natural resources and the relevant technological know-how can obtain any desired product. The monopoly of technological know-how is in practice very difficult to enforce; human ingenuity will eventually break through arbitrary artificial barriers. Therefore, the answer to the question we have posed boils down to the mode of access to natural resources.

Two diametrically opposed scenarios suggest themselves: resources are privately owned, or communally possessed and controlled. Intermediate forms are of course imaginable, but for the sake of analytic clarity we shall address the two pure forms. Each of them, while being pure fiction, points out a possible trend latent in the present real world.

Let us assume that all resources to which access can be restricted are privately owned. It does not take a great deal of mental effort to see that this leads to extreme dystopia, a system of pure monopolistic rentier extractivism. There must be a market in which the monopolists, a small minority of the population, sell resources they extract, and use their rentier revenue to buy resources they require as inputs for automatically producing consumer goods and robot-provided services, including protection of their property by armed robots. The majority of the population, owning no natural resources, are reduced to foraging in areas that the monopolists cannot control, to charity, and to rendering personal non-productive services to the rentiers.

Such a system is likely to be unstable, due to the enormous gulf between the rentiers and the majority, and the obvious temptation of the former to over-exploit their property. (Here, we are extrapolating from a tendency prominent even in capitalism, in which natural resources are by no means the only source of wealth.)

A very different picture emerges if we assume that all natural resources are possessed and husbanded as public goods. This may not necessarily lead to a utopia, but does provide a substructure for it: the possibility of a society all whose members enjoy material sufficiency, freedom, and creative leisure, and in which natural resources are not over-exploited.

[^41]
### 5.4 Connection to Price Measures of Productivity

Finding empirical evidence to evaluate LDLC is not straightforward. This requires large amounts of econometric data spanning decades. In the economic literature, there are few direct data on material labor productivity. ${ }^{18}$ In order to test this law, we will need to take an indirect approach: use widely available price and labor data and infer from them consequences regarding changes in material productivity. In this section we establish a connection between the growth of material productivity of large sample baskets $\mathcal{B}$ and the conventional price measure of aggregate productivity.

The connection between labor content and market price, derived in Chap. 3, gives us the following excellent approximation for the L -content of a sample basket $\mathcal{B}$ in a given year, $t$ :

$$
L_{t}(\mathcal{B}) \simeq \frac{M_{t}(\mathcal{B})}{\mathrm{E} \Psi_{t}}
$$

(Note that L-values and prices, hence also the specific price random variable $\Psi$ depend on time. This dependence is indicated by the subscript $t$.) Using this relation in equation (5.1), we obtain an approximation for the material productivity growth rate:

$$
\begin{equation*}
\delta \simeq 1-\frac{M_{t+1}(\mathcal{B})}{M_{t}(\mathcal{B})} \cdot \frac{\mathrm{E} \Psi_{t}}{\mathrm{E} \Psi_{t+1}} \tag{5.10}
\end{equation*}
$$

This provides a good approximation of $\delta$ using the nominal change in the price of $\mathcal{B}$ and the change in average specific price from period $t$ to the next. Note that the average specific prices are readily estimated using the average wage rate and the wage share: see (3.8). Thus, (5.10) establishes a connection between rates of change in prices and L-content.

Let us now consider how labor productivity is conventionally measured. It is defined over a given collection of firms. Suppose it produces an aggregate output basket $\mathcal{B}$ over, say, a year. The standard productivity measure is then defined as the ratio between the value added accrued to the firms through the sale of $\mathcal{B}$ and the amount of labor directly employed in producing it. We may denote this price measure of productivity as

$$
\begin{equation*}
P=\frac{\text { monetary value added of } \mathcal{B}}{\text { direct labor time taken to produce } \mathcal{B}} . \tag{5.11}
\end{equation*}
$$

By definition, the value added is obtained by deducting the total cost of the nonlabor inputs used up in the production of $\mathcal{B}$ from the total sale price of $\mathcal{B}$. Briefly, it equals sales minus non-labor costs. Note that $P$ has the dimension of money per worker-time; for instance, dollar per worker-hour. ${ }^{19}$

[^42]Clearly, this price-based measure and the material measure of labor productivity differ in their perspective on production and in the kind of questions they address. An obvious difference is that the former applies to a specified body of workers employed in production: it focuses on the contribution of this labor force to the monetary value added of the firms in which it is employed. In contrast, the material measure $\delta$ applies to products. It measures the physical amount of a given product or basket of products produced per unit of labor (one worker-hour) performed by workers belonging to diverse labor forces. Moreover, the price measure (5.11) is concerned only with the direct contribution of the given labor force. It is designed to assess the effectiveness of the utilization of that labor force in terms of its direct contribution to the revenue accruing to the capital it sets in motion. In contrast, the material measure takes into account the total-direct as well as indirect-labor inputs that go into the given product by considering the entire production process. Some of the indirect inputs may well be remote in place and time from the final point of production.

Thus, for example, an increase in the material productivity of sugar-producing labor need not be associated with any changes in the operation of sugar refineries, or in the conventional measure of the productivity of their labor force, or in the way their value added is divided between profit and wages. The increase may be entirely due to changes occurring somewhere upstream in the flow of inputs to the production of sugar. Moreover, even if during a given period the standard productivity of labor in, say, the US is stagnating in certain sectors, or across the economy, the labor content of products may still decrease, not only worldwide but even of those produced in the US. Over such periods, global improvements in material productivity have not been translated into US monetary value-added terms due to its regional share of employment and local production efficiency.

The fact that the labor content of, say, a one-minute trans-Atlantic phone call has become minuscule over the last half century has no obvious and direct parallel in the standard notion of productivity of labor, measured in terms of net income of a firm per hour worked. A one-minute call is not produced directly by a single process in a single firm: there are many ways to get a voice connection, using a variety of devices and appliances, each of which can perform many tasks. Still, the notion of the labor needed to obtain the voice connection is sufficiently well defined even in this mesh of joint production. ${ }^{20}$ It is in any case clear that the labor needed today is a tiny fraction of what was needed a century ago. This substantial decline in L-content is a crucial characteristic of developments in productive capacities.

Let us now consider the price measure of aggregate productivity (5.11) in the economy using the basket of net output products $\mathcal{B}_{\text {out }}$ in a given year. Then (5.11) may be rewritten as

$$
\begin{equation*}
P=\frac{M\left(\mathcal{B}_{\mathrm{out}}\right)}{L\left(\mathcal{B}_{\mathrm{out}}\right)} \tag{5.12}
\end{equation*}
$$

since by definition the labor content of $\mathcal{B}_{\text {out }}$ is the amount of labor expended directly in its production. Using the connection between price and L-content above, we therefore

[^43]have the following excellent approximation of aggregate productivity at year $t$ :
\[

$$
\begin{equation*}
P_{t}=\mathrm{E} \Psi_{t} \tag{5.13}
\end{equation*}
$$

\]

In order to be able to track the change of $P_{t}$ over time, from one year to the next, the monetary quantity appearing in the numerator of equations (5.11) and (5.12) must be standardized in some way to adjust for nominal changes of price levels and purchasing power across regions. This is usually done by considering a fixed heterogeneous sample basket for reference, which we denote as $\mathcal{B}_{\text {ref }}$. Then all prices are rescaled to measure in units of $M\left(\mathcal{B}_{\text {ref }}\right)$ for a given year. This ensures that the price of the reference basket is fixed:

$$
\begin{equation*}
M_{t+1}\left(\mathcal{B}_{\mathrm{ref}}\right)=M_{t}\left(\mathcal{B}_{\mathrm{ref}}\right) . \tag{5.14}
\end{equation*}
$$

Such prices are often referred to as real prices. ${ }^{21}$ Using them, we can now derive a connection between price and material measures of productivity for the heterogeneous sample basket $\mathcal{B}_{\text {ref }}$.

Consider the rate (5.10) for $\mathcal{B}_{\text {ref }}$, and note that by definition its real price does not change (see (5.14)). By inserting the relation between monetary productivity and specific prices, (5.13), we can express the connection (5.10) in the following way:

$$
\begin{equation*}
\delta \simeq \frac{P_{t+1}-P_{t}}{P_{t+1}} \tag{5.15}
\end{equation*}
$$

This establishes a bridge between two different measures: on the left-hand side we have the rate of decrease in L-content, reflecting the aggregate growth of material labor productivity of a large heterogeneous sample basket; and on the right-hand side is the relative rate of increase of the real price measure of labor productivity. ${ }^{22}$

It now transpires that although the material and standard measures of labor productivity are conceptually rather different, their rates of increase are the same (to an excellent level of approximation) for a large economic region and a representative diverse basket produced in that region.

[^44]
### 5.5 Empirical Evidence of LDLC

In this section we present some data regarding the rate of decrease of labor content, both globally and in certain large regions. The mere fact of the secular fall in labor content is widely accepted. However, the numerical value of this rate for large sample baskets $\mathcal{B}$, denoted above by $\delta$, is important for many considerations in coming chapters: from wage increases to capital accumulation, and interest rates. The derived result (5.15) shows that we can in fact estimate it using real price measures of aggregate output. This gives an estimate for $\delta$ over a given period, say, 5-10 years.

The rate of decrease for a large selective basket $\mathcal{B}^{\prime}$, made up of products from a particular group such as agricultural products or personal services, is denoted as $\delta^{\prime}$. The average rate over $T$ years is related to the relative change of its L-content as

$$
\begin{equation*}
\delta^{\prime}=1-\left(\frac{L_{t+T}\left(\mathcal{B}^{\prime}\right)}{L_{t}\left(\mathcal{B}^{\prime}\right)}\right)^{1 / T} \tag{5.17}
\end{equation*}
$$

using the general relation in (5.1). Without detailed data on integrated production processes, the relative change in L-content can only be estimated from price and labor data in a rather indirect way. Here we discuss a simple method for doing so that we will use below. Recall that we can relate L-content to specific prices $\Psi\left(\mathcal{B}^{\prime}\right)$ in (3.10). Now introduce the simplifying assumption that changes in the specific prices of the selective basket $\mathcal{B}^{\prime}$ track the changes in the (overall) average specific price, that is, $\bar{\Psi}_{t}\left(\mathcal{B}^{\prime}\right) / \mathrm{E} \Psi_{t}$ is roughly constant throughout the period. Then the ratio of labor contents in (5.17) can be approximated using price and wage data, ${ }^{23}$ and we arrive at an approximate expression:

$$
\begin{equation*}
\delta^{\prime} \simeq 1-\left(\frac{\bar{M}_{t+T}\left(\mathcal{B}^{\prime}\right)}{\bar{M}_{t}\left(\mathcal{B}^{\prime}\right)} \cdot \frac{\omega_{t+T}}{\omega_{t}}\right)^{1 / T} \tag{5.18}
\end{equation*}
$$

where $\bar{M}_{t}$ denotes prices in units of the average wage $\mathrm{E} W_{t}$. In other words, the average rate of decrease for a selective basket can be estimated using price changes relative to average wages and the wage shares under the assumption above.

Alternatively, we can approximate the average rate $\delta^{\prime}$ using real price data instead of wage data. The expression can then be given a more symmetric form:

$$
\begin{equation*}
1-\delta^{\prime} \simeq(1-\delta) \cdot\left(\frac{\bar{M}_{t+T}\left(\mathcal{B}^{\prime}\right)}{\bar{M}_{t}\left(\mathcal{B}^{\prime}\right)}\right)^{1 / T} \tag{5.19}
\end{equation*}
$$

[^45]Table 5.1 Monetary productivity using real price of aggregate output per hour worked. This approximates changes in labor content of a fixed large sample basket $\delta$; see (5.15). As expected, in the post-war recovery period, 1950-73, there is a local peak but rarely exceeds the bound in (5.6). Source Frankel and Kendrick (2014) and Maddison (1987).

|  | Change per year, $1870-1984[\%]$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Country | $1870-1913$ | $1923-50$ | $1950-73$ | $1973-84$ |
| France | 1.7 | 2.0 | 5.1 | 3.4 |
| Germany | 1.9 | 1.0 | 6.0 | 3.0 |
| Japan | 1.8 | 1.7 | 7.7 | 3.2 |
| Netherlands | 1.2 | 1.7 | 4.4 | 1.9 |
| UK | 1.2 | 1.6 | 3.2 | 2.4 |
| Average | 1.6 | 1.6 | 5.3 | 2.8 |

where $\bar{M}_{t}$ now denotes prices in units of a chosen reference sample basket $\mathcal{B}_{\text {ref. }}{ }^{24}$ We see that the average rate of decrease for a selective basket is given by relative changes in its real price and the average rate that holds for large sample baskets, $\delta$. Thus, if the real price of a selective basket is falling, it implies that its labor content is decreasing faster than that of a representative sample basket. Similarly, rising real prices imply that the labor content is decreasing slower than the reference. Both (5.18) and (5.19) provide an indirect estimate of $\delta^{\prime}$ for selective baskets of products that will be used below. Formula (5.19) provides a way of relating changes of L-content across economic sectors in terms of changes in real prices.

## Changes Over Longer Periods

In view of the discussion in Sect. 5.4 and the derived result (5.15), it is instructive to first consider data on changes in monetary productivity. Table 5.1 summarizes the measures for a few leading capitalist economies over long periods. We can therefore estimate the average rate of decreasing labor content of large sample baskets to be in the range of $2-3 \%$ per year in the present era.

Thus, by this estimate, the labor content of a typical diverse basket is roughly cut by about half every $25-30$ years. This parameter seems to be typical of modern capitalism over a wide range of time and regions. We see that apart from the exceptional period of post-war recovery and reconstruction, $\delta$ is confined to a rather narrow range. The brief exceptions that exceeded the derived bound (5.6) were Germany and Japan, two defeated powers of the Second World War whose productive capacities underwent great destruction and reconstruction. Maddison (1987) provides a wider survey of the empirical data, broadly in line with the present analysis.

[^46]

Fig. 5.2 Direct labor inputs per Cwt (=1001b) in various regions in the US, over a ten-year period. Source Agriculture (2016); and Wang et al. (2015)

## LDLC in US Capitalist Agriculture

Agricultural product-types, such as wheat, rice, and cotton, remain unchanged for long periods of time, and their production is quantified in standardized units with relative ease. Given their importance for the reproduction of human labor power and of the economy at large, it is therefore of interest to study LDLC in this sector of production.

Let us first look at rice and cotton lint produced in US agriculture. ${ }^{25}$ The direct labor time required per unit of rice and cotton lint is traced over 10-20 years in Figs. 5.2 and 5.3, respectively. For these crops, we observe a varying but persistent decrease of direct labor time. For rice, the rate of decrease ranges from about $2 \%$ (in the Mississippi Delta) to 6\% (in the Gulf Coast) per year across different regions. For cotton lint, the fall was even more dramatic, at a rate of about $15 \%$ per year.

The US department of agriculture keeps longer records of direct inputs into agricultural production. Figure 5.4 illustrates the fall of its overall direct labor input over a span of six decades. During this period, the labor directly expended in the agricultural sector decreased to nearly one-fourth, at an average rate of $2.4 \%$ per year, even as its outputs increased. This means that the direct labor input to a fixed basket of agricultural products decreased even more rapidly. This, of course, occurred through mechanization and a total increase of non-labor inputs. To assess the fall in indirect labor in agricultural products, we must take into account the falling L-content of their non-labor inputs.

Figure 5.5 shows the price changes of such inputs when indexed by average wage costs. We see that the indexed prices fall by nearly a factor of three over 60 years. That is, the price for a selective basket of input products $\mathcal{B}^{\prime}$ in 2008 was roughly

[^47]
## Implied Time to Produce Cotton Lint per Planted Acre and per Pound (United States, 1990-2011)



Fig. 5.3 Direct labor input in US cotton production during a 30 -year period, given in hours per product (right vertical axis) and hours per planted area (left vertical axis). The secular decrease in direct labor inputs is clear. Source Agriculture (2016); and Wang et al. (2015)


Fig. 5.4 Proportional changes in labor and other inputs in total agricultural production. Source Wang et al. (2015, Fig. 8)
one-third of the price in 1948. Using (5.18) and the averaged levels of wage shares reported by Elsby et al. (2013, pp. 6-7), we can therefore approximate the average rate of decrease for input products by

$$
\delta^{\prime} \simeq 1-\left(\frac{1}{3} \cdot \frac{0.58}{0.60}\right)^{1 / 60} \simeq 1.9 \% \text { per year }
$$

Input prices indexed by labor 1948-2008


Fig. 5.5 Prices of non-labor inputs compared to wages. We infer that there was a secular decline of the L-content of material inputs to agricultural production using (5.18). Source Wang et al. (2015, p. 23, Fig. 16)

The evidence from US agricultural production is in other words consistent with LDLC. Almost all large-scale agricultural production across Europe and China exhibit very similar development. ${ }^{26}$

## Tracing Rates of Decrease Using Real Price Data

The question we consider here is: Can we assess quantitatively the operation of LDLC using real price data for various selected baskets of products? Recall that real prices are defined in units of the price of a fixed sample basket of products $\mathcal{B}_{\text {ref }}$; see (5.14).

Consider first the real price of a basic commodity-product such as crude oil, as shown in Fig. 5.6. Over a period of about 150 years, the real price per barrel of crude oil has hovered around the $\$ 50$ level (2010 prices), apart from some rather wide shortterm variations. This is for a seemingly limited resource, whose price often includes a large rent component derived from monopoly on its geographical location. This observation is not very surprising, considering that oil is itself a significant component in the sample basket $\mathcal{B}_{\text {ref }}$ used to define the price level. This tends to make the real price of oil stable over many decades.

We now turn to real price changes for a range of product-types, which can be used to infer changes in their labor contents. Table 5.2 and the graph lines in Fig. 5.7 provide several examples of the movement of real prices of various products. The table covers a period of five decades in the US, and the figure depicts a collection real-world data in the US and the EU over roughly a decade. The products can be roughly divided into two categories: labor-intensive services (e.g., haircut and health care); and mass-produced goods (e.g., cars and TV sets).

[^48]

Fig. 5.6 Real prices of crude oil have not changed much for 150 years despite wide short-term variations. The rate of decrease in the labor content of crude oil can be deduced as explained below. Source Ro (2014)

We see that the real prices of the services gradually rise, while the prices of massproduced goods fall more or less rapidly, depending on the product. We estimate the real price of a selective basket of labor-intensive services in the US to increase by about $2.2 \%$ a year. ${ }^{27}$ We also estimate the real price of a selective basket of massproduced goods to fall by roughly $4-4.5 \%$ per year. ${ }^{28}$ These diverging trends are illustrated schematically by the red and blue lines in Fig. 5.8 (left). The middle green line shows a (constant) real price that serves as an approximation for large sample baskets, or products with very diverse production methods using many diverse inputs, such as crude oil.

We can infer from this data that the average rate of decreasing L-content for many mass-produced goods is greater than that of the reference sample basket $\mathcal{B}_{\text {ref }}$ using the real price data in the approximation (5.19). We can also use the price trend for labor-intensive services as a reference, by assuming the L-content of a basket such services exhibits virtually no decrease so that $\delta^{\prime} \simeq 0$ in (5.19). Since its real price is estimated to increase by $2.2 \%$ per year, we obtain an approximation of the rate $\delta \simeq 2.2 \%$ per year for sample basket $\mathcal{B}_{\text {ref. }}{ }^{29}$ Using this reference rate in (5.19) gives a rough and independent means of converting the relative real price changes of selective baskets into rates $\delta^{\prime}$. The method is illustrated by the two diagrams in Fig. 5.8.

We can now estimate the L-content of a selective basket of mass-produced consumer and industrial goods to have fallen at a rate of about $4-7 \%$ per year, whereas that of crude oil and gas products have fallen at a rate of about $2 \%$ per year. Our

[^49]Table 5.2 Real price average percentage yearly change in various categories of products during 1970-2020 in the US. A clear difference is evident between the two categories: prices of standard products fell, while the real price of health care rose sharply. The table was compiled by the authors, using standard sources: Claxton et al. (2018) for healthcare data; OECD (2021b) for teachers' wages; NCES (2021) for university fees

| Product-type | Change per year [\%] |
| :--- | :--- |
| Car | -0.3 |
| Watch | -1.3 |
| Television | -4.3 |
| Crude mineral oil | $\pm 0.0$ |
| Gas | -0.3 |
| Trans-Atlantic flight | -2.0 |
| Shipping | -2.9 |
| Haircut | +0.8 |
| Education | +2.5 |
| Health care | +6.0 |
| Fast food | +1.0 |

Fig. 5.7 Movement of real (inflation-adjusted) prices of various types of products. Prices of labor-intensive services either rise or remain stagnant while those of mass-produced goods tend to decline rapidly. Source Lowrey (2014)

Change in Consumer Prices 2005-2014



Fig. 5.8 Changes in real prices and L-content for three large product baskets. These figures illustrate the relations of price change to L-content change as expressed by (5.19) above. Left: schematic depiction of real price changes in the US and elsewhere, based on the assumption that during a 30year period the price of a basket of labor-intensive services increases by about $2.2 \%$, whereas that of a basket of mass-produced products decreases by about $4 \%$ per year. We assume here that services constitute about $60 \%$ of the economic activity. On the right: the inferred trend of L-content. We assume that the L-content of a typical basket of labor-intensive services (red line) remains virtually unchanged. Against this baseline, we can convert the relative price changes of large sample baskets and mass-product baskets into a rough estimate of the rate of decrease of their L-content
assumptions and findings are consistent with those presented in Table 4 and Fig. 2 in Flaschel et al. (2013, pp. 17-19) for the German economy.

## Chapter 6 Wages and Class Divisions

We now turn to the important topic of wages, which affect the survival and material well-being of billions of people. Following the probabilistic framework of Chap. 3, we consider individual wages to be indeterminate and unpredictable from a macroeconomic point of view. But what are the central factors that determine the variation of wages? How does the law of decreasing labor content (LDLC) affect the distribution of wages? How does one account for the stagnation of wage growth in parts of the Global North and its acceleration in regions of East Asia? Despite all structural variations of labor markets, why have total wage incomes remained remarkably stable as a share of aggregate output? How is the distribution of wages bound up with the appropriation of labor content by different economic classes? These are some key questions that we address in this chapter, which we conclude with an assessment of some of our results using wage and income data from a range of various countries.

### 6.1 Purchasing Power of Wages

Since nominal price and wage levels change across time and regions, it is necessary to measure wages by what they can purchase. To compare the distribution of wages in, say, the United Kingdom in 1920 and 2020, we must measure wages using a common unit in terms of their purchasing power. A standard choice is to use the price of a fixed sample basket $\mathcal{B}_{\text {ref }}$ as a reference, which gives a unit of real wages.

This fixed basket has certain limitations, since the products purchased by wage incomes change across time and regions. We will instead consider any sample basket $\mathcal{B}$ of diverse products that the economy can produce. Specifically, we measure wages by the size-measured in worker-hours-of sample baskets that they can purchase. This measure standardizes wages across time and regions with respect to the prevailing productive capacities of the economy. Recalling the connection between price and labor content established in Chap. 3, we measure the wage rate $W$, defined in

Sect. 3.2, in terms of its L-content purchasing power:

$$
\begin{equation*}
\bar{W}=\frac{W}{\mathrm{E} \Psi}, \tag{6.1}
\end{equation*}
$$

using the average or 'expected' specific price $\mathrm{E} \Psi$. We shall refer to $\bar{W}$ as the $L$-wage rate for brevity. ${ }^{1}$

Let us illustrate the meaning of L-wages by considering a person working for a week at a firm for a typical duration of $T_{i}=40$ hours. Suppose she receives a nominal pay of $\$ 20$ per hour and that the average specific price $\mathrm{E} \Psi$ in the economy is $\$ 50$ per worker-hour. Then this labor contract yields an L-wage rate of

$$
\bar{W}_{i}=\frac{20}{50}=0.40 .
$$

This means that her week's wage income can purchase a basket of diverse products $\mathcal{B}$ whose labor content is approximately ${ }^{2}$

$$
\bar{W}_{i} T_{i}=0.40 \cdot 40=16 \text { worker-hours. }
$$

In other words, at an L-wage rate of 0.40 , she can purchase a sample basket of products with an L-content that is $40 \%$ of the time she labored.

We take L-wages to be the appropriate measure of wages in the analysis of capitalism and consider the distribution of L-wage rates $\bar{W}$ below. In anticipation of things to come, we note that the bulk of employed labor time earns rates in the range from 0.25 to 0.75 . That is to say,

$$
\mathrm{P}(0.25 \leq \bar{W} \leq 0.75) \text { high. }
$$

### 6.2 Structural Features of the Wage Distribution

Using L-wage rates, we can now study certain stable structural features of the wage distribution in capitalist market economies.

First, if we consider the expected L-wage rate, $\mathrm{E} \bar{W}$, as averaged over all labor time, we find that it is well-approximated by the wage share:

$$
\begin{equation*}
\mathrm{E} \bar{W}=\frac{\mathrm{E} W}{\mathrm{E} \Psi} \simeq \omega, \tag{6.2}
\end{equation*}
$$

using (6.1) and (3.8). Recall that the wage share $\omega$ is the portion of the total value added in an economic region that is allotted to wages of all employed workers. This

[^50]

Fig. 6.1 The distribution of L-wage rates $\bar{W}$ illustrated using a density function. The horizontal axis shows possible $L$-wage rates ranging from 0 to 2 , where the expected rate $E \bar{W}=0.5$ is marked with a cross. Due to (6.2), this rate is bounded between 0 and 1 . The blue-shaded tail area equals the proportion of labor time that earns L-wage rates above 1 . That is, $\mathrm{P}(\bar{W}>1)$. The red shaded area to the left corresponds to the worst-paid half of labor. By definition, this half earns no more than the median L-wage rate, which in this case equals 0.35 . The skewed form of a log-normal distribution implies that the median rate is always lower than $E \bar{W}$, where the ratio $0.35 / 0.50=70 \%$ is a measure of wage equality
fact bounds the expected L-wage rate:

$$
0<\mathrm{E} \bar{W}<1,
$$

since all employees collectively can only earn a fraction of the aggregate output measured in monetary terms.

Second, wage rates are outcomes of a multiplicity of independent or weakly interdependent factors. As shown in Chap. 5, the structure of the labor market and the labor contracts on offer change persistently as a consequence of LDLC and shifting demands on the commodity market. Firms are destroyed in competition and entire economic sectors decline, while new firms are formed and the demand for new products rises. On this shifting terrain, individuals who do not own productive assets seek employment equipped with differing skills and credentials across regions with varying local prospects. Workers struggle for higher wages with varying degrees of success. If the resulting wage rates are considered as outcomes of many independent multiplicative factors, then an educated guess is that $\bar{W}$ has a 'log-normal' distribution. Its form is skewed, as illustrated in Fig. 6.1, which reflects the fact that $E \bar{W}$ is not the typical wage rate for most workers. ${ }^{3}$ That is, a few workers earn exceptionally high wages that raise the average rate beyond what is typical for most workers. Put differently, half of employed labor time receives less than $70-80 \%$ of the average wage rate.

It turns out that this distributional form provides a good approximation of wage incomes within a capitalist economy. ${ }^{4}$ We shall therefore take the distribution of

[^51]$\bar{W}$ to be log-normal as a working hypothesis. What are its main determinants? It is a question that not only relates to the livelihoods of many millions of workers but also, as we shall see, to the development of capitalist economies and their persistent inequalities.

We will next discuss two factors-the tail end of the distribution that earns L-wage rates above 1 and the relative position of the bottom half of employed labor-that jointly constrain the shape of the entire distribution of $\bar{W}$.

## Upper Limits on L-Wage Rates

We begin by considering L-wage rates above 1 . Earning such rates, workers can purchase baskets of products with an L-content that exceeds the time they labor. In other words, a labor contract for $T_{i}$ hours of work attains wages that can purchase sample baskets whose L-content is greater than $T_{i}$ worker-hours.

The proportion of labor time that earns such wages is necessarily bounded. By definition, the bottom half of labor earns wage rates in the ranges of 0 to $\bar{W}_{\text {median }}$, the median wage rate. But due to the skewed form of the wage distribution considered above, the median rate is strictly less than 1 , since

$$
0<\bar{W}_{\text {median }}<\mathrm{E} \bar{W}<1
$$

It follows that the proportion of labor time that earns L-wage rates above 1, viz., $\mathrm{P}(\bar{W}>1)$, cannot exceed $50 \%$. Put simply, the wage rates received at this tail end are economically denied to the majority of workers. These wages can then be viewed as a form of rent. Let us now consider the type of labor contracts that can receive very high wage rates.

First, there are sudden moments of labor shortages in various firms, responding to rapidly changing market conditions. This may happen when new unexpected natural resources are discovered or there is an unanticipated surge of consumer demand. Then firms are ready to bid up wages to attract new workers. But such 'windfall wages' are temporary, as more workers seek similar opportunities and firms adapt their production. Second, there are sectors of the labor market with more persistent shortages of certain labor skills. This includes doctors, lawyers, and some categories of engineers. The supply of such labor can be restricted by the need for extensive training or guild-type barriers to obtaining formal credentials. This improves the bargaining power of certain skilled or credentialed workers and enables a small portion of labor to obtain a 'skill rent' that achieves $\bar{W}>1 .{ }^{5}$ Third, firms hire employees to manage and supervise others' labor. These manage firms so as to implement the profitability criterion in daily operations. But managerial and supervisory employees are themselves difficult to monitor and control by the owners of the firm. Instead, the commitment of managers to the goals of the organization

[^52]is induced by higher earnings linked to careers and promotion along its hierarchy. ${ }^{6}$ Such favorable locations in firms enable a small portion of labor to obtain a 'loyalty rent' such that $\bar{W}>1$.

In summary, we conclude that the proportion of labor with an L-wage rate that exceeds 1 must be quite small, that is,

$$
\begin{equation*}
\mathrm{P}(\bar{W}>1) \text { small. } \tag{6.3}
\end{equation*}
$$

Since approximately one-tenth of the labor force occupies credentialed, managerial, or supervisory positions, we would guesstimate $\mathrm{P}(W>1)$ to be in the order of $10 \%$. This imposes a constraint on the distribution of L-wage rates.

## Downward Pressure on L-Wage Rates

We now turn to the lower range of the wage distribution. Suppose the nominal pay of a worker was unchanged. Then her L-wage would decrease in one year by a factor

$$
\begin{equation*}
(1-\delta) \cdot \frac{M_{t}(\mathcal{B})}{M_{t+1}(\mathcal{B})}, \tag{6.4}
\end{equation*}
$$

where $\mathcal{B}$ is a large sample basket of diverse products measured in nominal prices. ${ }^{7}$ This is the outcome of two different processes: LDLC, which accounts for the factor $(1-\delta)$, and price inflation of the sample basket. Note that, even under no inflation, the L-content purchasing power of her wage decreases at an annual rate of about $2-3 \%$ due to LDLC! This means there is a persistent downward pressure that acts to compress the entire distribution of $\bar{W}$ toward zero. ${ }^{8}$ This is counteracted by workers who struggle to keep wages catch up with price inflation and productivity. However, the ability of individual workers to keep up their wage gains varies substantially.

First, some firms are less efficient than others, which gives their workers less room to negotiate better wages if the businesses are to remain viable. Such firms are also often more labor-intensive, so that wages constitute a larger share of total costs. We have identified a relative shift of employment toward such sectors with a very low rate of decreasing labor content: specifically, various service sectors (see Chap. 5). Second, insufficient demand for consumption and investment products will under LDLC swell the pool of unemployed and thus exert additional downward pressure on L-wages, especially for the worst-paid half of labor. Third, the geographical spread of efficient firms, advanced sectors, and expansive demand for consumer and investment goods are highly unequal. These geographical differentials drive the migrations of the unemployed toward regions with more favorable labor markets. As competition

[^53]intensifies, in the absence of strong labor unions the workers' bargaining position weakens, especially over labor contracts that require few skill credentials and are typically located at the bottom half of the wage distribution. (This labor is subject to further downward pressure when it is demarcated on the labor market on the basis of legal status or identifiable ethnic markers.) Fourth, due to LDLC, a given consumption basket needed for survival can be met by an ever-smaller L-wage, which lowers its bottom floor at the lower rungs of the labor market.

On a global scale, the more favorable labor markets were for long concentrated in North America and Northern Europe. These partly shifted to East Asia as a result of developmental state institutions in the mid- to late-twentieth century. Huge parts of South America, Africa, South and South-East Asia-home to a substantial portion of the world population-remain almost devoid of such developmental paths. The result has been manifested in a net migration flow from the Global South to the Global North. But also within individual countries, such as the United States or South Africa, the differential conditions of labor markets can be enormous, which gives rise to intracountry migrations and in the absence of strong labor unions undermines economic security.

In summary, the above factors affect the relative position of the worst-paid half of labor, which we should expect to exhibit considerable variation. We quantify this position in terms of the ratio of the median to the expected wage rate,

$$
\begin{equation*}
0<\frac{\bar{W}_{\text {median }}}{\mathrm{E} \bar{W}}<1, \tag{6.5}
\end{equation*}
$$

which is found to be in the range of $60 \%-85 \%$ in advanced capitalist economies. This ratio thus measures the inequality of wage rates and imposes a rather weak constraint on the distribution of L-wage rates. ${ }^{9}$

### 6.3 Control and Appropriation of Labor Effort

We have now considered two determinants of the wage distribution in terms of its tail end (6.3) and the position of the worst-paid half of labor (6.5). We now show how these determinants do not merely reflect the inequality among workers, but also affect the polarization between workers and non-workers whose primary incomes derive from property ownership.

[^54]
## Class Polarization

Recall that the average of the L-wage rate distribution, $\bar{E} \bar{W}$, is very close to the wage share of the value added in a capitalist market economy; see (6.2). Suppose that $E \bar{W}=0.50$, then the remaining half of the total value added is taken in the form of property incomes - profits, interest, and rent. But only a tiny portion of the population earns its living from property, which is mainly concentrated in the hands of a small class of capitalists and rentiers. ${ }^{10}$ It follows that the determinants of the L-wage distribution play a central role in determining also the overall level of income inequality in the economy.

By measuring wages in terms their L-content purchasing power, it becomes more transparent how income inequality is bound up with the control and appropriation of labor effort. Consider, for instance, a capitalist economy with ten million workers laboring on average eight hours per day. This amounts to approximately $\mathbb{L}=20$ billion worker-hours of labor added over a year that is controlled and directed by managers within firms. Now, if the average L-wage rate is $E \bar{W}=0.50$, half of the added labor content, about 10 billion worker-hours, can be appropriated by firms, capitalists, and rentiers through market exchange. They control to what extent and in what direction this collective labor effort materializes as additional equipment and new machinery, infrastructure, schools, and healthcare centers, or is frittered away in the form of luxury goods, domestic services, and mansions, not to mention hugely expensive weapons, military equipment, and private security services. In other words, $\mathbb{L}$ multiplied by $E \bar{W}$ quantifies a real global division of resources between working classes and propertied classes, measured in terms of worker-hours. But it emerges through a decentralized process of profit-driven firms that control the efforts of individual workers, and purchases by the main recipients of property income. The durations of work $T$ and the L-wage rates $\bar{W}$ at each firm are thus two principal factors through which the material interests of workers and the owners of capital clash. We turn next to two key determinants of the global division.

## Relative Stability of Wage Share

Writing in mid-nineteenth-century England, Marx estimated the wage share to be approximately $50 \%$, or in other words $E \bar{W}=0.50$, which we used above. ${ }^{11}$ More than a century and a half later, there appears to be relatively little change from this level. Indeed, despite significant local variations of labor markets and workers’ bargaining powers, the wage shares across a wide range of national economies have remained fairly stable and rarely exceeded $65 \%$ nor dropped below $40 \% .{ }^{12}$ Standard

[^55]

Fig. 6.2 Relative stability of the wage share. The horizontal axis shows a wide range of values for the ratio $\bar{W}_{\text {median }} / E \bar{W}$. This ratio quantifies the relative position of the worst-paid half of labor. The dotted line shows the wage share that follows when the tail proportion $\mathrm{P}(\bar{W}>1)$ is fixed to $12 \%$. The blue region shows a band of wage shares compatible with a tail proportion in the range $10 \%$ (bottom dashed line) to $15 \%$ (top dashed line). Note that over a typical span of wage equality levels $(60-80 \%)$, the range of compatible wage shares is compressed to a range around $50-60 \%$, or $0.50 \leq \mathrm{E} \bar{W} \leq 0.60$
economic theory has even gone so far as taking a 'constant' wage share as a stylized fact known as 'Bowley's law'. ${ }^{13}$ But what accounts for the relative stability of the wage share amid all structural changes? And what determines its range of variation?

Our conjecture is that the constraints on the wage distribution considered in (6.3) and (6.5) impose fairly narrow constraints on $E \bar{W} .{ }^{14}$ Suppose the employees earning wage rents at the tail end of the distribution constitute a stable minority. Let us assume that the proportion of labor time earning these rates is in the order of $10-15 \%$. Then, under the hypothesized log-normal distribution, it follows that the wage share given by $E \bar{W}$ remains in a fairly narrow band around $50-60 \%$, even as the position of the worst-paid half of labor (6.5) changes significantly! We illustrate this result in Fig. 6.2. ${ }^{15}$

Under the conjecture above, the primary path through which firms and owners of capital increase their collective share of aggregate output is by increasing wage inequality. Specifically, this occurs through the weakening of the position of labor, so that workers at the bottom half of the distribution find it harder to counteract the downward pressures on L-wages discussed above. As their relative position (6.5) declines, capital's share of income rises. Thus, mounting levels of income inequality in a capitalist market economy is bound up with the degradation of labor. ${ }^{16}$

When workers at the bottom half of the wage distribution exert effort against the disorder of competitive labor markets and organize a collective bargaining power, they have a chance of improving their conditions. In doing so, they intensify the

[^56]conflict between their material interests and those of the propertied classes. The workers at the bottom end do not, however, have interests that readily align with the earners of wage rents at the tail end, who benefit precisely from the scarcity of their credentials or their loyalty to the owners of capital.

### 6.4 Remarks on Relation to Theory of Exploitation

The use of the L-wage rates to analyze the extraction and control of labor effort is related, of course, to the analysis presented by the early labor movement. Marx famously labeled the process by which added labor content is appropriated by capitalists as exploitation and defined a quantitative 'rate of exploitation' (or 'rate of surplus value'), which is related to the expected L-wage rate:

$$
\begin{equation*}
\text { Rate of exploitation }=\frac{1-\mathrm{E} \bar{W}}{\mathrm{E} \bar{W}} . \tag{6.6}
\end{equation*}
$$

This rate is thus obtained by a mere arithmetical transformation of the wage share. For instance, when the share is $50 \%$, the rate above equals $100 \%$.

An important difference between our analysis and that of a more traditional labor theory of value is that we do not posit some hypothetical 'value of labor power'. Instead, we take the irreducible heterogeneity of wages as fundamental and quantify wages in terms of their L-content purchasing power. We take there to be a global 'pool' of L-content produced by the entire workforce, which is locally allotted to workers and appropriated by firms and owners of capital via market exchange. Unlike the presentation in Marx (1867), we do not project the global economy and its workforce onto the model of a single 'factory'.

One corollary can be derived if we consider that the aggregated net savings of the entire workforce is very small. ${ }^{17}$ Then total wage income is spent on a large collection $\mathcal{B}_{\text {workers }}$ of products, so that the wage share equals

$$
\omega=\frac{M\left(\mathcal{B}_{\text {workers }}\right)}{M\left(\mathcal{B}_{\text {out }}\right)} \simeq \frac{L\left(\mathcal{B}_{\text {workers }}\right)}{L\left(\mathcal{B}_{\text {out }}\right)},
$$

using (3.12). Thus the average L-wage rate is close to the labor share of (net) output in (2.1):

$$
\begin{equation*}
\mathrm{E} \bar{W} \simeq \frac{L\left(\mathcal{B}_{\text {workers }}\right)}{L\left(\mathcal{B}_{\text {out }}\right)} . \tag{6.7}
\end{equation*}
$$

[^57]
### 6.5 Empirical Data on Wages

In this section we present some data on wages and incomes across the world, in the last half century. The central points and conclusions discussed above are demonstrated using standard data sources, such as the International Labor Organization (ILO). Note that in what follows, real wages are wages adjusted for inflation.

The presentation of the available data addresses two principal points. First, the range within which L-wages can move. Second, the degree of flexibility within these bounds. An open question that requires further study is the socioeconomic forces through which the bounds are imposed. ${ }^{18} \mathrm{We}$ also address the relative stability of the global wage share $\omega$ for people employed by firms (as opposed to being selfemployed).

We take the range of L-wages,

$$
0.25<\bar{W}<0.75
$$

to hold for a large majority of workers worldwide. But wage distributions can also be considered locally in an economic region such as North America or China. In this consideration, the actual wage range in each region is narrower, as shown by Guerriero (2019).

Below we will first look at the global wage share and then turn to regional changes in wages over the last half-century. We conclude with a discussion on the minimum L-wage in the United States.

## Relative Stability of the Global Wage Share

The division of value added between firms and workers can be considered on various scales, regional and global. The data depends also on the definition of labor share: whether the income of the self-employed is part of the labor share or not. At least $10 \%$ of working persons are self-employed. Here we focus on workers employed by firms and their wage share $\omega$.

Guerriero (2019, pp. 6-7) provides estimates of $\omega$ for many countries for the 45 -year period 1970-2015, and finds that its average value is $49.7 \%$ (and a median value of $49.0 \%$ ), in other words very close to the $50 \%$ value used in our theoretical analysis. Note, however, that the variability among workers is very pronounced and $E \bar{W}=0.50$ is far from reflecting the real-life condition of workers with less than median wages. This requires further study.

[^58]

Fig. 6.3 A bar diagram comparing employee compensation to gross value added in the UK, 2013. The ratio hovers around 0.5 , one half, with several deviations, e.g., in the real estate sector where gross value added is a poor indicator of added labor content. Source UK, Office of National statistics, October 2015

## Selected Countries

Data on wage shares is abundant and here we can only provide a few selected samples. ${ }^{19}$ They all point to variations of L-wages within stable bounds.

Sweden: Söderberg (2010, pp. 458-9) shows that L-wages for popular layers of the working class have been remarkably stable over the period of intense capitalist development in the twentieth century, from 1900 to 2000, as nominal wages kept pace with both price inflation and productivity growth.

United Kingdom: In Fig. 6.3, we see the wage shares within large sectors of the UK economy, which vary around the $50 \%$ mark. The greatest deviations from this level occur in the government and real estate sectors, where we have no reason to believe value-added operates as in other sectors. Note, however, that the wage share for the whole economy is still close to $50 \%$.
China: Qi (2014, p. 2, Fig. 1.1) shows that throughout 30 years of rapid state-led capitalist growth, the wage share $\omega$ varied between $45 \%$ and $55 \%$. There has been a

[^59]substantial increase in wages-multiplying the average real wage five-fold over 11 years. Despite this rapid growth, the average L-wage rate has been rather stable, as real wages have paralleled the growth in productivity, converging toward the world average of $E \bar{W}=0.50$. The rise came partly as a result of persistent struggles by the Chinese working class, with roughly a million labor court disputes per year. ${ }^{20}$

## Regional Variations in Wages

Within the relatively stable global wage share of approximately $50 \%$, there are of course important considerable variations across regions and time. Large segments of the older, developed regions have experienced wage stagnation over the last 50 years or so. ${ }^{21}$ By contrast, wages grew considerably faster in the newer capitalist countries in Asia and some parts of Africa. ${ }^{22}$

Figure 6.4 shows the movement of real wages over a short period of 5 years in various world regions. It captures the stable and relatively higher wage growth in Asia and East Europe, as compared to developed capitalist countries. It also illustrates the effects of the economic crisis of 2008, which brought about a more rapid convergence of wages as workers in older capitalist countries suffered falling L-wages. Their prior, historically given, wages reflected in part the local concentration of high skills and modern production techniques. The battering of their bargaining power by the crisis and rise of the East Asian economies have together eroded their relative advantage among workers worldwide. Note that worldwide real wages have been rising by $2-3 \%$ per year, in line with the rate of decreasing labor content for sample baskets, so as to maintain the approximate 1:1 division of added labor content between employees and firms. This has occurred while the developed as well as Middle Eastern economies experienced regional wage stagnation.

We can observe the effects of these macroeconomic shifts if we look at the growth of real incomes on a longer time scale, from 1970 to 2010. Figure 6.5 reproduces the so-called 'Elephant Curve' from Milanovic (2019) ${ }^{23}$ and shows the changes in real incomes across the global income distribution. We see that incomes at both ends of the global distribution have grown much faster than in the 75-90th percentiles, which are nearly stagnant and by and large represent the older capitalist economic regions.

The reasons in terms of labor power are not hard to work out. In the era of globally integrated capitalist production, enabled by the cheap movement of goods and efficient means of communication, the allocation of the workforce should be viewed from a global perspective. The regional concentration of advanced production technologies and matching skills has been altered on a massive scale. Capital,

[^60]

Fig. 6.4 Real wage growth in various regions. The variation by regions reflects both loss of technological monopoly by older centers and variation in changes in material productivity, due partly to the growing share of the service sectors. Source ILO (2015)


Fig. 6.5 Real growth of global incomes over four decades (Lawson, 2017; Milanovic, 2019). The horizontal axis shows the relative income ranks arranged from the poorest to the richest of the global population. The vertical axis shows the real growth of incomes that a given rank accrued from 1970 to 2010. The 20th-70th percentiles of the global income distribution have experienced real income increases of $40-80 \%$, and this is also the case for the top percentile. However, the 75th90 th percentiles have experienced nearly stagnant incomes. These comprise a substantial portion of workers in the once-dominant Global North, whose wages have evidently not kept pace with the global rise in material productivity. Consequently, their L-wages have been falling
both financial and material, has moved to new, vastly dispersed areas, so as to take advantage of the local reserves of labor, without which it cannot preserve or improve its economic position, as will be explained in Chap. 7. This displaced old production centers, creating many new ones away from the Global North. Workers concentrated in East Asia have acquired skills that had previously been monopolized by the older, more advanced capitalist regions of the Global North. The decimation of these older cohesive production centers-eroding their workers' bargaining power in the process-has been accompanied by a significant shift of employment toward service sectors. ${ }^{24}$ The result has been a substantial fall of L-wages for large sections of workers.

## Minimum L-Wage Rates in the US

In the US, the federally mandated (real) minimum wage rate was $\$ 7.25$ dollars per hour in 2019. This was $31 \%$ lower than the rate 50 years earlier. ${ }^{25}$ This basic fact tells us something about the workers at the bottom of the US labor market. The biologically determined food and shelter basket allotted to many millions of lowest paid US workers in 2020 was the same as in $1940 .{ }^{26}$ At the average rate of LDLC, we estimate the labor content of this basic minimal basket is cut by half every 20-30 years.

Thus, despite all economic growth, the legal minimum wage measured in Lcontent purchasing power has been persistently declining. In 2021, it corresponds roughly to an L-wage rate of $\bar{W}=0.15 .{ }^{27}$ In other words, workers who earn this rate may work 40 hours in a week, but receive a pay that can purchase diverse products corresponding to only 6 worker-hours. This confirms the basic claim by labor-movement economists that a bottom floor of wages exists in which the workforce is barely kept alive to satisfy the needs of firms and the propertied classes.

[^61]
## Chapter 7 <br> Limits to Growth and Accumulation

We have seen how the profit imperative propels the capitalist economy toward exponential growth, while the law of decreasing labor content (LDLC) has made this tendency a material reality, albeit periodically punctuated by severe economic dislocations, depressions, and wars. The real growth of monetary incomes and wealth are central to the institutions of capitalist market economies.

What are the limits to this growth in capitalism? Why is it unevenly spread across economically integrated regions of North America, Europe, and East Asia? Why has it stagnated in once rapidly expanding regions? Why is only a limited portion of the growing output reinvested into greater productive capacity? Are there any bounds to the level of material wealth accumulated from this growth? What constrains the growth of financial assets that are recorded in bankbooks and on computers?

These are questions that concern the future path of capitalist development. In this chapter, we will attempt to shed light on them using the concept of labor content. Price-based models of aggregate output and productivity offer no explanations derived from the real material constraints in the sphere of production. ${ }^{1}$ Indeed, in standard economic theory, growth can be achieved by improving the 'productivity' of any 'factor of production'-including capital! By contrast, our starting point is labor as the primary source that powers capitalism; not just people using their laboring capacity in local workplaces, but also their added labor content that is accessible through global market exchange. We conclude the chapter with an assessment of empirical evidence on growth.

[^62]
### 7.1 Growth of Output and of Workforce

Let $\mathcal{B}_{t}$ denote the basket of products that constitute the (net) output of the capitalist sector in year $t$. Then the growth rate of output is conventionally measured as the following rate

$$
\begin{equation*}
\frac{M\left(\mathcal{B}_{t+1}\right)-M\left(\mathcal{B}_{t}\right)}{M\left(\mathcal{B}_{t}\right)}, \tag{7.1}
\end{equation*}
$$

using real prices. The growth rate fluctuates over business and inventory cycles. In certain regions, the local rate may also change dramatically due to contingent factors such as sharp shifts in global demand of locally produced product-types, natural disasters, and armed conflicts. Over longer periods, the growth rate (7.1) averages to about $1-3 \%$ per year in most advanced economies. Countries that have sustained longer periods of high growth rates have been the envy of the world, e.g., the United Kingdom, the United States, and Germany at the time of their zenith. During the second half of the twentieth century, East Asian economies, such as Japan, Taiwan, South Korea, and China, experienced very rapid growth rates.

Conversely, faltering growth rates in old capitalist economies continue to bedevil social commentators and pundits. To quote a recent example:

More than 151 million Americans count themselves employed [...] The question is this: What are they doing all day? The number of hours Americans worked rose 1.9 percent in the year ended in March. New data released Thursday showed that gross domestic product in the first quarter was up 1.9 percent over the previous year. Despite constant advances in software, equipment and management practices to try to make corporate America more efficient, actual economic output is merely moving in lock-step with the number of hours people put in, rather than rising as it has throughout modern history. ${ }^{2}$

Meanwhile, output growth in China was more than three times the rate in the US. Why can't the growth of the old economies match the growth of the newly industrializing ones? In 2012, several Nobel laureates and 'other leading thinkers'- including twotime president George W. Bush-assembled to address the problem of 'slow' growth, writing that they

[^63]The question of growth is better understood in terms of the ultimate input of labor. Using the average specific price $\mathrm{E} \Psi$ as a bridge between price and labor content of sample baskets (3.12), the monetary quantities in (7.1) are well-approximated by

$$
\begin{equation*}
M\left(\mathcal{B}_{t+1}\right) \simeq \mathbb{L}_{t+1} \cdot \mathrm{E} \Psi_{t+1} \quad \text { and } \quad M\left(\mathcal{B}_{t}\right) \simeq \mathbb{L}_{t} \cdot \mathrm{E} \Psi_{t} \tag{7.2}
\end{equation*}
$$

[^64]where $\mathbb{L}_{t}$ is the labor added in year $t$. This quantity of worker-hours grows at a rate $\ell$ (which may be negative) so that we may express the labor added during the subsequent year as
\[

$$
\begin{equation*}
\mathbb{L}_{t+1}=(1+\ell) \mathbb{L}_{t} . \tag{7.3}
\end{equation*}
$$

\]

Leaving aside any significant changes of the average number of work hours per worker, $\ell$ is essentially the growth rate of the workforce in the capitalist market economy. The average specific price, $\mathrm{E} \Psi_{t}$, also changes from one year to the next. Using (5.10), with real rather than nominal prices, we can express the change in a simpler form as

$$
\begin{equation*}
\mathrm{E} \Psi_{t+1} \simeq \frac{1}{1-\delta} \mathrm{E} \Psi_{t} \tag{7.4}
\end{equation*}
$$

By inserting (7.4) and (7.3) in (7.2), the growth rate in (7.1) can be expressed by the following approximation: ${ }^{4}$

$$
\begin{equation*}
\ell+\delta \tag{7.5}
\end{equation*}
$$

which is the sum of two different rates: the growth of the workforce and the decreasing labor content. This is a basic relation that eschews any need to measure quantities of other putative 'factors of production'. ${ }^{5}$

Let us return to the examples of US and Chinese economies in the light of the basic growth rate formula (7.5). As we have seen in Sect. 5.2, LDLC leads to a shift of employment toward labor-intensive services. When the share of such employment rises in a regional economy, its local basket of outputs will be subject to ever-tighter limits on $\delta$ due to technical limitations of labor-saving changes. Moreover, as the bargaining power of workers is weakened, the incentives to invest in labor-saving technologies-to the extent that they exist-are also weaker. In the example quoted above, the labor added grew by $1.9 \%$; and if the average rate $\delta$ approaches zero, the growth rate is constrained to

$$
0.019+0=1.9 \% \text { per year. }
$$

Under such limiting conditions on material productivity, output growth is simply constrained by the socio-demographic factors that determine the growth of the workforce $\ell$. In China, the workforce grew far more rapidly due to the massive scale of urbanization and migration from the rural regions. The urbanization rate in 1980 was about $20 \%$ of the population and tripled over the subsequent four decades. ${ }^{6}$ During the same period, the overall population increased by nearly $50 \%$, so that the growth of the workforce $\ell$ was roughly $4 \%$ per year. Moreover, Chinese labor was shifting toward mechanized and industrial production processes, with a postulated average rate of decreasing L-content around $\delta=2.7 \%$ per year. Then (7.5) yields

[^65]

Fig. 7.1 Range of output growth rates compatible with growing workforce, using (7.5). The horizontal axis shows a range of values for $\ell$. Note that in the advanced economies, $\ell$ is lower than $2 \%$ per year, whereas the Chinese economy expanded its workforce at nearly $4 \%$ per year. The dotted line shows the resulting output growth rate when the rate of decreasing labor content $\delta=2.7 \%$ per year. The shaded region shows the range of output growth rates when varying $\delta$ from $0 \%$ per year (bottom dashed line) to 5\% per year (top dashed line)

$$
0.04+0.027=6.7 \% \text { per year, }
$$

that is, a growth rate that is three and a half times greater than that derived above for the US.

Given the bounds on $\delta$, we can now conclude that such rapid rates of output growth are demographically unsustainable in mature capitalist economies! After depleting labor reserves from non-capitalist agriculture and petty production, the growth of the workforce in the capitalist sector can simply not exceed population growth for very long. Figure 7.1 provides an illustration of the range of output growth rates compatible with a growing workforce. Population growth in large regions, such as the United States and Europe, is below $0.5 \%$ per year due to declining fertility and thus aging populations. Locally, in countries such as Italy or Japan, the populations are even shrinking. On a global scale, the population growth rate is about $1 \%$ per year, which means that the global output growth is bounded to stay below $4 \%$ per year.

Tightening growth constraints do not merely slow down the growth of material living standards, but they also affect the path of development. Persistent growth has the important effect of displacing immediate economic trade-offs and conflictsbetween wages and profits or consumption and investments-across time. For instance, an organized collective of workers could choose to restrain wage demands under the assumption that more profits will be reinvested to expand productive capacity. The resulting growth could then yield greater real wage gains in the future. ${ }^{7}$ But as the growth constraint tightens, such relative gains become less plausible which sharpens conflict over distribution.

[^66]
### 7.2 Accumulation of Productive Capital

A macroeconomist steeped in standard theory may wonder what role capital plays in the basic relation (7.5). We contend that productive capital-the stock of machines, equipment, and buildings that the workers use-can play no role that is quantitatively independent of, or substitutable for, labor. Rather, this basket of heterogeneous products, denoted by $\mathcal{B}_{\text {capital }}$, primarily enables and enhances the productive capacity of labor in the capitalist sector.

For its workforce to grow at a rate $\ell$ and labor content of large sample baskets to fall at a rate $\delta$ in (7.5), a certain share of the labor content added each year must be accumulated as investments in productive assets rather than being consumed. An increased per-capita demand for many investment goods-such as steel, pipelines, aircraft, excavators, and so on-also acts as an important factor of employment growth. ${ }^{8}$ Capital accumulation is, however, determined by uncoordinated firms with owners that demand 'decent' rates of return on assets. Let us therefore study the limits to accumulation set by the profit imperative.

For notational convenience, let

$$
\mathbb{K}=L\left(\mathcal{B}_{\text {capital }}\right)
$$

denote the L-content of the total capital stock used in production and let $\alpha$ be the relative share of labor added $\mathbb{L}$ that is accumulated in the form of new investment goods. The accumulation share $\alpha$ is of course bounded by $1-\omega$, i.e., the share of value added that accrues to firms and the owners of capital. That is, we have the following bounds on the accumulation share:

$$
\begin{equation*}
0<\alpha<\alpha_{\max }=1-\omega \simeq 50 \% \tag{7.6}
\end{equation*}
$$

As we have seen, the upper bound remains fairly stable across time over large regions. The expected profit rate averaged across all capital shown in (3.14) can thus be expressed as

$$
\begin{equation*}
\mathrm{E} R \simeq \alpha_{\max } \cdot \frac{\mathbb{L}}{\mathbb{K}} \tag{7.7}
\end{equation*}
$$

We will now study how this relation constrains the distribution of profit rates dynamically.

If $\mathbb{K}_{t}$ is the L-content of the capital stock at year $t$, then after wear-and-tear and accumulation, its L-content in the following year equals

$$
\begin{equation*}
\mathbb{K}_{t+1}=\alpha \mathbb{L}_{t}+(1-\delta)(1-d) \mathbb{K}_{t}, \tag{7.8}
\end{equation*}
$$

where the first term is the newly accumulated labor content and the second term reflects the material depreciation at the rate $d$, as well as the decreasing L-content

[^67]of products (see also Sect. 5.2). The average profit rate in year $t+1$ is
$$
\mathrm{E} R_{t+1}=\alpha_{\max } \cdot \frac{\mathbb{L}_{t+1}}{\mathbb{K}_{t+1}}
$$
where $\mathbb{K}_{t+1}$ and $\mathbb{L}_{t+1}$ can be replaced by the expressions in (7.8) and (7.3), respectively. Then, after some rearranging, we find that the average profit in year $t+1$ is a function of the average rate in the previous year, which we can express as
\[

$$
\begin{equation*}
\mathrm{E} R_{t+1}=f\left(\mathrm{E} R_{t}\right) \tag{7.9}
\end{equation*}
$$

\]

It can then be shown ${ }^{9}$ that at any given steady accumulation share $\alpha$, the average profit rate is dynamically constrained to converge toward its steady-state value, approximated by

$$
\begin{equation*}
\mathrm{E} R \simeq \frac{\alpha_{\max }}{\alpha}(\ell+\delta+d) \tag{7.10}
\end{equation*}
$$

What does this dynamic constraint imply? We see that while a high accumulation share $\alpha$ enables rapid growth of output, it has (other things being equal) the paradoxical effect of depressing the distribution of profit rates as $E R$ is lowered. This is because the average rate of profit (7.7) is inversely proportional to the labor content of the total capital stock.

To illustrate this point, let us contrast again mature capitalist regions, such as North America and Europe, with industrializing regions, such as China. In the former regions, the accumulation share $\alpha$ has stayed around $20 \%$ of labor added for more than a decade, while in China it remained above $40 \% .^{10}$ In the mature regions, the workforce growth hovers around $1 \%$ per year and $\delta$ is in the range of $1-2 \%$ per year. Then the average profit rate has a steady-state level at

$$
\mathrm{E} R \simeq \frac{0.50}{0.20}(0.01+0.015+0.05)=18.8 \% \text { per year }
$$

assuming a rate of material depreciation at 5\% per year. Suppose instead that the accumulation share were to match that of China. This would cut the average rate of return in half, so that more than half of the capital invested in the economy would earn profit rates below

$$
\mathrm{E} R \simeq 9.4 \% \text { per year, }
$$

given that the distribution of profit rates is skewed. While a substantial portion of the capital would still yield 'decent' rates of return, other non-productive assetssuch as financial and real estate assets-would appear increasingly attractive, to the point of lowering the share $\alpha$ that is accumulated as additional capital goods.

[^68]

Fig. 7.2 Steady-state level of average profit rate $E R$ in (7.10) versus accumulation share $\alpha$. Most mature capitalist economies exhibit $\alpha$ in the range of $20-25 \%$. The dotted line shows the steady-state level when L-content of products decreases at the average rate $\delta=2.7 \%$ per year and the workforce grows at $2 \%$ per year. The region in magenta shows the range of $E R$ as the workforce growth ranges from $0 \%$ per year (bottom dashed line) to $4 \%$ per year (top dashed line). The region in pink shows the same range but when $\delta$ has slowed down to $0 \%$ per year. Throughout, we assume that the material depreciation rate is 5\% per year and use (7.6)

By contrast, consider an industrializing economy like that of China. With a rapidly growing industrial workforce at a rate of $\ell \simeq 4 \%$ per year, we see that it can sustain an extraordinarily high accumulation share of $\alpha=40 \%$ at an average profit rate of

$$
\mathrm{E} R \simeq \frac{0.50}{0.40}(0.04+0.027+0.05) \simeq 14.6 \% \text { per year } .
$$

In Fig.7.2, we illustrate the range of steady-state average profit rates that arise from capital accumulation with a growing workforce and decreasing labor content of products. We can now infer a dynamical limit to accumulation and growth: When the growth of the workforce becomes constrained by slow population growth and employment shifts toward labor-intensive service sectors, a high share of accumulation becomes unsustainable under the profitability criterion. The appropriated share of labor added that is not accumulated as productive capital fritters away instead in the hands of the propertied classes.

### 7.3 Accumulation of Material Wealth

A signature of capitalist growth is the increasing amount of private wealth that is amassed. The very premise of market dependence entails the concentration of this wealth in the hands of a few private institutions, firms, and households. In nominal terms, there is no apparent limit to the amount of wealth that an individual can accumulate, whether as durable assets or as numbers on a bank record. But what are the real constraints to this process?

We begin with the collection of material assets accumulated via market exchange, which in addition to productive capital stock include housing stock, precious metals, and other produced durable resources. We let $\mathcal{B}_{\text {wealth }}$ denote the basket of these products and derive a constraint on the size of the total wealth $M\left(\mathcal{B}_{\text {wealth }}\right)$.

First, we consider the L-content of this basket, denoted as

$$
\mathbb{W}=L\left(\mathcal{B}_{\text {wealth }}\right)
$$

and now let $\alpha$ be the total share of new labor content that is accumulated as material assets. The L-content in a given year $\mathbb{W}_{t}$ then changes to

$$
\begin{equation*}
\mathbb{W}_{t+1}=\alpha \mathbb{L}_{t}+(1-\delta)(1-d) \mathbb{W}_{t} \tag{7.11}
\end{equation*}
$$

in the following year (compare (7.8)). Next, consider the ratio $\mathbb{W}_{t} / \mathbb{L}_{t}$ of accumulated L-content to the new L-content added. To obtain its numerical value in the subsequent year, we divide (7.11) by $\mathbb{L}_{t+1}$. It can then be shown, using the growth rate of the workforce (7.3), ${ }^{11}$ that for any given accumulation share $\alpha$ the ratio has a steady-state level that is well approximated by

$$
\begin{equation*}
\frac{\mathbb{W}}{\mathbb{L}} \simeq \frac{\alpha}{\ell+\delta+d} \tag{7.12}
\end{equation*}
$$

This means that the L-content accumulated as wealth through time cannot exceed a certain multiple of the labor added each year.

Now note that the total accumulation share on a world scale is of the order of $\alpha=25 \%$. Then an upper bound on the ratio $\frac{\mathbb{W}}{\mathbb{L}}$ is obtained when the growth of the workforce and the decrease of L-content are both zero, i.e.,

$$
\begin{equation*}
\frac{\mathbb{W}}{\mathbb{L}} \leq \frac{0.25}{0.05}=5 \tag{7.13}
\end{equation*}
$$

assuming a material depreciation rate of 5\% per year. We now connect this result to the total wealth in monetary terms. Since $\mathcal{B}_{\text {wealth }}$ is a sample basket of heterogeneous products, the relation between labor content and price (3.12) implies that the ratio is well approximated by

$$
\frac{\mathbb{W}}{\mathbb{L}} \simeq \frac{M\left(\mathcal{B}_{\text {wealth }}\right)}{M\left(\mathcal{B}_{\text {out }}\right)}
$$

Then, using (7.13), we have the upper bound on wealth:

$$
\begin{equation*}
M\left(\mathcal{B}_{\text {wealth }}\right) \leq 5 \cdot M\left(\mathcal{B}_{\text {out }}\right) \tag{7.14}
\end{equation*}
$$

[^69]In other words, the value (in real price) of material wealth accumulated through the capitalist sector cannot exceed five times the real price of its aggregate output per year, under the assumed rate of material depreciation. ${ }^{12}$

### 7.4 Accumulation of Financial Assets

Financial assets constitute a fundamentally different but central form of wealth in capitalism. They comprise stocks, bonds, bank deposits, loans, and other contracts that yield unearned incomes in the form of interest, dividends, and rents. They do not depreciate materially, as do real produced assets, and their yield is only indirectly connected to the immediate production and sale of commodities produced by workers in firms.

The returns on financial assets have no apparent limits and seem rather to be governed by the scarcity of credit or other market factors. But over nearly three centuries of capitalist development, average interest rates have not deviated far from the rates found in records from ancient Rome! ${ }^{13}$ How are we to understand this constraint on financial profits? To provide a basis for analysis, let us consider a financial asset $F$ that yields returns through interest, dividend, or rent claims. In a given year, it has a nominal price of $M(F)$ and yields a rate of profit $r$ such that the asset plus profit income in the following year equals $(1+r) \cdot M(F)$. From a macroeconomic vantage point, the rate $r$ is of course indeterminate and unpredictable. We will derive here some basic constraints on the average profit rate over a large collection of financial assets.

In terms of purchasing power, the asset commands

$$
\frac{M(F)}{\mathrm{E} \Psi_{t}}
$$

worker-hours of L-content in market exchange. Let us now consider a large portfolio of financial assets, denoted by $\mathcal{F}$, with an average rate of profit $\bar{r}$. ${ }^{14}$ Suppose its total value is $\$ 10,000,000$ and the average specific price in the economy was $\$ 50$ per hour, then it amounts to

$$
\frac{10000000}{50}=200000 \text { worker-hours. }
$$

This total L-content purchasing power changes from one year to the next as

$$
\begin{equation*}
\frac{M(\mathcal{F})}{\mathrm{E} \Psi_{t}} \rightarrow(1+\bar{r}) \cdot \frac{M(\mathcal{F})}{\mathrm{E} \Psi_{t+1}}, \tag{7.15}
\end{equation*}
$$

[^70]which corresponds to a growth rate of
\[

$$
\begin{equation*}
(1+\bar{r}) \cdot \frac{\mathrm{E} \Psi_{t}}{\mathrm{E} \Psi_{t+1}}-1 \tag{7.16}
\end{equation*}
$$

\]

per year. To express this rate in a more convenient form, ${ }^{15}$ let $p$ denote the price inflation of a large sample basket $\mathcal{B}$. Then we can approximate ${ }^{16}$ the growth rate of L-content purchasing power in (7.16) by

$$
\begin{equation*}
\bar{r}-\delta-p \tag{7.17}
\end{equation*}
$$

In other words, the portfolio of financial asset $\mathcal{F}$ has a purchasing power over Lcontent that grows at the approximate rate (7.17), which is the result of three different rates: the rate of profit $\bar{r}$ minus the rates of decreasing labor content $\delta$ and of average price inflation $p$. This clearly constrains their aggregate profit rate $\bar{r}$ in two ways. First, if the rentier share of aggregate output is not to decline, $\bar{r}$ must exceed $\delta+p$ in (7.17), or else the purchasing power over the newly added labor content would vanish over time. Second, this purchasing power cannot persistently exceed the growth of new labor added per year. The power acquired by rentiers cannot, after all, claim new labor content greater than $(1-\omega) \mathbb{L}$ each year. But the labor added increases at a rate $\ell$ in (7.3), which must bound the financial growth rate (7.17) from above. These facts impose together the following constraint on the average rate of profit

$$
\begin{equation*}
\delta+p \leq \bar{r} \leq \ell+\delta+p \tag{7.18}
\end{equation*}
$$

If we assume the growth rate of labor added equals the global population growth of $1 \%$ per year and use $\delta=2.7 \%$ and a price inflation of $2 \%$ per year, typical of many mature regions, then (7.18) yields

$$
4.7 \% \leq \bar{r} \leq 5.7 \% \text { per year. }
$$

Thus, while apparently limitless, the accumulation of financial assets is ultimately constrained by the flow from the source that powers capitalism.

From this point of view, financial assets are an entitlement to present and future labor content. Each debt-based asset held by rentiers has an equivalent liability held by firms, households, or states. The debt is settled over time (often indefinite) through the transfer of purchasing power to the rentiers. Some of the most powerful rentiers are of course private banks. They create pairs of credit and debt that temporarily enable individual firms to expand their operations; households consume beyond their current cash constraints; and states expend non-sovereign money. But the resulting interest payments in return enable the rentiers to appropriate massive amounts of labor content each year. If the entitlement to future L-content is breached by debtors

[^71]Table 7.1 Employees' share in total workforce, given in percent points, in 1999 and 2013, respectively. Notice that the change reflects the ratio of the two percent figures, thus, e.g., a growth from 20 to 30 percentage points is $a+50 \%$ growth of share of employed persons. (ILO data)

| Region | 1999 | 2013 | Approx. proportional <br> change, in \% |
| :--- | :--- | :--- | :--- |
| Africa | 24.6 | 26.2 | 7 |
| Asia | 30.7 | 35.4 | 15 |
| Latin America and the <br> Caribbean | 59.0 | 62.8 | 6 |
| Middle East | 71.9 | 80.3 | 12 |
| Eastern Europe and <br> Central Asia | 74.9 | 78.3 | 5 |
| Developed economies | 84.1 | 86.4 | 3 |

who fail to transfer this tribute, legal institutions or brute state coercion are set in motion to enforce the contracts. This has been exemplified in numerous debt-crises of the Global South and, recently, in Greece. ${ }^{17}$

### 7.5 Empirical Evidence on Growth and Accumulation

In this section we present data that illustrates the key determinants of growth and accumulation. We will use Gross Domestic Product (GDP) as a proxy measure of the aggregate output of the capitalist sectors. ${ }^{18}$

## Global Growth and Growth of Regional Workforces

Let us start with figures concerning global growth in the decade leading to the year 2015. The principal claim we made in Sect. 7.1 is that the yearly growth of net output product cannot exceed the growth of working population by more than the rate $\delta$ of decrease in labor content. Our overall estimate $\delta$ in Chap. 5 was $2-3 \%$.

Figure 7.3 shows the global growth of output during a period of 9 years. Using the end-points of the curve, we estimate the average growth rate to be $3.5 \%$ per year. Noting that the growth rate of the world population is slightly above $1 \%$ a year, this is entirely in line with our estimate of an upper bound of the rate of growth: $\ell+\delta \simeq 0.01+0.027=3.7 \%$ per year.

[^72]World Gross Domestic Product in \$US Trillion 2007-2015


Fig. 7.3 Global economic output measured as a gross domestic product in $\$$ US trillion. On such a large scale, the global rate of the fall in labor content, coupled with roughly $1 \%$ average population growth, gives an excellent estimate of the growth in world economic output. Similar correspondence holds on longer time scales, and in large regions in general. Compare (statista, 2021)

## Share of Employed Workforce

The rate of participation in the employed workforce is important for understanding and estimating the present and newly created products and possibilities for further growth. It is important to not conflate population growth with a growing workforce employed in capitalist market production. In the second half of the twentieth century, there was a large movement of population in Asia, Africa, and elsewhere from subsistence, mostly rural living, to a fast-developing capitalist production, so that in several important regions-such as China, India, Vietnam, and Indonesia-the workforce employed by capital grew considerably faster than the population. According to the labor theory of capitalism, this accelerated output growth rates.

Table 7.1 shows the rates of participation of working-age population in the employed workforce in various regions. The low rates in many African countries are notable in the first two columns. This allows for a greater potential of growth in the total product, which in fact is realized in several African countries. Note that the pace of the growth of the employed workforce is somewhat higher in the first four regions listed in the table, where the initial participation was relatively lower. For example, in Africa as a whole the average yearly growth during the 14 years is about $7 \%$, (since $26.2 / 24.6 \simeq 1.07$ ) whereas in Asia it grew by $15 \%$. These rates are results of many causes, such as increasing women's participation in the employed workforces, mass eviction and migration away from agricultural lands, reduced prices of some basic food products, weather changes, shortage of water in rural areas, wars, and other social, political, and economic factors-many of which, as often under capitalism, are results of disastrous dislocations.

## Relation Between Labor Hours and Growth

To continue the discussion above of the global rate of growth, we give several typical examples of the estimate, made in previous sections of this chapter, relating output per year expressed in monetary terms to the size of the employed working forces, in relatively mature capitalist regions. A more careful analysis of data relating to the total workforce is not made here, but would reveal similar upper bounds to growth.

Table 7.2 illustrates the very close relation between the size and growth of the number of working hours and the corresponding growth in total output. When considering a large region, the total number of hours worked, generating new labor content materialized in products, is largely sufficient to explain and predict observed growth. The data here is typical, as can be easily checked in the references cited below.

In mature capitalist regions, growth of output per working hour, measured in real (inflation-indexed) prices, is generally no more than the LDLC permits: $2-3 \%$. However, this rate is a global average. More detailed values depend on both region and era and are not necessarily constant in every region at every point in time. As we pointed out in Chap. 5, with the growth of the local service sectors the local rate of decrease in labor content may well also be reduced. As a result, overall gains in material productivity decrease; and eventually-in accordance with our postulatesso do gains in monetary-terms productivity.

Table 7.2 gives examples of regional output growth rates over the last decades leading to the year 2015. The table illustrates the bounds on these rates set by (7.5), namely the growth of the workforce and of material productivity. We see that the rate of fall of labor content, $\delta$, stays within the bounds discussed in Sect. 5.2, specifically (5.6). It was shown in (5.15) that this rate is closely related to the monetary growth of output in real prices. To corroborate (7.5), we use here an independent approximation of $\delta$ in a large region over long periods based on the method and discussed around Fig. 5.8 and (5.19). That is, the rate is approximated from real prices of personal services using home-care wages or teachers' wages as proxies. This gives at most a rough indication. Data on teachers' wages given in OECD (2021b) record an estimate of their changes. The Trends field of this reference shows differences between say France and Germany. This indicates the relatively slower pace of productivity gains in France as seen in Table 7.2. Trends in prices of home care show a relative decline in the growth of productivity also in the US and the UK. ${ }^{19}$

France and many other countries have moved toward a service economy in the last few decades, hence according to our account in Chap. 5 we would expect the local decrease in L-content to be considerably slower than the worldwide rate of $2-3 \%$. This is indeed the case. The history of the relevant parameters in France in the last $50-60$ years is given in (Durand, 2018) and in (Tradingeconomics, 2021). Similar trends are evident also in the US.

[^73]Table 7.2 Approximate values of principal growth parameters in selected countries, in the 20-30 years ending in 2015. All figures are in percentage points per year. The second and third columns report estimates of $\ell$ and $\delta$, respectively. The fourth column reports the growth of real output in monetary terms, which is strongly bounded by the growth rate of the workforce and the rate of decreasing L-content of sample baskets produced in a given region; see (7.5). Estimates using data from OECD (2021b), Tradingeconomics (2021), OECD (2021b), and FRED (2021). (The decline in yearly working hours is slow compared to the above rates, less than $0.1 \%$ per year, and is ignored here.)

| Country | Growth in workforce | Change in productivity | Output growth |
| :--- | :--- | :--- | :--- |
| Worldwide | 1.4 | $2-3$ | 3.5 |
| US | 1.5 | 1.0 | 2.5 |
| China | 7 | $2-3$ | $8-9$ |
| France | $\leq 0.5$ | 0.5 | $\leq 1$ |
| Sweden, Belgium | 0.6 | 2.5 | 3.1 |

Table 7.3 The data is given in trillions of 2020 US\$. The numerical ratio of total wealth to yearly aggregate net output is normally bounded by 4-5

| Country | Output (GDP) | Total wealth | Wealth to GDP ratio |
| :--- | :--- | :--- | :--- |
| World | 86.5 | 360 | 4 |
| US | 21 | 106 | 5 |
| China | 14 | 64 | 4.5 |
| Germany | 3.8 | 14.6 | 4 |
| France | 2.8 | 13.7 | 5.1 |
| Japan | 5.1 | 25 | 5 |
| Sweden | 0.5 | 2.5 | 5 |
| Taiwan | 0.6 | 4 | 6.6 |
| Switzerland | 0.7 | 3.9 | 5.5 |

## Accumulated Wealth

We conclude this section with a table demonstrating the relation calculated in formula (7.14), between the total material wealth of a large region and its net yearly product. Measuring wealth is not simple, because the standard estimates typically also contain financial assets and liabilities. Globally, however, the value of these pairs must sum to zero. Hence for a rough estimate, we can rely on standard sources. Looking at the level of global wealth in Table 7.3 , we see that it is approximately 4 times the aggregate output produced in a year. This corroborates our estimated upper bound in (7.14). Note that most developed countries have locally accumulated wealth in that order. In less developed countries, estimates are hard to come by. It turns out, however, that the ratio is bounded by a number smaller than 5 .

## Part III <br> Futures

## Chapter 8 <br> Limits to Capitalist Development

As the productive capacities of labor are enhanced, capitalist development exhibits a dual tendency of growth and inequality. As workers succeed in organizing and enhancing their bargaining power, they are able to improve material living standards on a mass scale. However, the global distributions of productive capacities and of employment opportunities are highly uneven. Both factors keep changing as a result of the law of decreasing labor content (LDLC) and lead to structural shifts of the global capitalist economy.

Building on the results from the second part of the book, this chapter highlights these shifts, which we argue raise the obstacles to improving the human well-being for an increasing share of the world population: Degradation of working conditions; widespread misery and despair; deteriorating social services and infrastructure; and environmental damage. We show here why the narrowing limits of development are inseparable from the institutions of capitalist market economies. We conclude that alternative development paths that can improve human well-being will require a different configuration of the political economy.

### 8.1 Obstacles to Improving Human Well-Being

Let us take stock of some basic variables in our analysis. We have argued that firms' profit imperative drives economic development toward exponential growth. The growth of output, however, is subject to certain demographic and technical constraints: On the one hand, it depends on the growth of the workforce employed in the capitalist sector. Once the rural reserves of labor are depleted, this workforce is constrained by overall population growth-which is low. On the other hand, the rate of decreasing labor content is constrained by technical limitations that differ across sectors. As we have seen in Sect. 5.2, these differentials lead to a persistent shift of employment toward more labor-intensive service sectors.

In what follows, we show that the exponential growth imperatives and the tightening constraints combine to deepen the problems of capitalist development on a global scale. These problems are subject to important regional variations but they result in undermining human well-being and flourishing for an increasing share of the population.

## Problems of Persistent Unemployment

In Sect. 5.1, we showed that unemployment is reproduced when the demand of various product-types does not balance the effects of LDLC. The anarchic and highly unequal allocation of purchasing power cannot ensure that these factors are balanced. Not only is unemployment persistent but the pool of unemployed swells and shrinks with the shifting demand for capital goods. However, as the demographic and technical constraints tighten, a significant rise of the share of capital accumulation becomes unsustainable under the profitability criterion. The resulting path of development is deepened unemployment and underutilization of skills.

In the US, over the past seventy years less than $65 \%$ of the working-age ( $25-70$ years old) adult population are employed, while in Argentina and Brazil, this rate is in the $50-60 \%$ range. In the EU, the rate is generally below $75 \% .^{1}$

The lack of gainful employment does not only cripple individuals and families, but also undermines the bargaining power of those workers who are employed. This results in worsening of working conditions-increased intensity, working hours, and job insecurity-and weak wage growth (especially at the bottom half of the L-wage distribution). The flow of ununionized migrant workers into saturated labor markets only aggravates their downward pressure. Of course, the global workforce did increase massively with the incorporation of the East Asian economies into global capitalism. But as industrial employment concentrated in this region, it has only served to accelerate the shift toward service production underway elsewhere. Ironically, the weakened bargaining power of workers in labor-intensive sectors also lowers the incentives for firms to save labor directly, thus further tightening the growth constraints.

Public investment and demand for capital goods could rise to improve both employment and modernize regionally uneven economies. But this path has been severely constrained by prerogatives of the capitalist class in most parts of the world.

## Problems of Reproducing Population

Population growth is an outcome of three factors: fertility, mortality, and migration. As agricultural societies transform into modern industrial societies, fertility rates

[^74]are dramatically moderated. ${ }^{2}$ Moreover, the effects of persistent unemployment and degradation of working conditions, described above, make it harder to form and sustain families. This has exerted downward pressure on fertility rates, even pushing them below basic reproduction levels in many regions. At the same time, the ensuing despair of unemployment has even increased mortality rates in certain sections of the population. These changes have tightened the demographic constraints sharply and have only been locally offset by net migration into the advanced economies. Meanwhile, in less developed regions entangled in global capitalism, such as Africa, India, and South and Central America, the continued decrease in the labor content of basic life-sustaining products, basic foods, and medicine reproduced large masses of working poor under abysmal poverty, with little prospect of integrating in modern production centers.

Families in industrialized societies depend on the provision of vital reproductive services, such as child rearing, education, and healthcare. For the unemployed and workers at the bottom of the wage distribution, adequate services are not affordable through market exchange. Indeed, market-based provision means that the quality of such services is distributed on the basis of buyers' purchasing power. Firms driven by the profit imperative, therefore, produce an allocation of vital services that are inefficient for the population's needs.

Market-based allocation of foods, medicine, housing, and vital services has in most capitalist societies been supplemented by public provision. Since most lifepreserving services are persistently labor-intensive, their share of total employment is rising. This would necessitate an increase in the public share of the labor added at the expense of capital incomes. But the prerogatives of the capitalist class blocks such development, which leads to persistently underfunded public services and deteriorating quality of service. The problem is aggravated by the fact that aging populations increase the needs for such services.

## Problems of Increasing Amounts of Waste By-Products

The imperatives of exponential growth lead to an exponential increase in the utilization and extraction of several natural ultimate inputs. When natural replenishment rates cannot keep up, the effects of growth are visible as deforestation, loss of arable land, overfishing, and depleted mines. But production and consumption of the products into which these inputs enter also result in growing quantities of waste and pollutants spewed into the surrounding environment.

Many of these by-products are chemical compounds that are toxic to human beings and other species. The most significant challenge, of course, is the growing quantity of greenhouse gas emissions in the atmosphere and the ensuing climate changes. The rising temperature leads to deteriorating physical working conditions and increased

[^75]risks of exhaustion for large numbers of workers. ${ }^{3}$ It also increases risks for all life forms through the increased frequency of forest fires, flooding, and storms, which destroy crops and damage homes as well as productive assets. The increased amounts of such by-products can, therefore, only aggravate the problems of reproducing the population discussed above.

### 8.2 Requirements to Overcoming Obstacles

The three problem domains listed above combine to impair human well-being on an increasing scale and undermine the reproduction of the source that powers capitalism: labor. Only certain types of measures can overcome these obstacles. But as we argue below, the measures are fundamentally at variance with the operation of capitalist market economies.

## Regulated Utilization of Ultimate Inputs

The utilization of gifts of nature and labor by firms cannot be left to market-based allocation according to purchasing power. Instead, utilization must be regulated by societal needs and the long-run stability of economic reproduction. This necessitates firm political intervention that prevents the exhaustion of workers and continually coordinates the reallocation of labor toward gainful and meaningful employment. Labor time is to be allocated as a scarce common resource, not to be squandered on the production and advertising of toxic consumption goods, private security services, or luxury articles accessible only for the very rich.

Similarly, gifts of nature with low replenishment rates cannot be squandered and their resulting waste by-products dumped onto the surrounding environment. This means keeping many large fossil fuel reserves in the ground. Indeed, this is the implication of a recent court order on Royal Dutch Shell to cut its global carbon emissions by $45 \%$ by the end of 2030 .

## Reduction of Total Working Time

The conversion of LDLC into exponential output growth cannot proceed without strong constraints. This growth is tightly coupled with the growing utilization of gifts of nature and increasing amounts of waste by-products. Thus, the composition of output products will have to adapt and the aggregate growth rate, governed by (7.5), will have to be reduced to stay within the long-run limits of natural replenishment

[^76]rates and waste efficiency rates. If LDLC were to proceed at a rate of $\delta=2.7 \%$ per year, then total working time will have to be gradually lowered toward $-2.7 \%$ per year.

Under coordinated reallocation of labor, this means reduced working weeks and extended vacation time for workers, depending on types of employment. At the same time, the resulting job vacancies will need to be filled which reduces the number of unemployed. Such changes have only reached an experimental stage on very small scales within capitalist societies:

A New Zealand firm that let its employees work four days a week while being paid for five says the experiment was so successful that it hoped to make the change permanent.[...] reducing work hours as a way of improving individual productivity. In Sweden, a trial in the city of Gothenburg mandated a six-hour day, and officials found employees completed the same amount of work or even more. ${ }^{4}$

Of course, under the profit imperative, only a few firms can afford such reductions without substantial wage cuts.

## Replacement of Profitability Criterion

The allocation of resources and accumulation of productive assets can no longer be decided in terms of rates of return on investments. The share of accumulation $\alpha$ will have to rise to modernize machinery, equipment, power plants, and transportation for more efficient utilization of ultimate inputs, especially fossil fuels. Such investments would increase the demand for labor in many sectors. At the same time, the pressures of climate change will necessitate transformations of infrastructure on a scale that private investment cannot and will not match. Replacement of the profitability criterion means that preserving the value of capital assets, measured in purchasing power of labor content, would no longer be ensured.

Thus, the allocation of surplus labor content in the form of investment products will have to be subjected to other socioeconomic criteria than profitability. Since total private consumption can no longer expand at exponential rates, increased leisure time and expansion of collective consumption and investments will have to become the primary economic means of improving human well-being. This would mean a substantial expansion of free child and elderly care, education, and healthcare. But also investment in public parks, libraries, movie theaters, and other recreational facilities.

### 8.3 Socialized Political Economy

To overcome the rising obstacles to improving human well-being identified above will require regulated utilization of ultimate inputs; a reduction of total working

[^77]time; and a replacement of the profitability criterion. These are measures that are incompatible with the institutions of capitalist market economies.

Recognition of the limits to capitalist development is not new of course. The waste of labor and the crippling effects of unemployment on the worker and unemployed worried many writers and commentators in the past, including the great physicist Albert Einstein who in 1949 wrote:

I am convinced there is only one way to eliminate these grave evils, namely through the establishment of a socialist economy, accompanied by an educational system which would be oriented toward social goals. In such an economy, the means of production are owned by society itself and are utilized in a planned fashion. A planned economy, which adjusts production to the needs of the community, would distribute the work to be done among all those able to work and would guarantee a livelihood to every man, woman, and child. The education of the individual, in addition to promoting his own innate abilities, would attempt to develop in him a sense of responsibility for his fellow men in place of the glorification of power and success in our present society. ${ }^{5}$

The measures discussed above all highlight the need for a coordinated form of the economy with a substantial degree of collective control over its productive assets. But finding the appropriate institutional forms for such a socialized political economy remains an open problem, as Einstein recognized:

> A planned economy as such may be accompanied by the complete enslavement of the individual. The achievement of socialism requires the solution of some extremely difficult sociopolitical problems: how is it possible, in view of the far-reaching centralization of political and economic power, to prevent bureaucracy from becoming all-powerful and overweening? How can the rights of the individual be protected and therewith a democratic counterweight to the power of bureaucracy be assured? Clarity about the aims and problems of socialism is of greatest significance in our age of transition. ${ }^{6}$

Finding solutions to such open problems will take a great deal of research and experimentation. However, it is clear that for macroeconomic coordination and planning to enable human flourishing, it will need a radically democratic institutional form in which citizens collectively decide over the major questions that affect their lives. ${ }^{7}$ But is there any basis for a socialized political economy within present-day capitalism? We turn to this open problem in the final chapter.

[^78]
## Chapter 9 <br> Open Problems of Transition

We have argued that overcoming the narrowing limits to human development under capitalism will require measures that are incompatible with institutions based on profitability and exponential growth. Instead, economic reproduction would have to be carried out through some form of socialized political economy. What are the material bases for such an economy within present-day capitalism? And which forces can be formed to steer a transition from latent possibilities to material reality? Rather than providing answers to these open problems, we will in this final chapter suggest some avenues of future research.

### 9.1 Material Bases for Transition

For a socialized political economy to emerge materially from current market-based societies, we must identify latent forms of coordinated, cooperative, and collective production that exist within them. If they could form into a self-reproducing nucleus, they could serve as the productive basis for a transition toward an economy that no longer allocates labor and resources with a view to profitability. Below we highlight four latent forms that could play a transitional role.

## Centralization of Capital

Writing in the 1860s, Marx observed a tendency of capitalist development to 'centralize' productive assets into fewer hands. The qualitative changes that accompany this process, he thought, could form the basis of a socialized or 'associated' form of political economy:

Hand in hand with this centralisation, or this expropriation of many capitalists by few, develop, on an ever-extending scale, the cooperative form of the labour process, the conscious technical application of science, the methodical cultivation of the soil, the transformation of the instruments of labour into instruments of labour only usable in common, the economising of all means of production by their use as means of production of combined, socialised labour, the entanglement of all peoples in the net of the world market, and with this, the international character of the capitalistic regime. Along with the constantly diminishing number of the magnates of capital, who usurp and monopolise all advantages of this process of transformation, grows the mass of misery, oppression, slavery, degradation, exploitation; but with this too grows the revolt of the working class, a class always increasing in numbers, and disciplined, united, organised by the very mechanism of the process of capitalist production itself. The monopoly of capital becomes a fetter upon the mode of production, which has sprung up and flourished along with, and under it. Centralisation of the means of production and socialisation of labour at last reach a point where they become incompatible with their capitalist integument. ${ }^{1}$
Large parts of this scenario have in fact stood the test of time very well indeed. Centralization of capital has proceeded to much greater extremes than ever before. In each major sector of the economy, the global market is dominated by a small number of large firms. This is equally true in traditional industries such as mining and shipbuilding; in branches that were in their infancy at the beginning of the twentieth century such as car manufacture, aircraft building, telecommunication, and pharmaceuticals; and in very new spheres such as computer software, in which the lion's share of the market has been hogged by a couple of firms.

The conscious technical application of science has been greatly stepped up and has become more consciously planned, systematized, and institutionalized, particularly since the middle of the twentieth century. Formerly, scientific research was for the most part carried on in publicly financed academic institutions and other non-business research foundations. Now big capitalist firms have their own research and development sections, geared directly to profit-seeking, and at the same time, big business has also become the chief paymaster of academic scientific research. Science as the pursuit of truth without regard to its business applicability is regarded as old-fashioned and is poorly supported out of the public purse. The methodical cultivation of the soil has spread to all parts of the planet. Agriculture, the last major refuge of petty commodity production, has become the domain of big agro-business, which not only aims to control the activity of the direct producers but also uses the most advanced scientific techniques to modify, subjugate, and dominate nature itself.

At the same time, cooperative forms of labor processes have in recent decades broken out of their old local confinement. Previously, human beings could not normally engage simultaneously in a consciously synchronized and coordinated productive activity unless they were physically brought together to one place. Now, with the advent of new modes of communication, they can synchronize and coordinate their productive activity in real time, across continents. The socialization of the labor process has also intensified in recent decades. Since the most modern instruments of labor have become dependent upon centrally supplied power, rapid transport, and continual online instantaneous communication with remote computers, they are

[^79]really only usable by direct and explicit cooperation of each user with many others. The economizing of all means of production by their use in production by combined, socialized labor has advanced much further.

The world market itself is of course not new. But its scope and degree of integration, the entanglement of all peoples in the net of the global market, have gone very much further in recent decades. Capitalism now truly encompasses the whole planet. And as remaining national economic barriers come tumbling down, the transnational character of capitalist production-its globalization-has reached a degree much greater than before. However, it remains an open question how its locally interconnected units of production should be jointly coordinated. ${ }^{2}$

## Cooperative and Artisanal Production

Marx also observed changes in the ownership structures of capital, which he thought opened a path toward a socialized or 'associated' political economy:

> The credit system is not only the principal basis for the gradual transformation of capitalist private enterprises into capitalist stock companies, but equally offers the means for the gradual extension of co-operative enterprises on a more or less national scale. The capitalist stock companies, as much as the co-operative factories, should be considered as transitional forms from the capitalist mode of production to the associated one, with the only distinction that the antagonism [between the social and private character of wealth] is resolved negatively in the one and positively in the other. ${ }^{3}$

In hindsight, the credit system did indeed form the basis for transforming capitalist firms, but it is clear that very little of the 'positive' resolution has come to pass. Producer cooperatives employ only a tiny minority of the global workforce and are largely absent from the basic sectors of the economy. For such non-capitalist firms to become feasible, the L-content of important capital goods would have to fall very fast for groups of workers to acquire them and stay afloat in competitive markets.

While such a steep fall has not occurred for productive assets in key economic sectors, it has taken place with respect to general-purpose computing machines and adaptable manufacturing devices. This has opened up new possibilities for setting up small-scale production of services, software, and customized manufacturing items. The expansion of the credit system has indeed led to the proliferation of many socalled 'start-up' firms. But these are dependent on financial 'venture' capital and largely subordinated to the profit imperative. In so far as they survive, they do not form self-sustaining networks of production and exchange, but are rather highly dependent on major capitalist firms.

Nevertheless, small-scale cooperative producers could emerge in the fringes of the economy and direct themselves toward specialized, local needs of the communities in

[^80]which they are embedded. This is most salient in regions laid to waste when capitalists seek more profitable investments elsewhere. In so far as the economic imperatives of cooperatives and self-employed artisanal producers can be subordinated to the needs of their communities, they could play an important complementary role in a socialized political economy.

## Public Provision

The inability of capitalist firms to secure long-run needs for education, healthcare, and infrastructure has led nation-states to undertake the provision of such basic products. After their mobilizations for two World Wars and concomitant mobilizations of working-class and national liberation movements, there was a massive expansion of public institutions. These were not subject to the profit imperative, but organized or coordinated the production of important non-commodity products: major infrastructure projects, roads, communication lines, basic social services, medical services, basic and higher education, research, and so on. In other words, there are extensive non-capitalist means of organizing production within present-day economies.

However, public institutions suffer from two main weaknesses. First, they rarely form a viable self-sustaining economic sector but are to a great extent dependent on taxable incomes from the capitalist sector. Second, they are often subject to bureaucratic control, rather than being governed by their workers and beneficiaries. Nevertheless, the increasing needs for investments in infrastructure and the provision of vital services are only likely to heighten the political contestation over the future of public institutions under capitalism.

## Production of Non-commodities

The decreasing L-content of computing machines and 3D-printers, along with the global expansion of Internet connections, has led to increased production of non-commodities outside of public institutions. It has emerged from communities engaged in open-source software and 3D-printing designs. Some of these products play a central role in running Internet servers worldwide. (The present book, like many others, was typeset by its authors using $L^{A} T_{E} X$, a sophisticated scientific typesetting freeware!) They are produced in disregard of the profitability criterion and indeed expect little or no remuneration. Instead, their production and maintenance are driven by the creative zeal of many thousands of individuals.

This form of production cannot, of course, coalesce into a viable self-reproducing sector. But it could challenge the dominance of certain software firms and even erode the dependence on the market of important types of software infrastructure and designs.

### 9.2 Forces of Transition

For each of the latent material bases listed above, we may identify a potential agent of change. From the centralization of capital, the early labor movement identified workers; in cooperative and artisanal production, we have associated producers; for public institutions, we identify state managers; and in non-commodity production, globally networked communities. However, as we argued, the last three bases are either highly dependent on capitalist production or cannot form self-reproducing development paths by themselves.

Furthermore, any measures in a direction that challenges capitalist prerogatives threaten the interests of those who make their living from owning and managing capital. Could the interests of such a tiny minority carry any weight when the wellbeing of an increasing share of the world population is at stake? State managers in parliamentary nation-states could after all use their coercive powers as a force to implement necessary transitional policies, given a strong enough electoral mandate. But this overlooks three basic limits. First, state managers do not have the capacities to run complex economies. Second, as long as economic reproduction is largely carried out through capitalist firms, state institutions are dependent on their taxable revenues and commodity-products which constrain state managers seeking to stay in power. Third, state managers are organized into nation-states competing for capital investments. This market dependence disqualifies them as a principal force of transition. ${ }^{4}$

What is then the constellation of forces that could steer a reorganization and transition toward a socialized political economy? It will likely depend on politicaleconomic conjunctures, but as we will argue next, the source of labor content and discontent-namely, workers-would necessarily form an indispensable component of such a constellation.

## Collective Capacities of Workers

Marx famously thought the central agents of change were workers, who were 'increasing in numbers, and disciplined, united, organized by the very mechanism of the process of capitalist production itself.' But are not workers just as market dependent as state managers, if not more? Indeed, few workers and their families could sustain themselves for long without a wage income. Nevertheless, socialist labor movements maintained that the working class would play an indispensable role due to its specific location in capitalism:

> As the exploited class, it is caught in a systematic clash with capital, which cannot generally and permanently satisfy its needs. As the main producing class, it has the power to halt-and within limits redirect-the economic apparatus of capitalism, in pursuit of its goals. And as

[^81]the collective producer it has the objective capacity to found a new, non-exploitative mode of production. This combination of interest, power and creative capacity distinguishes the working class from every other social or political force in capitalist society, and qualifies it as the indispensable agency of socialism. ${ }^{5}$

This assessment turned out to be accurate. From the mills of Yorkshire to the mines of Johannesburg, organized workers with a capacity to disrupt economic processes have been an indispensable force of egalitarian change in capitalism. ${ }^{6}$ However, this force has by itself been largely insufficient to set in motion deep structural changes.

Firstly, organizing disruptive capacities is hard work, full of risks. It involves selfsacrificing labor organizers who are instrumental in forging solidaristic collectivities in workplaces and neighborhoods. Only by affirming mutual trust, with a reasonable chance of winning concessions, are fellow workers willing to take on the risks of collective mobilization and action. Given the high personal costs and efforts of sustained mobilization, there is no wonder it is difficult to organize even beyond a single workplace. Secondly, disruptive capacities vary significantly across different locations of the economy. The greatest potentials are in large, capital-intensive firms, typically involved in mining, industry, or transport. Thirdly, mobilization of disruptive capacity not only pits workers against capitalists and their paid lackeys, but the very risk of capital investment strikes and economic downturn pits them also against the interests and coercive powers of state managers. Instead, the greatest advances by organized workers have occurred in conjunctures when the threat of investment strikes is weakened: severe economic dislocation, war, disasters, and pandemics. In other words, the structural changes that any given coalition of workers can bring about depend on conjunctural factors that are largely external to it.

## Open Problems of Organization

The early labor movement's anticipation of organized workers growing in numbers along with industrial capitalism was prescient and a widespread tendency for almost one hundred years. Growing capital-intensive firms increased the latent disruptive capacities of workers, while larger workplaces homogenized interests among them. Both factors lowered the barriers for labor organizers to build solidaristic collectivities and political mass organizations. These organizations played a decisive role in the dramatic restructuring of capitalism that followed the disaster of the Second World War, and the establishment of full employment policies in the advanced economies. This process enhanced their disruptive capacities. It proceeded until the high postwar accumulation share could no longer be sustained and moderated worker demands could no longer be maintained. ${ }^{7}$

The structural changes of capitalist economies since the 1970s have shifted the terrain for organizers. The concentration and centralization of capital, with some

[^82]important exceptions, are no longer coupled with larger workplaces. Due to cheapening of transportation, even industrial production tends to be dispersed. Moreover, the shift from industrial to service employment has tended to decentralize workplaces. The abandonment of full employment policies by nation-states lowered the bottom floor in many labor markets and helped reassert the prerogatives of capitalists. As anticipated by Marx, these owners were increasingly taking the form of financial rentiers through a globalized credit system divorced from locally organized units of production. Together these changes have weakened the latent disruptive potentials that give workers collective leverage and raised the barriers for labor organizers.

If organized workers remain an indispensable agent of transition from capitalism, this leaves us with several open problems:

- In which economic sectors do workers possess the greatest disruptive capacities? Given the process of deindustrialization in large parts of the Global North and South, the efforts of labor organizers will need to be directed toward alternative stable and critical sectors, such as logistics, transportation, and energy.
- Which coalitions of workers can be formed beyond individual firms or sectors? The tail portion of labor that receives wage rents has few material interests in egalitarian social change. By contrast, workers located at the opposite end have greater interests but their transformative capacities depend on how their employment is allocated across the economy. Since it is almost certain that skilled workers will need to play an important role in economic coordination, this will depend on the extent to which their material interest in change overlaps with that of wider sectors of the working class. This points at the vital role of political mass organizations, which can bring together workers of various levels of skill, as well as individual members of other classes who are committed to egalitarian social change.
- How can trade-union and political coalitions of workers coordinate their capacities across nation-states? Any collectivity of workers is rooted in a national or regional political culture. Given the global character of capitalism, strategies of their organizations will have to be coordinated across political cultures shaped by significant economic differences: Northern and Southern Europe; deindustrializing US and industrializing Vietnam, and so on.
- What types of conjunctural factors are likely to open paths toward structural changes? In the twentieth century, geopolitical rivalry and conflict played a decisive role. Will the growing rivalry between the United States and China open up a space for independent regional action? At the same time, we can anticipate that the management of more frequent and severe climate disasters will pit the basic imperatives of economic reproduction against those of profitability and stable business confidence. How should organizations of workers build capacities in preparation for such conjunctures?

The open problems raised above require further analysis of trends and conjunctural shifts. However, they cannot be resolved from one's armchair. Only those who engage in struggles for a more humane society will have a chance of constructing relevant answers.

## Appendix A Open Theoretical Problems

The discussion in Chaps. 5, 6, and 7 raises several problems that are open to interrogation and research. Some of these are internal to the labor theory of capitalism; others are related to overcoming of the deep predicaments that are inseparable from the present capitalist system and cannot be effectively solved within it. Here, we list a few problems of the first kind. For those of the second kind, see Chap. 9.

Let us state some theoretical problems that are only partially addressed in the preceding chapters. They all relate to exploring theoretical bounds on the variation of crucial parameters of the economy that seem to be remarkably stable over the long term, moving within a rather narrow range.

1. Stability of the worldwide wage share of value added and its rather narrow range of variation. This presents an intriguing puzzle. It appears that the global share $\omega$ of wages in value added is empirically more stable and moves within a narrower range than explained so far by theoretical considerations; see Sect. 6.5. This calls for a more careful discussion of the plausible form of the distributions of wages rates and their political-economic implications. Several authors have in recent years provided some theoretical insights and empirical observations about the distribution of labor incomes. ${ }^{1}$
2. Narrow range of the rate of decrease of L-content. It is clear that an increase of the share of services-especially personal hands-on services-tends to slow down the overall fall of L-content of the total output. Moreover, increasing automation, requiring less human labor input per product, while reducing directly the L-content of products, also drives more workers into services, where reduction of L-content of their output is much slower. On balance, the question of theoretically determined lower and upper bounds to the rate of decrease in L-content remains open. Compare Chap. 5.
3. Treatment of material depreciation. This represents a theoretical challenge whether using monetary or labor-content accounting. According to the basic

[^83]properties of L-content listed in Sect. 2.3, the reduction of L-content of machines due to wear-and-tear is part of the inputs used up in production and must be subsumed in the L-content of the product.

However, beyond the general LDLC and wear-and-tear, machines lose L-content also by being outmoded, replaced by improved, more efficient counterparts, etc. This too is sometimes captured by the term 'depreciation'. Firms tend to overestimate the depreciation of their material capital for tax purposes. But how is this depreciation to be measured in reality? How should it be defined more precisely? This is related to estimates of loss or reduction of labor content, which we addressed in Chap. 5.

Thus, for example, there was a massive elimination of L-content in the vacuumtube industry. Here, it was not a matter of finding new, more efficient, methods for producing vacuum tubes; rather, a superior replacement in the form of semiconducting transistors was introduced. Due to its simplicity, the replacement has a far lower L-content. In other words, LDLC often operates not merely by producing the same product-type in a more efficient manner, but by replacing it with a new product whose applications include and generally surpass those older one. The question is: How exactly to account for the lost L-content? Is it destroyed? Due to the rapid circulation and replacement of all tools, this process may not have, by itself, a significant impact; but the totality of all such ongoing replacements might well be substantial.

It seems clear that, as opposed to wear-and-tear in production, the reduction of L-content due to the general trend expressed by the LDLC is not transferred to the new products, even though it occurs behind the back of the firm in each production area. (See Sect. 4.4 and Chap. 5.) But the same reasoning may not apply to other causes of reduction in the L-content of tools, which are due neither to wear-and-tear nor to LDLC.

## Appendix B <br> Mathematical Technicalities

## B. 1 Possible Measure of L-Content

The analysis in this book is carried out using any measure of labor content $L(\mathcal{B})$ over product baskets $\mathcal{B}$ that satisfy the basic properties (L1-L4) in Sect. 2.3. In this section of the technical appendix, we will discuss a particular choice that can be estimated using supply and use tables from national accounting data; see, e.g., Eurostat (2008). The measure is constructed using principles discussed in Zachariah and Cockshott (2020) and differs from conventional formalizations given in, e.g., Pasinetti (1977), Steedman (1977), and Roemer (1981).

We consider an economy capable of reproduction with a workforce that can be trained and reallocated across concrete tasks. The economy has $m$ different units of production that jointly produce a (net) output $\mathcal{B}_{\text {out }}$ during a given period (say, a year). By property (L2), the labor added during the year is

$$
\begin{equation*}
\mathbb{L}=L\left(\mathcal{B}_{\text {out }}\right) \tag{B.1}
\end{equation*}
$$

The workforce is collectively allotted a basket of products $\mathcal{B}_{\text {workers }}$ that is part of $\mathcal{B}_{\text {out }}$. The L-content of $\mathcal{B}_{\text {workers }}$ must thus be a portion of the labor added in (B.1), which we call the labor share of output, denoted

$$
\begin{equation*}
\lambda=\frac{L\left(\mathcal{B}_{\text {workers }}\right)}{L\left(\mathcal{B}_{\text {out }}\right)} . \tag{B.2}
\end{equation*}
$$

In other words, in the production of $\mathcal{B}_{\text {out }}$, a certain portion $\lambda$ of its L-content is necessarily allotted to the workforce. We will construct a measure $L(\mathcal{B})$ starting from the real distribution of products to the workforce under a given set of conditions of production.

## Mapping Baskets to Vectors

For a formal treatment, we partition all products in a given period into $d$ distinct product-types. The level of resolution is of no matter here; one may adopt a few hundred product-types available in national accounting data or a fine-grain classification into many millions of distinct types of goods and services. The partition enables us to map any basket to a $d \times 1$-dimensional vector,

$$
\mathcal{B} \Leftrightarrow \mathbf{b}
$$

which records the quantities of each product-type in the basket in an ordered list. ${ }^{2}$ Then the L-content of $\mathcal{B}$ has an equivalent expression,

$$
\begin{equation*}
L(\mathcal{B}) \equiv \mathbf{c b} \tag{B.3}
\end{equation*}
$$

where $\mathbf{c}$ is a nonnegative $1 \times d$-vector that specifies the L-content per unit of each product-type.

For instance, in a toy economy with $d=3$ product-types, we may have a basket

$$
\mathcal{B}=\{2 \text { units of iron, } 9 \text { units of corn, } 4 \text { units of sugar }\} \quad \Leftrightarrow \quad \mathbf{b}=\left[\begin{array}{l}
2 \\
9 \\
4
\end{array}\right],
$$

and its L-content will be

$$
L(\mathcal{B})=2 \cdot c_{\mathrm{iron}}+9 \cdot c_{\mathrm{corn}}+4 \cdot c_{\mathrm{sugar}}
$$

where $c_{\text {iron }}, c_{\text {corn }}$, and $c_{\text {sugar }}$ specify the labor content of a unit of iron, corn, and sugar, respectively.

The choice of L-content measure, $L(\mathcal{B})$, is then determined by the vector $\mathbf{c}$, which we will now derive from first principles.

## Products Distributed to Workforce

We represent the basket of net outputs by

$$
\mathcal{B}_{\text {out }} \Leftrightarrow \mathbf{b}_{*} .
$$

Let the $d \times 1$-dimensional vector $\mathbf{w}$ denote the quantities of products distributed to the workforce, divided by total hours of worked during the given year. That is, the vector

[^84]$$
\mathcal{B}_{\text {workers }} \Leftrightarrow \mathbf{w} \mathbb{L}
$$
represents the allotted basket of goods and services and can be thought of as the workers' material wage vector. It determines the labor share in (B.2):
\[

$$
\begin{equation*}
\lambda=\frac{\mathbf{c w} \mathbb{L}}{\mathbf{c b}_{*}} \equiv \mathbf{c w} \tag{B.4}
\end{equation*}
$$

\]

since $\mathbb{L}=\mathbf{c b}_{*}$ in (B.1). This is the case for any measure of L-content. We shall now connect the products distributed to the workforce with the products that it produces across $m$ distinct units of production. Here, we take a 'firm' to be a unit of production that jointly produces an assortment of products using a multitude of direct inputs.

Let us begin with inputs to production. Firm $i$ employs $x_{i}$ hours of direct labor in a given year, and the products that it uses up in the process are given by

$$
\mathbf{a}_{i} x_{i},
$$

where $\mathbf{a}_{i}$ is a $d \times 1$-vector of coefficients that specifies the amount of each producttype used up per unit of direct labor. All $m$ firms then employ

$$
x_{1}+\cdots+x_{m}=\mathbf{1 x} \equiv \mathbb{L}
$$

units of labor, where $\mathbf{1}$ is a row vector $[1, \ldots, 1]$ of $m 1 \mathrm{~s}$, and use up a basket of products given by

$$
\mathbf{a}_{1} x_{1}+\cdots+\mathbf{a}_{m} x_{m}=\mathbf{A x}
$$

where $\mathbf{A}$ is an $d \times m$ nonnegative matrix of the product input coefficients to each firm.

Next, we turn to the output products. The collection of firms produces a gross quantity of product-type $j$ denoted $q_{j}$ and we let $\mathbf{q}$ denote the $d \times 1$ vector of gross quantities of all product-types. Thus, the vector of (net) output products can be expressed as

$$
\begin{equation*}
\mathbf{b}_{*}=\mathbf{q}-\mathbf{A x}, \tag{B.5}
\end{equation*}
$$

after deducting the input products. The gross production of each product-type $j$ is distributed differently across the $m$ firms. A given firm $i$ uses $x_{i}$ worker-hours to jointly produce $d_{i}$ distinct product-types. Specifically, it produces $p_{i j} q_{j}$ units of type $j$, where $p_{i j}$ denotes firm $i$ 's proportion of the gross output of the economy. It employs $\ell_{i j}=x_{i} /\left(p_{i j} q_{j}\right)$ hours of labor per unit of product-type $j$. In other words, for each of the $d_{i}$ product-types, we have that $\ell_{i j} p_{i j} q_{j}$ equals $x_{i}$ hours. We can then write

$$
x_{i}=\frac{1}{d_{i}} \sum_{j=1}^{d} \ell_{i j} p_{i j} q_{j},
$$

where the summation is over all product-types. Since a firm $i$ produces only a subset of all types, $p_{i j}=0$ for most types $j$. The labor employed in each firm can then be written in matrix form

$$
\begin{equation*}
\mathbf{x}=\mathbf{L} \mathbf{q} \tag{B.6}
\end{equation*}
$$

where $\mathbf{L}$ is the $m \times d$ labor allocation matrix whose $i j$ th element is $\frac{\ell_{i j} \cdot p_{i j}}{d_{i}}$.
In summary, the pair of matrices $(\mathbf{A}, \mathbf{L})$ describes the technical conditions of production and can be estimated using data from supply and use tables. ${ }^{3}$ For compactness, we combine relations (B.5) and (B.6) into a single expression

$$
\left[\begin{array}{c}
\mathbf{b}_{*}  \tag{B.7}\\
\mathbf{0}
\end{array}\right]=\left[\begin{array}{cc}
\mathbf{I}_{d} & -\mathbf{A} \\
-\mathbf{L} & \mathbf{I}_{m}
\end{array}\right]\left[\begin{array}{l}
\mathbf{q} \\
\mathbf{x}
\end{array}\right] .
$$

This relation can now be used to tie the quantities of products allotted to the workforce to the products that it produces. The vector of allotted products can be expressed as

$$
\begin{align*}
\mathbf{w} \mathbb{L} & =\mathbf{w}(\mathbf{1} \mathbf{x}) \\
& =\mathbf{w}\left[\begin{array}{ll}
\mathbf{0} & 1
\end{array}\right]\left[\begin{array}{c}
\mathbf{q} \\
\mathbf{x}
\end{array}\right] \\
& =\mathbf{w}\left[\begin{array}{ll}
\mathbf{0} & 1
\end{array}\right]\left[\begin{array}{cc}
\mathbf{I}_{d} & -\mathbf{A} \\
-\mathbf{L} & \mathbf{I}_{m}
\end{array}\right]^{-1}\left[\begin{array}{c}
\mathbf{b}_{*} \\
\mathbf{0}
\end{array}\right]  \tag{B.8}\\
& =\underbrace{\left(\mathbf{w} \mathbf{1} \mathbf{L}\left(\mathbf{I}_{d}-\mathbf{A L}\right)^{-1}\right)}_{\equiv \mathbf{W}} \mathbf{b}_{*} \\
& =\mathbf{W} \mathbf{b}_{*},
\end{align*}
$$

where the fourth equality follows from applying the block-inverse formula to the matrix in (B.7). The $d \times d$ matrix $\mathbf{W}$ specifies the allotted products tied up with the production of $\mathbf{b}_{*}$. With the workers' allotment matrix $\mathbf{W}$ in place, we can now turn to the derivation of an L-content measure (B.3).

## Derived L-content Measure

Using (B.4) and (B.8), the labor share (B.2) can be expressed alternatively as

$$
\begin{equation*}
\lambda=\frac{\mathbf{c W b}_{*}}{\mathbf{c b}_{*}} \tag{B.9}
\end{equation*}
$$

[^85]where $\mathbf{c}$ is yet to be specified. However, according to (B.4), the labor share $\lambda$ is determined by the average allotment of products per hour of work under any given set of conditions of production (and is independent of the specific quantities in $\mathbf{b}_{*}$ ). Using this fact uniquely determines an L-content measure and $\lambda$.

To see this, rearrange the equality in (B.9) as $(\lambda \mathbf{c}-\mathbf{c W}) \mathbf{b}_{*}=\mathbf{0}$, which must now hold for any $\mathbf{b}_{*}$. That is to say

$$
\begin{equation*}
\lambda \mathbf{c}=\mathbf{c} \mathbf{W} \tag{B.10}
\end{equation*}
$$

where we see that $\mathbf{c}$ is a left-eigenvector to the workers' allotment matrix $\mathbf{W}$ and the labor share $\lambda$ is given as a solution to $\operatorname{det}(\lambda \mathbf{I}-\mathbf{W})=0 .{ }^{4}$ Using the expression for $\mathbf{W}$ in (B.8) and the matrix determinant lemma, we have

$$
\left(1-\mathbf{1} \mathbf{L}(\mathbf{I}-\mathbf{A} \mathbf{L})^{-1} \mathbf{w} \lambda^{-1}\right) \lambda^{d}=0
$$

Since $\lambda=0$ corresponds to a workforce that does not consume anything, it is economically meaningless and only

$$
\lambda=\mathbf{1} \mathbf{L}(\mathbf{I}-\mathbf{A} \mathbf{L})^{-1} \mathbf{w}>0
$$

is a meaningful solution. Next, using the definition of $\mathbf{W}$ in (B.8), equation (B.10) can be written as

$$
\lambda \mathbf{c}=\mathbf{c W} \equiv(\mathbf{c w}) \mathbf{1} \mathbf{L}(\mathbf{I}-\mathbf{A L})^{-1} .
$$

But by (B.4) cw $=\lambda>0$. Thus, we end up with a nontrivial solution to (B.10),

$$
\mathbf{c}=\mathbf{1} \mathbf{L}(\mathbf{I}-\mathbf{A} \mathbf{L})^{-1}
$$

which uniquely determines an L-content measure in (B.3):

$$
\begin{equation*}
L(\mathcal{B}) \equiv\left[\mathbf{1} \mathbf{L}(\mathbf{I}-\mathbf{A} \mathbf{L})^{-1}\right] \mathbf{b} \tag{B.11}
\end{equation*}
$$

from basic properties of economic reproduction.

## Further Remarks on the Derived Measure

It can be seen that the derived L-content measure averages the labor expended across all units of production and $a d d s$ all intermediate labor requirements at each stage of production. Specifically, using a series expansion form of the matrix in (B.11), we obtain an alternative expression:

[^86]$$
L(\mathcal{B})=\left(\mathbf{c}_{0}+\mathbf{c}_{1}+\mathbf{c}_{2}+\cdots\right) \mathbf{b},
$$
where
$$
\mathbf{c}_{k}=\mathbf{1} \mathbf{L}(\mathbf{A L})^{k}
$$
determines the labor requirements at the $k$ th stage of production (counting backward). Thus, $L(\mathcal{B})$ aggregates all labor required to produce a basket $\mathcal{B}$. We can see that the L-content can decrease from reducing the direct labor input $\mathbf{c}_{0}=\mathbf{1 L}$ and/or the indirect labor input $\mathbf{c}_{k}$ where $k>0$.

The construction of the L-content measure above also sheds a new light on the classical-Marxian distinction between 'productive' and 'unproductive' economic activities. It can be used to identify units of production whose output enters directly or indirectly into $\mathcal{B}_{\text {workers }}$. Only technical changes in their production processes can reduce the L-content of $\mathcal{B}_{\text {workers }}$ and thereby alter the labor share $\lambda$ via productivity gains. See Zachariah and Cockshott (2020) for a connection to Marx's conception of 'relative' and 'absolute' surplus value.

Unlike conventional formalizations found in, e.g., Pasinetti (1977), Roemer (1981), and Chaps. 11-12 in Steedman (1977), the measure above remains economically meaningful also when units of production can produce several product-types jointly; see also Farjoun (1984).

## B. 2 Distribution of Wage Rates

Here, we discuss some properties of the distribution of wage rates $W$. Over a given period, $\mathbb{L}$ hours of labor are expended in production. A labor contract $i$ is paid at the rate of $W_{i}$ per hour over a duration of $T_{i}$ hours.

The set of all labor contracts constitutes a very large sample space and enables us to treat the wage rate as a random variable. We assign each contract $i$ a probability weight

$$
\pi_{i}=T_{i} / \mathbb{L}
$$

which together sum to 1 . Thus, the proportion

$$
\begin{equation*}
\mathrm{P}(a \leq W \leq b) \tag{B.12}
\end{equation*}
$$

can equivalently be interpreted as the probability of drawing at random a labor contract with a wage rate $W$ in between $a$ and $b$.

## Expected Wage Rate

The expected wage rate over all labor contracts is defined as

$$
\mathrm{E} W=\sum_{i} \pi_{i} W_{i}
$$

where $i$ ranges over the enumerated contracts $\{1,2, \ldots\}$. Let the wage paid in contract $i$ be denoted by

$$
V_{i}=W_{i} T_{i}
$$

Then the expected wage rate can be expressed as

$$
\mathrm{E} W=\sum_{i}\left(\frac{T_{i}}{\mathbb{L}}\right) \frac{V_{i}}{T_{i}}=\frac{\sum_{i} V_{i}}{\mathbb{L}}
$$

which is the ratio of total wages paid to total labor added-see (3.3).

## Uncoordinated Labor Market and Distribution of Wage Rates

Let us now consider possible distributions of wage rates, as characterized by $P(\cdot)$ in (B.12). For clarity, we simplify the analysis so that each labor contract is of the same duration.

Specifically, we take each worker-hour to be contracted individually so that there are $\mathbb{L}$ contracted hours that can receive different wage rates. Moreover, we partition the range of wage rates into $m$ separate intervals:

$$
\begin{equation*}
\left[\mathcal{W}_{1}\left|\mathcal{W}_{2}\right| \cdots \mid \mathcal{W}_{m}\right] \tag{B.13}
\end{equation*}
$$

Thus, $\mathbb{L}$ distinct worker-hours can be arranged into $m$ disjoint bins. Each arrangement is a microeconomic configuration and there is a very large number of such possible microstates. ${ }^{5}$ Since the market allocation of labor contracts is largely uncoordinated, we take there to be a ceaseless and turbulent change of microstates over time. Lacking any detailed knowledge about the labor contracts, we shall for now remain indifferent to which of these states we may find the economy in.

The proportion of the total labor time $\mathbb{L}$ that receives wage rates in a specific interval $\mathcal{W}_{j}$ is denoted

$$
p_{j}=\mathrm{P}\left(W \in \mathcal{W}_{j}\right)
$$

so that the $m$ parameters

$$
\begin{equation*}
\left(p_{1}, p_{2}, \ldots, p_{m}\right) \tag{B.14}
\end{equation*}
$$

[^87]characterize the overall distribution of wage rates $W$. In other words, a fixed set of parameters (B.14) specifies a macroeconomic configuration. Any given macrostate is compatible with several alternative microeconomic arrangements of labor into the $m$ bins in (B.13). Specifically, the relation between a macrostate and the number of compatible microstates, denoted $N$, can be expressed as
\[

$$
\begin{equation*}
\left(p_{1}, p_{2}, \ldots, p_{m}\right) \quad \rightarrow \quad N=\frac{\mathbb{L}!}{\left(\mathbb{L} p_{1}\right)!\left(\mathbb{L} p_{2}\right)!\cdots\left(\mathbb{L} p_{m}\right)!} \tag{B.15}
\end{equation*}
$$

\]

Now we can adopt the perspective of statistical mechanics and consider the macroeconomic configuration $\left(p_{1}^{*}, \ldots, p_{m}^{*}\right)$ that has the maximum number of compatible microstates $N^{*}$. When $\mathbb{L}$ is very large, it turns out that this number $N^{*}$ is vastly greater than that of any other macroeconomic configuration with $N$ compatible microstates. ${ }^{6}$ Thus, as the uncoordinated economic system exhibits a ceaseless and turbulent change of microstates, most of them will conform to the macroeconomic state $\left(p_{1}^{*}, \ldots, p_{m}^{*}\right)$. This macroeconomic distribution of wage rates describes a statistical equilibrium of the system.

What can be said about the shape of this distribution? If there were no macroeconomic constraints on the distribution, it can be shown that all $m$ parameters $\left(p_{1}^{*}, \ldots, p_{m}^{*}\right)$ are equal, so that labor is allocated equally across all wage intervals (B.13). However, the system is subject to macroconstraints, such as (3.2) and (3.3), and only some macroeconomic states $\left(p_{1}, \ldots, p_{m}\right)$ can satisfy them. ${ }^{7}$ The constraints, therefore, determine the distribution $\left(p_{1}^{*}, \ldots, p_{m}^{*}\right)$ with the maximum number of microstates. Constraints such as (3.2) and (3.3) result in skewed equilibrium distributions which were discussed in Sect. 3.2.

## Log-normal Approximation

The distribution of wage rates in Chap. 3 is constructed over such large sample spaces that $\mathrm{P}(\bar{W} \leq w)$ is essentially smooth function of $w$. That is, $\bar{W}$ has a distribution that is well-approximated using a continuous density function, such as the log-normal form illustrated in Fig. 6.1.

The log-normal distributions arise, for instance, when a positive random variable $\bar{W}$ is the outcome of several independent positive multiplicative factors. ${ }^{8}$ For the formation of L-wage rates $\bar{W}$, this is a reasonable working hypothesis, since L-wages

[^88]$$
a \leq \sum_{j=1}^{m} p_{j} f\left(\mathcal{W}_{j}\right) \leq b
$$
specified by functions $f(\cdot)$ and constants $a$ and $b$.
${ }^{8}$ See Wikipedia contributors (2021b) for further details.
vary with many factors such as labor market conditions, skill sets, price inflation, and productivity; see also Cowell (2011, Chap. 4). Therefore, our working hypothesis in Chap. 6 is
\[

$$
\begin{equation*}
\bar{W} \sim \log -\operatorname{Normal}\left(\mu, \sigma^{2}\right) \tag{B.16}
\end{equation*}
$$

\]

where $\mu$ and $\sigma^{2}$ are two parameters that determine the shape of the entire distribution. For our purposes, a more convenient parameterization of the distribution is to use the median L-wage rate and its level relative to the expected L-wage (6.5):

$$
\begin{equation*}
\bar{W}_{\text {median }}>0 \text { and } v=\frac{\bar{W}_{\text {median }}}{\mathrm{E} \bar{W}} \in(0,1) \tag{B.17}
\end{equation*}
$$

The log-normal in (B.16) then admits an alternative parametrization:

$$
\begin{equation*}
\bar{W} \sim \text { Log-Normal }\left(\ln \bar{W}_{\text {median }}, 2 \ln \frac{1}{v}\right) \tag{B.18}
\end{equation*}
$$

using the fact that the median is given by $\exp (\mu)$ and the mean by $\exp \left(\mu+\sigma^{2} / 2\right)$.

## Wage Inequality

A common measure of income inequality is the Gini coefficient, denoted $G$, which is bounded between 0 and 1 . At one extreme, $G=0$, all L-wage rates are equal and at the other, $G=1$, all wage incomes are essentially concentrated to one person. The Gini coefficient for the distribution of L-wage rates is given by

$$
G=\frac{1}{\mathrm{E} \bar{W}} \int_{0}^{\infty} \mathrm{P}(\bar{W} \leq w)(1-\mathrm{P}(\bar{W} \leq w)) d w \quad \in(0,1)
$$

and for the log-normal distribution, it simplifies to

$$
\begin{equation*}
G=\operatorname{erf}\left(\sqrt{\frac{1}{2} \ln \frac{1}{v}}\right) \tag{B.19}
\end{equation*}
$$

where $\operatorname{erf}(\cdot)$ is the Gauss error function. ${ }^{9}$ It follows that this measure of wage inequality is determined by the relative position of the worst-paid half of labor. The relationship is illustrated in Fig. B.1.

[^89]Fig. B. 1 The Gini measure of wage inequality $G$ in (B.19) and the relative position of the worst-paid half of labor $v$; see (B.17)


## Derived Wage Share

Let the proportion of labor time in the tail of the L-wage distribution (6.3) be denoted by

$$
\rho=\mathrm{P}(\bar{W}>1) \ll 1,
$$

which we can relate to the cumulative distribution function of (B.16):

$$
\mathrm{P}(\bar{W} \leq 1)=1-\rho \Leftrightarrow \operatorname{erf}\left(\frac{-\mu}{\sqrt{2 \sigma^{2}}}\right)=1-2 \rho
$$

Thus,

$$
\mu=-\sqrt{2 \sigma^{2}} \cdot \operatorname{erf}^{-1}(1-2 \rho)
$$

We can use this expression along with that of $\sigma^{2}$ in (B.18), to express $\mathrm{E} \bar{W}=$ $\exp \left(\mu+\sigma^{2} / 2\right)$ in terms of the ratio $v$ and the proportion $\rho$. Then the expected L -wage rate equals

$$
\begin{equation*}
E \bar{W} \equiv \exp \left(\ln \frac{1}{v}-2 \sqrt{\ln \frac{1}{v}} \cdot \operatorname{erf}^{-1}(1-2 \rho)\right) \simeq \omega \tag{B.20}
\end{equation*}
$$

which determines the wage share as a function of the position of the worst-paid half of labor $v$ and the proportion $\rho$ of labor earning wage rents. Figure 6.2 is constructed using the derived wage share in (B.20).

## B. 3 Distribution of Specific Prices

A central concept throughout this book is the specific price $\Psi$, which is a rate that varies from one transaction to the next but forms a bridge between random market prices and deterministic labor content. Recall that the specific price in a transaction
$i$ equals $\Psi_{i}=M_{i} / L_{i}$, where $M_{i}>0$ is the price fetched in transaction $i$ and $L_{i}$ is the labor content of the commodity-product sold in that transaction; see Sect. 3.3.

The set of all transactions over a given period constitutes a very large sample space and we, therefore, treat the specific price as a random variable, similar to the wage rate. We assign each contract $i$ a probability weight

$$
\pi_{i}=\frac{L_{i}}{\sum_{j} L_{j}}
$$

which is the proportion of labor content of the products out of all L-content that is bought-and-sold during the period.

## Expected Specific Price

The expected specific price rate over all transactions during a given period is defined as

$$
\begin{equation*}
\mathrm{E} \Psi=\sum_{i} \pi_{i} \Psi_{i} \tag{B.21}
\end{equation*}
$$

where $i$ ranges over the enumerated transactions $\{1,2, \ldots\}$.
The following derivation of $\mathrm{E} \Psi$ differs slightly from that in Chap. 5 of Loc. The price $M_{i}>0$ of a commodity-product can be decomposed into the value added from each firm involved directly or indirectly in producing it and bringing it to the market. We denote the sum of the value added across all these $f$ firms by

$$
M_{i}=A_{i, 1}+A_{i, 2}+\cdots+A_{i, f}=\bar{A}_{i}
$$

where some of the terms may be 0 . (Note that some of these firms may not have made any profit, or may have made a loss, in producing their share of the final product.) ${ }^{10}$ Each of the firms also expended a certain amount of labor time with wage costs that sum up to

$$
V_{i, 1}+V_{i, 2}+\cdots+V_{i, f}=\bar{V}_{i}
$$

We can now express the specific price as a product of two positive factors:

$$
\begin{equation*}
\Psi_{i}=\frac{\bar{V}_{i}}{L_{i}} \cdot \frac{\bar{A}_{i}}{\bar{V}_{i}} \tag{B.22}
\end{equation*}
$$

Thus, we can interpret the specific price as the product of two random factors: an averaged pay per hour of work and the (inverse) of an averaged wage share.

[^90]The mean (B.21) can now be written as

$$
\begin{aligned}
\mathrm{E} \Psi & =\sum_{i} \pi_{i}\left(\frac{\bar{V}_{i}}{L_{i}} \frac{\bar{A}_{i}}{\bar{V}_{i}}\right) \\
& =\sum_{i}\left(\frac{L_{i}}{\sum_{j} L_{j}}\right) \frac{\bar{A}_{i}}{L_{i}} \\
& =\frac{\sum_{i} \bar{A}_{i}}{\sum_{j} L_{j}} \\
& =\left(\frac{\sum_{k} \bar{V}_{k}}{\sum_{j} L_{j}}\right) \cdot\left(\frac{\sum_{k} \bar{V}_{k}}{\sum_{i} \bar{A}_{i}}\right)^{-1}
\end{aligned}
$$

The first factor is the ratio of the total amount of wages paid out to the total labor expended across all products in a transaction during a given period. This ratio will, therefore, be close to the expected wage rate $\mathrm{E} W$. The second factor is the reciprocal of the ratio of the total amount of wages paid out to the total value added during the same period. This ratio will, therefore, be close to the wage share $\omega$. We, therefore, conclude that

$$
\mathrm{E} \Psi \simeq \mathrm{E} W \cdot \omega^{-1}
$$

provides a good approximation of the expected specific price, as stated in (3.8).

## Specific Price of Product Basket

Now consider a basket $\mathcal{B}$ containing a random sample of $m$ commodity-products in transaction. Its specific price can be expressed as a weighted sum:

$$
\bar{\Psi}(\mathcal{B})=\frac{M(\mathcal{B})}{L(\mathcal{B})}=\sum_{j=1}^{m} w_{j} \Psi_{j}
$$

where the weights are $w_{j}=L_{j} / \sum_{k=1}^{m} L_{k}$. As the sample size $m$ increases toward the total number of transactions in the period under consideration, we have

$$
\bar{\Psi}(\mathcal{B}) \rightarrow \mathrm{E} \Psi
$$

## Log-normal Approximation

In Chap. 5 of LOC, it was hypothesized that specific price $\Psi$ is approximately normally distributed. Using estimates of specific prices for baskets of outputs of several industry sectors in Japan and Sweden, there is evidence that the distribution is slightly rightskewed, unlike a symmetric normal distribution; see Zachariah (2006, Sect. 4.2). Indeed, Fröhlich (2013) found the empirical distribution to be better approximated by a log-normal distribution.

The findings are consistent with the fact that $\Psi$ is a positive random variable that can be viewed as a product of two weakly dependent positive factors (B.22). This motivates a hypothesized log-normal distributional form, that is,

$$
\begin{equation*}
\Psi \sim \log -\operatorname{Normal}\left(m, s^{2}\right) \tag{B.23}
\end{equation*}
$$

where the parameter $s$ is positive and $m$ is real-valued. Let us now see how macroconstraints prescribed by (3.8) and (3.9) determine the shape of this hypothesized distribution.

Using the derived constraint on the expected specific price in (3.8), we obtain

$$
\begin{equation*}
\mathrm{E} \Psi=\frac{\mathrm{E} W}{\omega} \Leftrightarrow \exp \left(m+\frac{s^{2}}{2}\right)=\frac{\mathrm{E} W}{\omega} \tag{B.24}
\end{equation*}
$$

The bulk of specific prices realized in market exchanges must be sufficient to cover the wage costs. This viability constraint is partly captured in (3.9) regarding the proportion of bought-and-sold labor content with market price $M$ that fails to exceed the costs of the average labor time expended when paid at the average wage rate, i.e., $L \cdot \mathrm{E} W$. We denote this proportion by $\gamma$. Then

$$
\begin{equation*}
\mathrm{P}(\Psi \leq \mathrm{E} W)=\gamma \Leftrightarrow \operatorname{erf}\left(\frac{\ln \mathrm{E} W-m}{s \sqrt{2}}\right)=2 \gamma-1 \tag{B.25}
\end{equation*}
$$

Using (B.24), we can eliminate one of the distributional parameters:

$$
m=\ln E W-\ln \omega-\frac{s^{2}}{2}
$$

Substituting this in (B.25) results in a quadratic constraint on $s$ :

$$
\ln \omega+\frac{s^{2}}{2}=s \sqrt{2} \underbrace{\operatorname{erf}^{-1}(2 \gamma-1)}_{f(\gamma)} \Leftrightarrow s^{2}-s 2 \sqrt{2} f(\gamma)+2 \ln \omega=0 .
$$

This equation has two solutions but only one of which is positive:

$$
s(\omega, \gamma)=\sqrt{2} f(\gamma)+\sqrt{2 f^{2}(\gamma)+2 \ln \left(\omega^{-1}\right)}
$$



Fig. B. 2 Distribution of specific price $\Psi$ derived in (B.26), with wage share $\omega$, proportion $\gamma=$ $\mathrm{P}(\Psi \leq \mathrm{E} W)$, and average wage rate $\mathrm{E} W=\$ 25$ per hour. Solid curve: $\omega=50 \%$ and $\gamma=10 \%$. Dashed curve: $\omega=50 \%$ and $\gamma=1 \%$. Note that the mean $\mathrm{E} \Psi=\$ 50$ per hour in both cases. Dotted curve: $\omega=60 \%$ and $\gamma=10 \%$, which lowers the mean
since $-\ln \omega>0$.
This allows us, finally, to express the hypothesized distribution of specific prices (B.23) in terms of the average wage rate $\mathrm{E} W$, the wage share $\omega$, and the proportion $\gamma$ :

$$
\begin{equation*}
\Psi \sim \log -\operatorname{Normal}\left(\ln \frac{\mathrm{E} W}{\omega}-\frac{s^{2}(\omega, \gamma)}{2}, s^{2}(\omega, \gamma)\right) \tag{B.26}
\end{equation*}
$$

Figure B. 2 illustrates how this distribution is shaped by the parameters $\gamma$ and $\omega$ : The dispersion of specific prices increases with the proportion $\gamma=\mathrm{P}(\Psi \leq \mathrm{E} W)$ and decreases as the wage share $\omega$ rises. ${ }^{11}$

## B. 4 Probabilistic Law of Decreasing Labor Content

The probabilistic mechanism that in part drives LDLC was first derived in Chap. 7 of LOC. We restate the derivation here for completeness.

For an input replacement

$$
\mathcal{B}_{i-1} \rightarrow \mathcal{B}_{i}
$$

let $C_{i}>1$ denote the cheapening factor and

$$
D_{i}=\frac{L\left(\mathcal{B}_{i-1}\right)}{L\left(\mathcal{B}_{i}\right)}
$$

denote the resulting L-content ratio. Then taking the logarithm of the ratio, we have according to (3.17):

[^91]$$
\log D_{i}=\log C_{i}+\log \bar{\Psi}\left(\mathcal{B}_{i}\right)-\log \bar{\Psi}\left(\mathcal{B}_{i-1}\right)
$$

Using this alternative form, we can show that the probability that an input replacement reduces the L-content of non-labor inputs is greater than $1 / 2$ :

$$
\begin{aligned}
\mathrm{P}\left(D_{i}>1\right) & =\mathrm{P}\left(\log D_{i}>0\right) \\
& =\mathrm{P}\left(\log C_{i}+\log \bar{\Psi}\left(\mathcal{B}_{i}\right)>\log \bar{\Psi}\left(\mathcal{B}_{i-1}\right)\right) \\
& >\mathrm{P}\left(\log \bar{\Psi}\left(\mathcal{B}_{i}\right)>\log \bar{\Psi}\left(\mathcal{B}_{i-1}\right)\right) \\
& =\frac{1}{2}
\end{aligned}
$$

where the inequality in the third line follows from $\log C_{i}>0$. Since the input replacement occurs over a brief period in which structural change is assumed to be negligible, we take the distribution of specific prices to be unchanged. Thus, by symmetry, we have

$$
\mathrm{P}\left(\bar{\Psi}\left(\mathcal{B}_{i}\right)>\bar{\Psi}\left(\mathcal{B}_{i-1}\right)\right) \equiv \mathrm{P}\left(\bar{\Psi}\left(\mathcal{B}_{i}\right)<\bar{\Psi}\left(\mathcal{B}_{i-1}\right)\right),
$$

from which it follows that both sides equal $\frac{1}{2} .{ }^{12}$
Next, we examine the expected value

$$
\mathrm{E} \log D_{i}=\mathrm{E} \log C_{i}+\mathrm{E} \log \bar{\Psi}\left(\mathcal{B}_{i}\right)-\mathrm{E} \log \bar{\Psi}\left(\mathcal{B}_{i-1}\right),
$$

where the last two terms in the expression above are equal and cancel each other. This results in

$$
\mathrm{E} \log D_{i}=\mathrm{E} \log C_{i}>0,
$$

since $C_{i}>1$. Next, we apply Jensen's inequality to obtain

$$
\log \mathrm{E} D_{i} \geq \mathrm{E} \log D_{i}>0,
$$

from which we conclude that the expected L-content ratio is greater than 1, i.e.,

$$
\log \mathrm{E} D_{i}>0 \Leftrightarrow E D_{i}>1 .
$$

Finally, we consider a sequence of $m$ replacements. The resulting L-content ratio

$$
D_{m}^{*}=\frac{L\left(\mathcal{B}_{0}\right)}{L\left(\mathcal{B}_{m}\right)}
$$

equals the product of all independent ratios: $D_{1} D_{2} \cdots D_{m}$. Under the negligible structural change, these $m$ ratios are also identically distributed. Thus, taking the logarithm of $D_{m}^{*}$ yields a sum of $m$ independent and identically distributed terms:

[^92]$$
\log D_{m}^{*}=\log D_{1}+\log D_{2}+\cdots \log D_{m}
$$

Let $c=\mathrm{E} \log D_{i}>0$; then we can write

$$
\mathrm{E} \log D_{m}^{*}=m c>0
$$

By applying the Law of Large Numbers,

$$
\mathrm{P}\left(\left|\log D_{m}^{*}-m c\right| \leq m \varepsilon\right) \rightarrow 1
$$

as $m$ increases, for any positive constant $\varepsilon$. Therefore, the probability of reducing L-content after $m$ replacements, i.e.,

$$
\begin{aligned}
\mathrm{P}\left(L\left(\mathcal{B}_{m}\right)<L\left(\mathcal{B}_{0}\right)\right) & =\mathrm{P}\left(D_{m}^{*}>1\right) \\
& =\mathrm{P}\left(\log D_{m}^{*}>0\right)
\end{aligned}
$$

approaches 1 .

## B. 5 Dynamics of L-Content

We examine changes in the L -content of product baskets over time. To obtain readable final expressions, we will often make use of the following first-order approximation:

$$
(1+a)(1+b) \simeq 1+a+b
$$

when constants are small: $|a| \ll 1$ and $|b| \ll 1$.

## Demand for Products and Employment

Let $s$ denote the size of the population and $c$ be the per-capita demand for a given product-type. The basket of demanded products of the same type for the year $t$ is then denoted:

$$
\mathcal{B}_{t}=\left\{c_{t} s_{t}\right\} .
$$

In the following year, the basket is

$$
\mathcal{B}_{t+1}=\left\{(1+\kappa) c_{t}(1+g) s_{t}\right\},
$$

where $\kappa$ is the growth rate of per-capita demand and $g$ is the population growth rate.

Suppose the L-content of $\mathcal{B}_{t}$ decreases at a rate $\delta^{\prime}$. Then the total labor required to meet the demand is

$$
\begin{aligned}
L_{t+1}\left(\mathcal{B}_{t+1}\right) & =(1+\kappa)(1+g) L_{t+1}\left(\mathcal{B}_{t}\right) \\
& =\left(1-\delta^{\prime}\right)(1+\kappa)(1+g) L_{t}\left(\mathcal{B}_{t}\right) .
\end{aligned}
$$

In other words, the growth rate of total labor requirements is

$$
\left(1-\delta^{\prime}\right)(1+\kappa)(1+g)-1 \simeq-\delta^{\prime}+\kappa+g
$$

as stated in (5.3). Compare also the discussion in Pasinetti (1993, Chap. 4).

## Bounding the rate of LDLC

Suppose that the L-content of the basket of material wealth in a given year is a certain factor $\beta$ times the total labor added. That is,

$$
\mathbb{W}=\beta \cdot \mathbb{L}
$$

For this L-content not to fall persistently, (5.5) must be satisfied when the growth of the workforce is negligible. We can express this condition as

$$
(1-\delta)(1-d) \beta \mathbb{L}+(1-\omega) \mathbb{L} \geq \beta \mathbb{L}
$$

Solving for $\delta$ gives

$$
\delta \leq 1-\frac{\omega+\beta-1}{\beta(1-d)}
$$

which is used in (5.6).

## Decreasing L-content of a Selective Basket

In Sect. 5.5 we study large selective baskets made up of products from a particular group, such as agricultural products, mass-produced goods, or personal services. For a large selective basket $\mathcal{B}^{\prime}$, we have

$$
L_{t+1}\left(\mathcal{B}^{\prime}\right)=\left(1-\delta^{\prime}\right) L_{t}\left(\mathcal{B}^{\prime}\right)
$$

Iterating this recursion, we obtain the rate of decrease $\delta^{\prime}$ averaged over $T$ years by

$$
\begin{equation*}
\left(1-\delta^{\prime}\right)^{T} \equiv \frac{L_{t+T}\left(\mathcal{B}^{\prime}\right)}{L_{t}\left(\mathcal{B}^{\prime}\right)}=\frac{M_{t+T}\left(\mathcal{B}^{\prime}\right)}{M_{t}\left(\mathcal{B}^{\prime}\right)} \cdot \frac{\bar{\Psi}_{t}\left(\mathcal{B}^{\prime}\right)}{\bar{\Psi}_{t+T}\left(\mathcal{B}^{\prime}\right)} . \tag{B.27}
\end{equation*}
$$

Suppose we assume that changes in the specific price of $\mathcal{B}^{\prime}$ track that of the average specific price so that

$$
\begin{equation*}
\bar{\Psi}_{t}\left(\mathcal{B}^{\prime}\right)=\beta \cdot \mathbf{E} \Psi_{t} \tag{B.28}
\end{equation*}
$$

is a reasonable approximation. Then we can obtain an estimate of the average rate $\delta^{\prime}$ using two different approximations of the average specific price.

- In the first case, we use

$$
\mathrm{E} \Psi_{t} \simeq \frac{\mathrm{E} W_{t}}{\omega_{t}}
$$

and inserting it into (B.28) and (B.27), we obtain

$$
\left(1-\delta^{\prime}\right)^{T} \simeq \frac{\omega_{t+T}}{\omega_{t}} \cdot \frac{M_{t+T}\left(\mathcal{B}^{\prime}\right) / \mathrm{E} W_{t+T}}{M_{t}\left(\mathcal{B}^{\prime}\right) / \mathrm{E} W_{t}},
$$

and this yields (5.18).

- In the second case, we use

$$
\mathrm{E} \Psi_{t} \simeq \frac{M_{t}\left(\mathcal{B}_{\mathrm{ref}}\right)}{L_{t}\left(\mathcal{B}_{\mathrm{ref}}\right)},
$$

and inserting it into (B.28) and (B.27), we obtain

$$
\left(1-\delta^{\prime}\right)^{T} \simeq \underbrace{\frac{L_{t+T}\left(\mathcal{B}_{\mathrm{ref}}\right)}{L_{t}\left(\mathcal{R}_{\mathrm{ref}}\right)}}_{(1-\delta)^{T}} \cdot \frac{M_{t+T}\left(\mathcal{B}^{\prime}\right) / M_{t+T}\left(\mathcal{B}_{\mathrm{ref}}\right)}{M_{t}\left(\mathcal{B}^{\prime}\right) / M_{t}\left(\mathcal{B}_{\mathrm{ref}}\right)},
$$

and this yields (5.19).

## Growth of output

Recall that the monetary growth rate of output is

$$
\begin{equation*}
\frac{M\left(\mathcal{B}_{t+1}\right)-M\left(\mathcal{B}_{t}\right)}{M\left(\mathcal{B}_{t}\right)}=\frac{M\left(\mathcal{B}_{t+1}\right)}{M\left(\mathcal{B}_{t}\right)}-1, \tag{B.29}
\end{equation*}
$$

using real prices.
We have that $M\left(\mathcal{B}_{t}\right) \simeq \mathbb{L}_{t} \cdot \mathrm{E} \Psi_{t}$, as well as

$$
\mathbb{L}_{t+1}=(1+\ell) \mathbb{L}_{t} \quad \text { and } \quad \mathrm{E} \Psi_{t+1} \simeq \frac{1}{1-\delta} \mathrm{E} \Psi_{t} .
$$

Inserting these relations into (B.29) yields

$$
\frac{1+\ell}{1-\delta}-1=(1+\ell)\left(\sum_{k=0}^{\infty} \delta^{k}\right)-1
$$

using geometric series and the fact that $|\delta|<1$. Then the first-order approximation is given in (7.5).

## Change in L-content Purchasing Power

Consider a nominal sum of money $M$, which grows at a rate $r$ from one year to the next. Then its L-content purchasing power changes as

$$
\frac{M}{\mathrm{E} \Psi_{t}} \rightarrow(1+r) \frac{M}{\mathrm{E} \Psi_{t+1}},
$$

which implies, for the L-content purchasing power of $M$, a growth rate of

$$
(1+r) \frac{\mathrm{E} \Psi_{t}}{\mathrm{E} \Psi_{t+1}}-1 \simeq(1+r)(1-\delta) \frac{M_{t}(\mathcal{B})}{M_{t+1}(\mathcal{B})}-1
$$

using (5.10). Now let

$$
p=\frac{M_{t+1}(\mathcal{B})-M_{t}(\mathcal{B})}{M_{t+1}(\mathcal{B})}
$$

denote the rate of price inflation of a fixed sample basket $\mathcal{B}$. Then the growth rate of L-content purchasing power equals

$$
(1+r)(1-\delta)(1-p)-1 \simeq r-\delta-p
$$

as summarized in (7.17).

## Dynamics of accumulation

Consider E $R$, the average profit rate (7.7), and use the dynamic relations (7.3) and (7.8). Then using the notation of Sect. 7.2, we can express the average rate in year $t+1$ as

$$
\begin{align*}
\mathrm{E} R_{t+1} & =\alpha_{\max } \frac{\mathbb{L}_{t+1}}{\mathbb{K}_{t+1}} \\
& =\alpha_{\max } \frac{(1+\ell) \mathbb{L}_{t}}{\alpha \mathbb{L}_{t}+(1-\delta)(1-d) \mathbb{K}_{t}} \\
& =\alpha_{\max } \frac{\mathbb{L}_{t}}{\mathbb{K}_{t}} \cdot \frac{(1+\ell)}{\alpha \frac{\mathbb{L}_{t}}{\mathbb{K}_{t}}+(1-\delta)(1-d)}  \tag{B.30}\\
& =\mathrm{E} R_{t} \cdot \frac{(1+\ell)}{\frac{\alpha}{\alpha_{\max }} \mathrm{E} R_{t}+(1-\delta)(1-d)}
\end{align*}
$$

which is a recursive relation of the form described in (7.9).
Now let $x_{t}=\frac{1}{\mathrm{E} R_{t}}>0$ be the reciprocal of the average profit rate. Then we can express (B.30) as a linear difference equation:

$$
x_{t+1}=\frac{\alpha}{\alpha_{\max }(1+\ell)}+\frac{(1-\delta)(1-d)}{(1+\ell)} x_{t}=a_{0}+a_{1} x_{t} .
$$

This system is stable as long as the workforce, and thus output, is not dramatically shrinking:

$$
\left|a_{1}\right|<1 \Leftrightarrow \ell>(1-\delta)(1-d)-1 \approx-5 \% \text { per year, }
$$

to use a conservative figure. Then the steady-state value of $x_{t}$ is

$$
x^{*}=\frac{a_{0}}{1-a_{1}}=\frac{\alpha}{\alpha_{\max }}\left(\frac{1}{(1+\ell)-(1-\delta)(1-d)}\right) \neq 0
$$

that corresponds to a steady-state level of the average profit rate

$$
\begin{aligned}
\mathrm{E} R & =\frac{\alpha_{\max }}{\alpha}((1+\ell)-(1-\delta)(1-d)) \\
& \simeq \frac{\alpha_{\max }}{\alpha}(\ell-\delta-d),
\end{aligned}
$$

which is result (7.10). ${ }^{13}$
The same approach can be used to study the ratio

$$
\begin{aligned}
\frac{\mathbb{W}_{t+1}}{\mathbb{L}_{t+1}} & =\frac{\alpha \mathbb{L}_{t}+(1-\delta)(1-d) \mathbb{K}_{t}}{(1+\ell) \mathbb{L}_{t}} \\
& =\frac{\alpha}{1+\ell}+\frac{(1-\delta)(1-d)}{1+\ell} \frac{\mathbb{W}_{t}}{\mathbb{L}_{t}}
\end{aligned}
$$

[^93]Setting $x_{t}=\mathbb{W}_{t} / \mathbb{L}_{t}$, we obtain as the steady state of this ratio:

$$
\begin{aligned}
\frac{\mathbb{W}}{\mathbb{L}} & =\frac{\alpha}{1+\ell-(1-\delta)(1-d)} \\
& \simeq \frac{\alpha}{\ell+\delta+d} \\
& \leq \frac{\alpha}{d}
\end{aligned}
$$

This corresponds to the upper bound in (7.12).

## Glossary

Commodity A usable whose ownership is, or is intended to be, transferred in a transaction of sale-and-purchase. (See Sect. 2.1.)
Commodity-product A usable that is both a commodity and a product. (See Sect. 2.1.)
Distribution of a statistical variable A mathematical function that gives the proportion of the values of the statistical variable that lie within any given range.
Expected value of a random variable Its mean (or average) value.
Labor content, L-content A measure of products that assign to each product the total - direct and indirect - input of labor (say, number of worker-hours) needed for its production. (See Sect. 2.3.)
L-content purchasing power of a given sum of money is the L-content of a representative sample basket of products it can purchase. (See Sect. 3.3.)
L-wage rate Wage rate measured in terms of its L-content purchasing power. The L-wage rate per hour equals the L-content of a sample basket of products it can purchase. (See Sect. 6.1.)
Labor share (of L-content of total net product) The ratio, denoted by $\lambda$, between the L-content of the part of the net product allotted to the workers employed in its production and the total L-content of that net product.(See Sect. 2.3.)
Price, real (standardized) Price adjusted to take account of inflation, so as to keep the total price of a fixed reference basket of products constant in time.
Product A usable whose creation requires direct or indirect input of labor. (See Sect. 2.1.)
Productivity of labor, material The quantity of a given type of product whose Lcontent is one worker-hour. It is the arithmetical inverse (reciprocal) of the Lcontent of one unit of product. The rate of change of material productivity is defined also for a basket of products. (See Sect. 5.1.)
Productivity of labor, monetary The monetary value added per one worker-hour (or per one worker) employed in its production. (See 5.4.)

Random variable (statistical variable) A variable quantity whose numerical value depends on the outcome of a random event. In statistics a variable that has a numerical value for each member of a large population is regarded as a random variable.
Specific price A random variable, denoted by $\Psi$, whose value for an individual commodity-product is the ratio between its price and its L-content. The specific price of a basket of commodity-products is the ratio between its total price and its total L-content. (See Sect. 3.3.)
Usable Any object or service of use to human beings. (See Sect. 2.1.)
Value added The difference between the sale price of a commodity-product or a basket of commodity-products and the cost of the non-labor inputs used up in its production.
Wage share (of value added) The ratio, denoted by $\omega$, between the total pay of the workers employed in production of a given output and the value added of that output. (See Sect. 3.2.)

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[^0]:    ${ }^{1}$ See https://en.wikipedia.org/wiki/Econophysics. However, note that the term 'econophysics' also has other meanings.

[^1]:    ${ }^{1}$ Milanovic (2019, ch. 1.1).
    ${ }^{2}$ For one example among many, see Fernandez and Krauss (2014).

[^2]:    ${ }^{3}$ Keynes (1936, preface).
    ${ }^{4}$ Romer (2016, p. 1).

[^3]:    ${ }^{5}$ See Sect. 7.1.
    ${ }^{6}$ See Sect. 5.2.
    ${ }^{7}$ See Sect. 7.3.
    ${ }^{8}$ See Sect. 7.4.
    ${ }^{9}$ See Sect. 5.3.

[^4]:    ${ }^{10}$ Farjoun and Machover (1983), see also a review by Champernowne (1985).
    ${ }^{11}$ See Sect. 3.6 for further references.
    ${ }^{12}$ Hence the adjective 'probabilistic', from the noun 'probability', to reflect this indeterminacy.
    ${ }^{13}$ See e.g. Cottrell et al. (2009) and Cockshott (2020).
    ${ }^{14}$ The traditional term 'labor value' is avoided because 'value' has unrelated connotations and suggests a stronger, more rigid connection to prices and other indicators, which is avoided here.
    ${ }^{15}$ Early figures in this tradition were Thomas Hodgskin, William Thompson, John Gray and John Francis Bray; see Siméon (2020, pp. 179-180). Karl Marx became a prominent thinker in the tradition.

[^5]:    ${ }^{1}$ Compare (Marx 1875): "Labor is not the source of all wealth. Nature is just as much the source of use values (and it is surely of such that material wealth consists!) ..."

[^6]:    ${ }^{2}$ See for instance (Mauss et al. 2000).

[^7]:    ${ }^{3}$ Let $\mathcal{B}_{\text {total }}$ denote the basket of total products and $\mathcal{B}_{\text {input }}$ the products used up in production. Then under economic reproduction we may write the (net) output basket as a residual $\mathcal{B}_{\text {out }}=\mathcal{B}_{\text {total }} \backslash \mathcal{B}_{\text {input }}$.

[^8]:    ${ }^{4}$ The measure corresponds roughly to the notion of economic 'value' developed by Ricardo and Marx, see our remarks in Sect. 2.4.
    ${ }^{5}$ An example of an operational measure is given in Sect. B.1.

[^9]:    ${ }^{6}$ This is a consequence of compounding. If $L(\mathcal{B})$ is reduced by $1 \%$ per year, then after 70 years: $L(\mathcal{B}) \cdot(1-0.01)^{70} \simeq L(\mathcal{B}) / 2$. Similarly, at a rate of $10 \%$ per year, we have that after 7 years: $L(\mathcal{B}) \cdot(1-0.10)^{7} \simeq L(\mathcal{B}) / 2$.

[^10]:    ${ }^{7}$ Marx (1894, Chap. 47).
    ${ }^{8}$ See for instance (Weiss 1995).
    ${ }^{9}$ Smith (1776, Book 1, Chap. 5).

[^11]:    ${ }^{10}$ Ricardo (1817, Chap. 1).
    ${ }^{11}$ Ibid.
    ${ }^{12}$ Ibid.
    ${ }^{13}$ Marx (1865, Sect. IV).

[^12]:    ${ }^{14}$ Ricardo (1817, Chap. 1).

[^13]:    ${ }^{1}$ The wage is thus really a price, or more accurately a rent, for this entitlement rather than the purchase of the capacity itself, as under slavery. In Arabic, the words for wage or salary and for rent on property such as a dwelling are derived from the same root: a-j-r. The same holds for Hebrew, which uses the root $\delta$-k-r for both. Some forms of employment, such as indenture, are arguably intermediate between slavery and hire. A glorified and well-paid instance of this is the condition of professional football players: they are sold and bought by football clubs, which are of course capitalist firms.
    ${ }^{2}$ See, e.g., Aston et al. (1987) and Allen (2009) for debates among economic historians about the origins and boundary conditions for capitalism.

[^14]:    ${ }^{3}$ Thus $W$ can be treated as a random variable.

[^15]:    ${ }^{4}$ We may refer to such a stable state as 'statistical equilibrium' in which structural macroeconomic variables are (virtually) unchanged but microeconomic configurations can change. See also Sect. B. 2 for a more technical description.

[^16]:    ${ }^{5}$ This procedure dates back to Adam Smith's decomposition of price into total wage costs and total profits.
    ${ }^{6}$ Recall that the value added in production is the price at which the product is sold minus the costs of its non-labor inputs. It also equals the sum of wages paid for its production and the profit made through its sale. This is the basis for value-added taxes (VAT) levied on the price of goods and services in many countries.

[^17]:    ${ }^{7}$ See Sect. B. 3 for a more detailed explanation of the properties of specific prices.
    ${ }^{8}$ See Sect. B. 3 for further details.

[^18]:    ${ }^{9}$ This follows using a theorem known as the Law of Large Numbers, see also Sect. B.3.

[^19]:    ${ }^{10}$ The purchasing power of a monthly income has therefore the unit of worker-hours per month. In other words, income flows have the dimension [workers] which represents the number of people needed to produce sample baskets that income recipients can appropriate through market exchange.

[^20]:    ${ }^{11}$ In light of new data on US firm profitability after the 1980s, we revise the assessment in LOC; see Scharfenaker and Semieniuk (2017, p. 482).
    ${ }^{12}$ That is, if the initial capital stock is $K_{i}$, then after seven years it will be $K_{i} \cdot(1+0.10)^{7} \simeq K_{i} \cdot 2$, leaving aside depreciating factors.

[^21]:    ${ }^{13}$ Further consequences of the constraints will be discussed in Chap. 8.

[^22]:    ${ }^{14}$ This result and the following one are explained in more detail in Sect. B.4.
    ${ }^{15}$ Input replacements can be understood as a form of 'capital-saving and labor-preserving' change, while the labor-saving changes above are 'capital-preserving'. The analysis of both types of changes covers intermediate mixed forms of technical changes as well. Joint labor-saving and capital augmenting change, however, requires further analysis beyond the scope of this book.

[^23]:    ${ }^{16}$ See Samuelson and Nordhaus (2009) for a paradigmatic textbook.
    ${ }^{17}$ For a more technical example, see Sect. B.2.

[^24]:    ${ }^{18}$ Also known as the 'Cobb-Douglas' production function (Cobb and Douglas, 1928). See e.g. Samuelson and Nordhaus (2009, Appendix 7), Foley et al. (2019, Chap. 3) and Pasinetti (1977, Chap. 1) for further details.
    ${ }^{19}$ Clark (1908, preface).
    ${ }^{20}$ Given (3.19), $A$ is measured units of [dollars ${ }^{1-\beta} \cdot$ hours $^{\beta} /$ year].
    ${ }^{21}$ See Foley et al. (2019, Chap. 8.3).

[^25]:    ${ }^{22}$ This result was explicitly derived and strikingly demonstrated by Shaikh (1974). See also Felipe and McCombie (2014) for further discussion.
    ${ }^{23}$ Maxwell (1873).
    ${ }^{24}$ See Morishima (1973), Pasinetti (1977), Steedman (1977), Roemer (1981) for formal treatments from this period.

[^26]:    ${ }^{25}$ Even in highly idealized models of the economy, profit rate equalization is brought about only through a delicate balance of price and quantity adjustment mechanisms, see Dupertuis and Sinha (2009) and Wright (2017).
    ${ }^{26}$ Some related works do not, however, consider how a proportion of total capital is distributed across different profit rate intervals, but instead the proportion of firms. But this makes no distinction between the weight that a tea shop and an airplane factory has in a capitalist economy.

[^27]:    ${ }^{1}$ From a wider ecological point of view one should also consider the labor of cleaning-up or managing waste by-products.

[^28]:    ${ }^{2}$ Thus the concerns, raised by an astute economist, Joan Robinson, about quantifying 'labour of different degrees of skill in terms of a unit of "simple labour"" (Robinson 1966, p. 19) is simply irrelevant for our measure. See also Appendix II in LOC.

[^29]:    ${ }^{3}$ Kantor et al. (2021).
    ${ }^{4}$ This is a consequence of the second law of thermodynamics and common to all life forms.
    ${ }^{5}$ State of the Union Address, December 3, 1861.

[^30]:    ${ }^{6}$ See discussions in Pasinetti (1977) and Kurz and Salvadori (1997).
    ${ }^{7}$ Altman (2020).
    ${ }^{8}$ Laboring capacity in the abstract was a central concept used in Marx (1867).

[^31]:    ${ }^{9}$ The challenges of quantifying depreciation of durable products are raised in Appendix A.

[^32]:    ${ }^{1}$ We say 'material labor productivity' to distinguish this L-content-based measure from the conventional price-based measure of productivity discussed below in Sect. 5.4.
    ${ }^{2}$ We may express this rate as $\delta=\frac{L_{t}(\mathcal{B})-L_{t+1}(\mathcal{B})}{L_{t}(\mathcal{B})}$ which of course can vary with $t$.
    ${ }^{3}$ Repeated application of $(5.1)$ yields $L_{t+25}(\mathcal{B})=(1-\delta)^{25} L_{t}(\mathcal{B})$. Using (5.2), we can therefore identify $(1-\delta)^{25}=1 / 2$ and solve for $\delta$ to obtain an average rate of 0.027 or $2.7 \%$.

[^33]:    ${ }^{4}$ See Wikipedia contributors (2021d).

[^34]:    ${ }^{5}$ See Sect. B. 5 for derivation of the approximation (5.3).

[^35]:    ${ }^{6}$ Compare the problems posed in Appendix A.

[^36]:    ${ }^{7}$ For the dramatic increase in relative weight of the service sectors in the United States, see (Johnston 2021) documenting that employment in services reached almost $80 \%$ of labor force.
    ${ }^{8}$ Piketty (2014, p. 67). It is not entirely clear what measure of labor productivity is being used here, but as we shall see this makes little difference.

[^37]:    ${ }^{9}$ Baumol (1967) is an early comment on the effects of differential growth rates of productivity between many services and industrial products.
    ${ }^{10}$ See Table 7.3 as well as the derivation in Sect. 7.3.

[^38]:    ${ }^{11}$ This will be relaxed in Chap. 7.
    ${ }^{12}$ This estimate comes from the monetary depreciation rate of capital stocks, which provides a reasonable upper bound on $d$. We consider the complete accounting and estimation of the wear-and-tear of durable capital goods, both in monetary terms and in terms of labor content, to be an open research problem (see Appendix A).
    ${ }^{13}$ See Sect. B.5.

[^39]:    ${ }^{14}$ If we use a more realistic accumulation fraction of $25 \%$, the bound in (5.8) doubles to $10.8 \%$. See also Sect. 7.3. There are also expenditures paid for by credit with an L-content purchasing power that is to be recovered by creditors; see Sect. 7.4. From Yashiv and Merz (2004, Table 1, p. 1423), we can deduce that the fraction of labor input is roughly $15-17 \%$ across a wide range of firms. Consider, for instance, IBM as a typical large company, its dollar worth being close to $\$ 200$ billion in 2020. It has roughly 400,000 employees, earning, on average, somewhat less than $\$ 100,000$ a year, in total somewhat less than $\$ 40$ billion. A simple calculation leads to the ratio of monetary wage to capital being $18-20 \%$.

[^40]:    ${ }^{15}$ Keynes (2010).

[^41]:    ${ }^{16}$ In an early transitional 'putting-out' form of capitalism, workers were formally employed as sub-contractors. This formal fiction has reemerged from time to time, including the present, as a means of super-exploitation.
    ${ }^{17}$ Strictly speaking, these are not products as defined above in Chap. 2; but we shall still refer to them as such, for lack of a more convenient term.

[^42]:    ${ }^{18}$ See Flaschel et al. (2013) for an example.
    ${ }^{19}$ A variant of the standard measure uses the number of workers instead of the labor time taken in production. This variant has the dimension money/worker.

[^43]:    ${ }^{20}$ See also Sect. B.1.

[^44]:    ${ }^{21}$ Also known as inflation-adjusted prices. Similarly, if these prices involve more than one currency, then they better be converted to units of purchasing power parity (PPP).
    ${ }^{22}$ Since a positive rate, $\delta>0$, means that L-content decreases, a positive numerator in (5.15) implies increase in productivity $P$. An equivalent, but slightly more symmetric form is

    $$
    \begin{equation*}
    \delta=-\frac{\Delta L(\mathcal{B})}{\Delta t} \simeq \frac{\Delta P}{\Delta t} . \tag{5.16}
    \end{equation*}
    $$

    Here, $\Delta P$ denotes the change in productivity over a time period $\Delta t$, while $\Delta L(\mathcal{B})$ is the change in the L -content of this basket.

[^45]:    ${ }^{23}$ See Sect. B. 5 .

[^46]:    ${ }^{24}$ In the special case when $\mathcal{B}^{\prime}=\mathcal{B}_{\text {ref }}$, the indexed price of the basket does not change. Then the fraction in (5.19) is equal to 1 and $\delta^{\prime}=\delta$ as expected.

[^47]:    ${ }^{25}$ A good source is Wang et al. (2015).

[^48]:    ${ }^{26}$ Compare Fig. 2 in EU-commission (2016, p.4). This report contains a wealth of relevant data.

[^49]:    ${ }^{27}$ Using data in Lowrey (2014), OECD (2021b), NCES (2021).
    ${ }^{28}$ Using data from Rodrigue (2020) and Johnston (2021).
    ${ }^{29}$ This estimate suggests that the overall rate of LDLC has declined in the US economy over the last two-three decades. We shall return to this point in Chap. 7.

[^50]:    ${ }^{1}$ Note that $\bar{W}$ is a dimensionless number.
    ${ }^{2}$ Using (3.12).

[^51]:    ${ }^{3}$ The skewed form of a wage distribution was a structural feature pointed out in Chap. 5 of LOC.
    ${ }^{4}$ See, e.g., Cowell (2011), Souma (2001), and Arata (2013).

[^52]:    ${ }^{5}$ See also the remarks on p. 222 in LOC.

[^53]:    ${ }^{6}$ See, for instance, (Wright 1997).
    ${ }^{7}$ See Sect. B.5.
    ${ }^{8}$ Excluding here the rare occurrence of price deflation.

[^54]:    ${ }^{9}$ A commonly used measure of income inequality is the Gini coefficient, which is bounded between $0 \%$ and $100 \%$. For the log-normal distribution of $\bar{W}$, this measure is a function of the ratio (6.5) alone. That is, Gini coefficient $=f\left(\bar{W}_{\text {median }} / E \bar{W}\right)($ Cowell 2011, Table A.2). See Sect. B.2. Note that a variable wage inequality disqualifies the exponential distribution, commonly used to model incomes, as a model for wages.

[^55]:    ${ }^{10}$ As demonstrated by Piketty (2014) and many others.
    ${ }^{11}$ Note, for instance, his numerical examples in Marx (1867, Chap. 11) and Marx (1885, Chap. 20) and his collaborator Engels' access to factory wage data.
    ${ }^{12}$ ILO-OECD (2015, p. 4, Fig. 1).

[^56]:    13 (Wikipedia contributors, 2021a).
    ${ }^{14}$ Compare, the open question in Appendix A.
    ${ }^{15}$ See Sect. B.2.
    ${ }^{16}$ For a recent example among many, consider Amazon's treatment of its workers reported in Streitfeld (2021).

[^57]:    ${ }^{17}$ This is necessarily so if capitalism is to persist.

[^58]:    ${ }^{18}$ See Appendix A.

[^59]:    ${ }^{19}$ See, for example, (ILO-OECD, 2015). It should be noted, however, that there are various definitions of wage share that may differ from our $\omega$.

[^60]:    ${ }^{20}$ Harvey (2018).
    ${ }^{21}$ Mishel et al. (2015) provides a good summary for the US.
    ${ }^{22}$ Klein (2012).
    ${ }^{23}$ Permission granted by the author.

[^61]:    ${ }^{24}$ See Thompson (2021), who estimates that the share of labor-intensive services in employment has risen to about $50 \%$ in the mature economies. We will return to this point in Chap. 7.
    ${ }^{25}$ See summary in Cooper et al. (2019).
    ${ }^{26}$ Compare Wikipedia contributors (2021c) and references therein.
    ${ }^{27}$ Using the average wage rate of $\$ 25$ per hour (2021) and a wage share of $50 \%$.

[^62]:    ${ }^{1}$ Such is the commonly used Solow-Swan model using an aggregate 'production function'; see Sect. 3.6 and Foley et al. (2019), Chap. 10 .

[^63]:    agree with the conclusion of economist and former U.S. Treasury official David Malpass that a $4 \%[/ y r]$ "growth renaissance" is possible. A few see the historic trend as a kind of limit. Even $3 \%[/ y r]$ growth would be a significant improvement over what is expected, but we firmly believe that $4 \%[/ y r]$-which has been achieved in 23 of the past 60 calendar years-can become America's New Normal. ${ }^{3}$

[^64]:    ${ }^{2}$ Irwin (2016).
    ${ }^{3}$ The Bush Institute (2012, Introduction).

[^65]:    ${ }^{4}$ See Sect. B.5.
    ${ }^{5}$ Compare Table 7.2 below.
    ${ }^{6}$ Hamnett (2020).

[^66]:    ${ }^{7}$ See, for instance, the discussion in Przeworski (1985), Chap. 4.

[^67]:    ${ }^{8}$ See Chap. 5.

[^68]:    ${ }^{9}$ Using basic properties of linear dynamical systems; see Sect. B.5.
    ${ }^{10}$ See Eurostat (2021) and statista (2021).

[^69]:    ${ }^{11}$ See Sect. B.5.

[^70]:    ${ }^{12}$ Compare Table 7.3 below.
    ${ }^{13}$ Temin (2004) reports nominal interest rates on loans in the range of $5-12 \%$ per annum.
    ${ }^{14}$ For a portfolio $\mathcal{F}=\left\{F_{1}, \ldots, F_{n}\right\}, \bar{r}$ is the weighted average of each individual rates $r_{1}, \ldots, r_{n}$ each weighted by the value $M\left(F_{i}\right)$ of its respective asset.

[^71]:    ${ }^{15}$ Using (5.10).
    ${ }^{16}$ See Sect. B.5.

[^72]:    ${ }^{17}$ See Hauner et al. (2020) for the connection between growing financial assets and imperialism prior to World War I.
    ${ }^{18}$ For justification of this, see Sect. 5.4.

[^73]:    ${ }^{19}$ See Fig. 3 in Komlos (2019).

[^74]:    ${ }^{1}$ See BGE (2021), Fred (2021), and OECD (2021a).

[^75]:    ${ }^{2}$ See Livi-Bacci (2017).

[^76]:    ${ }^{3}$ See, e.g., Kjellstrom et al. (2019).

[^77]:    ${ }^{4}$ Paybarah (2021). A similar development took place in Iceland; see BBC (2021).

[^78]:    ${ }^{5}$ Einstein (1949).
    ${ }^{6}$ Ibid.
    ${ }^{7}$ See, e.g., Burnheim (1985), Cockshott and Cottrell (1993, Chap. 13); and Van Reybrouck and Annan (2018) for discussions on alternative radical democratic institutions.

[^79]:    ${ }^{1}$ Marx (1867, Chap. 32).

[^80]:    ${ }^{2}$ Some technical ideas can be found already in the writings of Otto Neurath. The later work by Victor Glushkov and Stafford Beer are two independent examples. See references in Cockshott and Cottrell (1993).
    ${ }^{3}$ Marx (1894, Chap. 27).

[^81]:    ${ }^{4}$ Not to mention the enormous ability of owners of capital to lobby politicians and bureaucrats and bribe them with lucrative job offers.

[^82]:    ${ }^{5}$ Mulhern (1984, p. 22).
    ${ }^{6}$ See, for instance, Therborn (1977), Perrone et al. (1984), Wallace et al. (1989), and Usmani (2018).
    ${ }^{7}$ The latter point was predicted on theoretical grounds by Kalecki (1943).

[^83]:    ${ }^{1}$ See, for instance, Shaikh and Jacobo (2020) and Scharfenaker (2020).

[^84]:    ${ }^{2}$ We may assume that the quantities are all rational numbers.

[^85]:    ${ }^{3}$ By combining (B.5) and (B.6), it can be seen that a necessary condition for economic reproduction is $\lambda_{\max }(\mathbf{A L})<1$, where $\lambda_{\text {max }}$ denotes the maximum eigenvalue of $\mathbf{A L}$.

[^86]:    ${ }^{4}$ See also the Perron-Frobenius theorems for nonnegative square matrices, such as $\mathbf{W}$.

[^87]:    ${ }^{5}$ Each contracted worker-hour could be in one of $m$ bins, independent of the rest. This results in $m^{\mathbb{L}}$ possible microstates.

[^88]:    ${ }^{6}$ See, for instance, Chaps. 11 and 12 in Cover and Thomas (2012).
    ${ }^{7}$ These macroproperties can, for instance, be expressed as constraints

[^89]:    ${ }^{9}$ This result is obtained from Cowell (2011, Table A.2), where $G=2 \Phi\left(\frac{\sigma}{\sqrt{2}}\right)-1 \equiv \operatorname{erf}\left(\frac{\sigma}{2}\right)$, and using the cumulative distribution function for a standard normal $\Phi(x) \equiv \frac{1}{2}(1+\operatorname{erf}(x / \sqrt{2}))$.

[^90]:    ${ }^{10}$ If a firm fails to cover its costs, then its value-added term is negative.

[^91]:    ${ }^{11}$ Specifically, the variance of $\Psi$ is given by $\left(\frac{\mathrm{E} W}{\omega}\right)^{2} \cdot\left[\exp \left(s^{2}(\omega, \gamma)\right)-1\right]$.

[^92]:    ${ }^{12}$ Since the smooth distribution function of $\Psi$ is taken to be effectively continuous.

[^93]:    ${ }^{13}$ See also (Zachariah, 2009) and Sect. 14.3 in (Cottrell et al., 2009).

