A Primer on Political Phenomenology

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In my room, the world is beyond my understanding;
But when I walk I see that it consists of three or four hills and a cloud.

Wallace Stevens
Of the Surface of Things

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Introduction

The present essay is best understood if situated against the background of a broader research project[1]. This contextualization is important because, so far, the propositions we put forward here deal almost exclusively with a reconstruction of key passages from the first volume of Marx's *Capital*[26] and this could give the wrong impression that our aim is merely exegetical. This is not the case - nor do we seek to contribute to the established trend of "confirming" Marx's ideas by presenting them in a more scientifically acceptable form. Instead, our purpose is to show the benefits of an alternative framework, from which a *very specific* reconstruction of his categories is not only possible, but natural. We argue that if such an effort is deemed consistent, then we gain considerably from this alternative framework's richer environment, situating Marx's critique of political economy in a conceptual space that is capable of expressing more than what is allowed by its original "grammar".

In a previous publication [40], we defined the contours of this broader conceptual space through the statement of our research project's two main objectives.

The first one is to construct a theoretical approach to emancipatory political thinking in which the distinction between political economy and political organization is no longer a substantial one, requiring two separate conceptual frameworks. This means, on the one hand, that the tools employed in producing structural analyses of capitalist social formations - amongst which we can include Marx's critique of political economy - should also be useful in equally structural analyses of Leftist organizations and organizational networks. Inversely, it also entails developing the formal means to consider large economic social structures as particular organizational types, thus allowing Leftist categories of political action to find interesting counterparts in broader discussions of alternative social formations and large-scale dynamics.

This first objective could, thus, be summarized as the attempt to render the following statement meaningful: "economies are large organizations; collectives are small economies". And, as this proposition already suggests, any attempt to establish a common theoretical framework capable of expressing properties of both economic structures as well as political collectives must include variables and transformations that are *scale-dependent*, allowing for different qualitative effects depending on matters of size, composition and social layering structures.

[1] This research is currently carried out by the *Subset of Theoretical Practice*, part of the *Circle of Studies of Idea and Ideology*, and all our meetings and relevant publications can be found at www.theoreticalpractice.com

Our second objective complicates this picture even further. We aim at not only situating the duality between economy and organization within a common underlying framework - we also want this theoretical backdrop to support the investigation of *more* social structures and forms than we can already conceive today. That is, such a framework should not only be broader than both political economy and political organization, but it should also allow us to express other forms of economic and collective organization still to be discovered and tested.

Just as the first objective requires us to construct a scale-sensitive conceptual grammar, this second proposition requires this framework to have a practical and informative counterpart - which entails making the following statement also meaningful: “social organizations can be treated as political experiments”. We can only make sense of such a statement, however, if we are able to treat the prescriptive dimension of political action – the tactical and strategic dimensions of politics – as something that can be “encoded” into our structural analysis as a *new hypothesis*, consequentially turning political organization into an artificial space where consequences can be drawn out from organizational premises, evaluated, and shared amongst political thinkers and projects.

When combined, the perspective that emerges from these objectives can be framed as (1) a compositional and scale-sensitive point of view where economy and organization emerge as relative social concepts, (2) in which political practice is itself endowed with an experimental capacity that could inform theoretical constructs and which (3) reframes the *tools* of the critique of capitalism as part of a larger, more affirmative, *toolkit* for the construction of alternative social projects.

Two additional remarks must be added here, as they mediate the passage from this larger project to the specific aims of this essay.

The first one concerns the use of certain mathematical formalisms. We are currently working under the assumption that the underlying challenge imposed by the two objectives described above – which we called a change in the “grammar” of political theorization – can be dealt with through an engagement with an area of mathematics called *category theory* [22]. It is our belief that current work done under the banner of “applied category theory” [7] suggests that this field offers an adequate and natural environment in which to deal with the questions of composition, scale-dependence and generative structures that we previously mentioned, while also allowing us to preserve many of the insights and constructions that were developed under other conceptual paradigms. This hypothesis, in fact, is not a novel one, as several contemporary philosophers have already explored the potential of a categorical approach to logic as means to reconstruct dialectical and critical thought within a richer and more expressive framework.

This leads us to a second point worth mentioning, which concerns how the text is informed by certain key philosophical ideas. Though this is not the place to justify the philosophical influence behind some of our conceptual strategies, it is useful to at least make an explicit reference to a small and heterodox set of thinkers who partially shape this proposal.

The first one is Alain Badiou [2], who developed an "objective phenomenology" that mobilizes category theory as a means to investigate the logical nuances of different self-contained domains – which he calls "worlds" – while also taking care to build within his theory enough room for novel developments that might immanently affect and transform these spaces. Badiou's specific use of category theory, as well as his ingenious solution for treating the analysis of structure and of novel procedures with the same conceptual machinery, greatly influenced the *Primer*. His project, however, remains a purely philosophical one, with no direct political import. This is why we also searched for other thinkers who could help us to find an alternative point of view that would be more intrinsically political while nevertheless coherent with Badiou's choice of formal machinery and with his unifying approach to the structure/action divide.

We found such a perspective in Alexander Bogdanov's "Tektology"[6]: though it is presented as a "general theory of organizations" that should be applicable to scientific, artistic and political matters alike, one of Bogdanov's essential contributions was to realize that the limits of the historical materialist thinking of his time stemmed from its giving precedence to *labour* over *organization* as a primitive concept. For him, a political theory of organization - a regional theory inside the broader field of Tektology - has less restrictions than the labour point of view and is therefore capable of encompassing both our current theory of political economy and our concepts that deal with political action and strategy – thus giving us the tools to recognize within the political sphere some of the basic insights of Badiou's philosophy. Still, such a tektological theory of political organization was never fully developed by Bogdanov, and some crucial intermediate steps are necessary to move from the discussion of social organization in general to the specificity of historically determined social organization in capitalism – steps without which we cannot reach the appropriate descriptive level to properly engage with Marx's *Capital*.

In conceptualizing these steps, which have to do with the organizational composition of social formations, and capitalism more specifically, we were aided by the heterodox reading of Marx provided by Kojin Karatani[12]. In the last decade of his work, Karatani has attempted to recast the Marxist theory of modes of production – in which labour is an essential, almost ontological, category – in terms of a multilayered theory of modes of *exchange*. "Exchange", for him, does not mean "commodity exchange", however, but *metabolic relations* [verkehr] – a term which could probably be better translated as modes of organization, in fact. Using this theory, Karatani reconstructs a richer conceptual space for discussing how communities, States and markets interact in a given social formation – allowing us to discuss with greater specificity the categories of *Capital*, while adopting an organizational grammar that is coherent with Bogdanov's tektology and, we claim, the fundamental insights of Badiou's philosophy.

It should be said, finally, that neither Badiou, Bogdanov nor Karatani go as far as using this expanded conceptual space to recompose and investigate how it affects our thinking of political organization on a practical and concrete level. Badiou stops at the claim that a new political thinking is both necessary and possible, Bogdanov links this possibility to an overarching "organizational point of view" and Karatani ratifies the tektological wager by showing that new political strategies can be thought using the same categories he employs in order to think large scale social organizations. Part of what makes the current project unique, we believe, is the conscious attempt to weave together these contributions into a consistent framework that is built, from the ground up, with the explicit aim of developing the practical, experimental and organizational dimensions that this enriched grammar should allow for.

If we now return to the underlying strategy behind our reconstruction of Marx's critique of political economy, we can give it a more precise formulation and thereby define a more appropriate way to judge its merits. Our aim here is to demonstrate that the categories of the critique of political economy presented by Marx in *Capital* can be recomposed within the grammar of a field of mathematics called "applied category theory" – which, as we have already suggested, is a natural environment for dealing with compositional logic and scale-dependent settings. And since it has already been shown in Alain Badiou's work that category theory can provide the means to deal, simultaneously, with large social structures as well as with more localized experiments in political organization, if we are successful in "embedding" key concepts of Marxian political economy in this new formal setting, then we should be one step closer to establishing a common theoretical approach which is able to conceptualize both issues pertaining to the critique of political economy as well as an experimental take on political organization and action.

This overview should be enough to dispel the impression that the present propositions should be evaluated either as a merely exegetical effort - after all, we admittedly desire to change the "grammar" in which Marx articulated his ideas - or as an attempt to make Marx more "rigorous" in a scientific sense - since our use of novel formal machinery explicitly aims at making the critique of political economy more compatible with political practice, rather than merely giving it a new scientific hue.

Finally, a remark on form: the *Primer* was conceived as a condensed presentation of our approach to Marx's *Capital* with the purpose of facilitating an initial interlocution with others and the subsequent evaluation of the consistency and pertinence of this particular theoretical venue. Because of this, we decided to divide the main bulk of the text into numbered propositions, each relatively self-contained, keeping conceptual digressions to a minimum. The last section, however, seeks to address some of the broader theoretical implications of these propositions and prepare the ground for future installments of the research - readers unfamiliar with the mathematical tools employed here might find there a useful map for navigating the numbered propositions that precede it.

Objective phenomenology

1. Economic phenomena are not directly given to us — there is nothing in empirical observable reality which we could call an economic fact, only, perhaps, economic effects. To render the logic of capitalist political economy intelligible one must, quite counter-intuitively, assume that commodities appear, first and foremost, *to one another*. We call this basic assumption the "objective phenomenology" standpoint, which we distinguish from the standpoint of *fetishism* by asserting that while the latter is defined as the way relations between things appear *for us*, the former concerns the way relations between things appear *to themselves*. We explore here one of the fundamental consequences of taking the objective phenomenological approach, namely, the possibility of locating the logic of commodities within an algebraically closed system, which we identify with an objective perspective prescribed by the limits of its operations. We draw this concept of phenomenology from the work of Alain Badiou[2].

1.1 As a thinker, Marx was a phenomenologist because he asserted the actuality of relations between commodities, an actuality which persists independently of the knowing individual. He shows how these relations can be viewed at different resolutions and are themselves governed by an implicative structure, what he calls a *law* and we call a *logic*, that regulates capitalist existence from within.

[2]In [47], one of the co-authors attempts to sketch a provisional bridge from Badiou to Marx. However, most of the formulations from there are supplanted or improved upon in the present work.

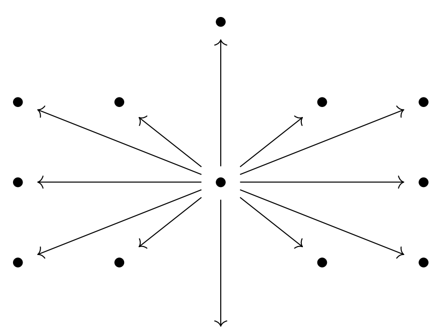
1.2 Our approach differs from Marx's in that we do not propose first and foremost a *critique* of political economy so much as an *embedding* of a particular mode of social organization within a class of others. In this we are still following the spirit of Marx's project by asserting that the laws and tendencies of capitalist accumulation are localized and rely on specific historical conditions and presuppositions, even if, unlike him, we approach these constraints from the standpoint of a richer theoretical framework that allows us to partially disentangle the limits of the social formation under analysis from the limits of the theory that expresses them [3].

1.3 This localization requires an extension of the language of Marx, and in some cases, discarding certain narrow conceptions of his which unduly constrain the expressive power of his critique of political economy.

The operational space of commodities

2. Let $\mathcal{C}$ be the set of all commodities. Following Marx, we define, for every commodity $C \in \mathcal{C}$, a set of ways of using it - which we will call its use-set C_U, and the set of other commodities it can be exchanged for, which we will call its exchange-set C_E. We also define "maps" $\mathcal{C}_U$ and $\mathcal{C}_E$ which assign to every commodity their respective use and exchange sets.

2.1 To assert that something is a commodity is to assert that its corresponding use-set and exchange-set must not be empty. Intuitively, we can rely on a visual metaphor where every commodity can "see" a set of other commodities, which includes those that are "more or less" equivalent to it.



[3]This approach is discussed in more detail in [40]

2.1.1 This seeing is transitive, such that if A sees B and B sees C, then A must see C. We can think of C_E as a restriction of this wider set to precisely those that C can "see most clearly", i.e. those that are strictly exchangeable with it.

2.1.2 The set C_U, on the other hand, can be thought as the ways in which the "commodity substrate" is either the output of antecedent operations (10.1) or the input of subsequent operations, productive or otherwise (10.3). The full information of a commodity C is captured in C_U and C_E, but what determines these two sets will require the entirety of sections 2 through 10 to develop.

2.2 A commodity itself can be loosely viewed as a "set of things"[4] that are produced in order to be exchanged, and as sets, they are subject to the familiar set operations. However $\mathcal{C}$ is not necessarily "closed" under all set operations, because not every part (subset) of a commodity will have a non-empty use and exchange value. We can therefore assert a difference between operations which preserve the commodity-form and those that do not. There are many examples of non-preserving operations, e.g. eating cake, demolishing a building, burying treasure, erasing debt, etc.

2.2.1 We assert that the amorphous form of a commodity can itself be viewed as nothing besides the set of commodity-preserving operations one could perform on it[5]. Since the output of such an operation can be the input of another, operations can be composed to form other operations, or likewise "factored" through certain commodity intermediates. Not only this, but each operation itself can be found to have certain further properties that restrict or expand the types of commodities that matter. The study of such spaces of operations is *category theory*.

2.2.2 We define a "list of commodities" C^n of length n as an element of $\mathcal{C}^n$. Let us say that $\psi : \mathcal{C}^n \to \mathcal{C}$ is an *n-ary commodity-preserving map* if and only if $\forall C^n \in \mathcal{C}^n \, \exists x, y : x \in \mathcal{C}_U(\psi)(C^n) \wedge y \in \mathcal{C}_E(\psi)(C^n)$. In other words, ψ is an operation which takes a list of input commodities and yield a certain output commodity (where commodities must have some use and exchange sets). Note that we do not say yet that these operations "produce" a commodity, as this

[4]We provide different definitions of the commodity throughout this text, each having a different didactic value. The most productive view for us is to treat the commodity not as a set, but as a *sheaf*, which we will define in the following sections.

[5]Operations here should be understood as consisting of many types of relations between inputs and outputs, and the character of these relations should be fully described by its relations with other relations. This is a "category-theoretic" way of viewing things, where "relations" are taken to be morphisms, and the collection of all morphisms targeting (or sourced from) a given object determines that object fully. Category theory has a much richer content, far beyond the scope of the text, but our intention is to show that it can be employed profitably here. We will selectively expound on certain topics from this field, but make no attempt to be rigorous or exhaustive. For a better introduction to this material, see [17] or [22]

word is reserved until 2.3.5.

2.2.2.1 Let us define the *composition operator* $\circ$ as a way to combine two operations f and g where the output of one becomes (one of) the input(s) of the other. This may specify an entirely new operation. Let $f \circ g$ be the operation of applying g first and then f next. Similarly $f \circ g \circ h$ is applying h, then g, then f. Composition is *associative*, so $(f \circ g) \circ h = f \circ (g \circ h)$.

2.2.2.2 We say that a set $\mathcal{O}$ of operations, or maps, is *closed* if any composition of two operations, if it exists, is part of $\mathcal{O}$. A closed space of operations may contain a smaller closed space.

2.2.2.3 A *diagram* is a pictoral representation of the maps in a category. Note that each arrow represents a set of possible operations, and each dot is a commodity.

$$\begin{tikzcd} \bullet \arrow[r, "f"'] \arrow[rr, bend left, "g \circ f"] & \bullet \arrow[r, "g"'] & \bullet \end{tikzcd}$$

2.2.2.4 We represent a condition or property of operations in our category in terms of a *commutative diagram*. Here, any two paths with the same start and ending are equivalent. For example, the following diagram says that $g \circ f = j \circ h$. Just as much as a diagram specifies operations, it also excludes those operations which do not obey its constraints. Here j cannot be just any operation, but must be consistent with our choice of f, h and g.

$$\begin{tikzcd} \bullet \arrow[r, "f"] \arrow[d, "h"'] & \bullet \arrow[d, "g"] \\ \bullet \arrow[r, "j"'] & \bullet \end{tikzcd}$$

2.2.3 In categorical terms, $\mathcal{C}_U$ and $\mathcal{C}_E$ are not only maps but ”functors” to the category of certain sets (we call them $\mathcal{U}$ and $\mathcal{E}$ respectively). Functors take not only objects but maps as inputs. Let us extend the definition of $\mathcal{C}_U$ to include a commodity preserving map as its input. That is, if ψ has inputs C^n and output B, $\mathcal{C}_U(\psi)$ yields a new function f between use-sets C^n_U and B_U. We can specify the input object of ψ to arrive at the formulation used in 2.2.2: $\mathcal{C}_U(\psi)(C^n)$ which is the same as writing B_U. The same applies to our definition of $\mathcal{C}_E$ but with the subscript U replaced with E.

2.2.4 To complete the definition of functor, we must also note that $\mathcal{C}_U$ and $\mathcal{C}_E$ preserve compositions. This is described as the *functorial* property: $\mathcal{C}_U(\psi \circ \gamma) = \mathcal{C}_U(\psi) \circ \mathcal{C}_U(\gamma)$, where ψ and γ are commodity-preserving operations. The property applies likewise for $\mathcal{C}_E$. The following commutative diagram between use-sets (the dots and arrows lie in the category of use-sets $\mathcal{U}$) illustrates this:

$$\begin{tikzcd}
\bullet \arrow[r, "\mathcal{C}_U(\gamma)"] \arrow[dr, "\mathcal{C}_U(\psi\circ\gamma)"'] & \bullet \arrow[d, "\mathcal{C}_U(\psi)"] \\
& \bullet
\end{tikzcd}$$

2.3 To say that the commodity-form is compositional means to equip $\mathcal{C}$ with those *commodity-preserving* operations $\mathcal{O}$. We can denote this with the pair $(\mathcal{C}, \mathcal{O})$, but for brevity we will often use $\mathcal{C}$ to denote both the set of commodities and its operations. For any set of input commodities $C_1, C_2, ...C_n$ and an output commodity B, there exists a set of commodity preserving operations, which we denote with $\mathcal{O}(C_1, C_2, ...C_n; B)$. We can also write $\mathcal{O}(*; C)$ to denote all preserving operations which have C as the output. Accordingly, $\mathcal{O}(C; *)$ is the set of all operations which have C as one of the inputs.

2.3.1 As anticipated in 2.1, the substrate that supports the commodity-form (e.g. a physical form) should be distinguished from the commodity itself. A commodity-preserving operation does not require that a commodity's substrate remains intact, nor does it require preserving a commodity's use and exchange sets - it only requires that the result of the operation is a commodity.

2.3.1.1 Our composition operator (2.2.2.1) can be viewed as acting on pairs of sets of operations where the output of one exists in the input list of the other. From the set of operations from A to B and the set of operations from B to C, we get the operations from A to C. Using the formalism from earlier, we write this as:

$$\circ : \mathcal{O}(C_i, ...C_j; B) \times \mathcal{O}(B, C_k, ...C_n; D) \to \mathcal{O}(C_i, ...C_j, C_k, ...C_n; D)$$

Generally when we see a labelled arrow in a diagram, we are referring to a set of operations.

2.3.2 The trivial commodity-preserving operation is that of identity. This is a unary function that does not change a commodity's uses or exchangeables: $\mathcal{C}_U(\mathrm{id}_C) = \mathrm{id}_{C_U}$ and $\mathcal{C}_E(\mathrm{id}_C) = \mathrm{id}_{C_E}$. Several actions can be identity operations: transporting, putting on display, exchanging, etc. although none of them are necessarily so.

2.3.3 A non-trivial but basic commodity-preserving operation is counting multiple commodities as one. If a set of commodities together has some use, and can be exchanged-as-one, that is, the exchange and use set of this collection is non-empty, then it is also a commodity. Items in a grocery cart are an example. Sometimes we refer to this as a *commodity bundle*.

2.3.3.1 We could describe the grocery cart categorically as a *coproduct*, which is a kind of *colimit*[6]. Given two commodities A and B, one says that C is a coproduct if it obeys the following diagram:

$$\begin{tikzcd} A \arrow[dr] \arrow[ddr] & & B \arrow[dl] \arrow[ddl] \\ & C \arrow[d, dashed] & \\ & D & \end{tikzcd}$$

One interprets this in the following way: C and D are both grocery carts containing items A and B. The special, or *universal*, condition which makes C the coproduct stems from the fact that D is allowed to contain A, B, and any other items, whereas C must contain A and B only. Here the dotted arrow denotes a unique operation that makes this diagram commute. We can alternately write C as $A + B$ to denote its relation to its constituents.

2.3.3.2 Because we want to conceive of commodity operations in the most general sense, we allow any possible combination of commodities to be treated as a single commodity. Therefore, we assert that $(\mathcal{C}, \mathcal{O})$ contains all *coproducts*.

2.3.4 It is very natural then to consider the reverse operation of "breaking up" a commodity into its constituents. This entails another kind of commodity-preserving operation, closely related to the previous example, which is to isolate some part of an existing commodity as a commodity. We will call this a *commodity projection*. Here, "part" should be interpreted as a formal condition on the use-set, so that π is a projection only if the following is true: $\mathcal{C}_U(\pi)(C^n) \subseteq \mathcal{C}_U(C^n)$[7].

2.3.4.1 Categorically, the input of a projection operation is called a *product*. It is *dual* to the coproduct mentioned in 2.3.3, in the sense that it is the same

[6]We elaborate on this more generally in 9.2, but it's certainly better to consult the standard category theory references such as [22].

[7]This includes, for example, selling some subset of items from the commodity bundle. Or perhaps exchanging a portion of one's time for food, provided there is some notion of the use-set of one's time (10. and 11.). However, it would not include isolating fresh water from salt water, which would introduce entirely new uses.

diagram with the arrows reversed. We write the product as $A \times B$ below:

$$\begin{tikzcd}
& D \arrow[dl] \arrow[dr] \arrow[d, dashed] & \\
& A \times B \arrow[dl, "\pi_A"] \arrow[dr, "\pi_B"'] & \\
A & & B
\end{tikzcd}$$

2.3.4.2 At this point, it is justifiable to wonder what is different about $A \times B$ and $A + B$. Indeed, at the level of the grocery cart metaphor, the two are the same, since we can just as well take things out of the grocery cart as put them back in. But this is not true for certain irreversible processes in the commodity world. For example, if A and B are now steel and human labour power, it is easy to conceive of their coproduct C as being the output of some concrete labour process. But it is not so easy to discern, for the same C, two projection arrows back to A and B.

2.3.4.3 Although it is not correct to say that every commodity is a coproduct or product, we can say that any pair of commodities determines a unique product and coproduct.

2.3.5 We call those commodity-preserving operations where the use and exchange of the output can be distinguished from that of the input(s) *commodity-producing*. Such a function ψ obeys $\mathcal{C}_U(\psi)(C^n) \neq \mathcal{C}_U(C^n)$ and $\mathcal{C}_E(\psi)(C^n) \neq \mathcal{C}_E(C^n)$.

2.3.6 Let us extend our definition of the functor $\mathcal{C}_U$ (2.2.3) to define the subset of those operations which are commodity-producing, namely:

$$\mathcal{C}_U(C_1, C_2, ...C_n; B) \subseteq \mathcal{O}(C_1, C_2, ...C_n; B)$$

A notational point here is that it is more general to write $C_1, C_2, ...C_n$ for a list of commodities if we do not require them to be products or coproducts. Otherwise, with the definitions from 2.3.3.1 and 2.3.4.1, we can write the above sets as $\mathcal{C}_{\mathcal{U}}(C_1 \times C_2 \times ... \times C_n; B)$ or $\mathcal{C}_{\mathcal{U}}(C_1 + C_2 + ... + C_n; B)$.

2.4 At this level of the analysis, commodities do not come with an explicit notion of quantity. That is, we treat x amount of commodity C as an altogether distinct commodity from any other amount y of the same commodity. In order to arrive at quantity as a relation between xC and yC (which Marx already starts with), we must interpret arithmetic operations as belonging to the larger class of commodity-preserving ones. For example, the operation of adding two of the same commodity together is a special case of a coproduct (2.3.3.1). In this special case, the coproduct is also a product (2.3.4.1 and 2.3.4.2).

2.5 We can identify commodities whose uses are *relatively homogenous* w.r.t. some set of operations. Example: pine and cedar wood share the property of being resistant to rot, so they would be interchangeable for certain building projects. In a different case, one may be more favored for aesthetic or traditional reasons. It is therefore useful to view $\mathcal{C}$ modulo a set of uses, which we call a *resolution*. Specifically, a Δ-resolution, also written as $\mathcal{C}_\Delta$, is the set of commodities in which those with uses existing in some set Δ are indistinguishable.

2.5.1 Categorically, the Δ-resolution is described by an *endofunctor*[8] $\Delta : \mathcal{C} \to \mathcal{C}$. This functor "forgets" differences by mapping distinct commodities to the same commodity of a more general "type", while maintaining the compositionality as detailed in 2.2.4. For example, pine and cedar wood would be mapped to rot-resistant wood. In fact, at an even "higher" resolution, pine wood itself is the result of forgetting the differences between subspecies of pine (e.g. white pine and sugar pine). This is useful because only certain properties are relevant for certain operations. For our purposes, there is no such thing as a resolution-independent commodity, and we assume that $\mathcal{C}$ contains commodities at all resolutions.

2.5.2 Given two Δ functors, let's call them Δ_A and Δ_B (where A and B denote different use-sets), there exists a set of operations α, one for each $C \in \mathcal{C}$ which starts from $\Delta_A(C)$ and goes to $\Delta_B(C)$. This set of operations is called a *natural transformation* between Δ_A and Δ_B, and each of its components we will write with a subscript for its associated commodity, i.e. α_C. Given any operation $\psi : C \to D$, we have the following *naturality square*:

$$\begin{tikzcd}
\Delta_A(C) \arrow[r, "\Delta_A(\psi)"] \arrow[d, "\alpha_C"'] & \Delta_A(D) \arrow[d, "\alpha_D"] \\
\Delta_B(C) \arrow[r, "\Delta_B(\psi)"'] & \Delta_B(D)
\end{tikzcd}$$

2.5.3 The natural transformations are just operations in $\mathcal{O}$, but their significance lies in how they commute with the Δ-resolution endofunctors. They show that if there are two different ways of "forgetting differences" between

[8] A functor (2.2.3) whose source and target are the same.

commodities, then there exists concrete operations that connect these different ways. This might be simply an identity operation 2.3.2 if there exists a commodity, e.g. wood, that possess the properties represented by Δ_A and Δ_B. However, there may be other more complex processes. For example, sturdiness and resistance to rot determine two kinds of resolutions, and a natural transformation between them might be to treat wood of one kind with certain chemical compounds to achieve the properties of the other (without necessarily preserving the properties in the initial resolution).

2.5.4 If we apply Δ to $\mathcal{C}$ twice, we get the same result as if applying it once. Therefore, resolution of the commodity world is here formalized as an *idempotent* endofunctor. This means there always exists an isomorphism between two commodities $\Delta\Delta C$ and ΔC. Let us define the set of these isomorphisms as a natural transformation $\mu : \Delta\Delta \to \Delta$. We can also specify a natural transformation from the *identity endofunctor* $1_{\mathcal{C}}$, which maps every commodity to itself, to any given resolution Δ as the set of commodity operations from $C \in \mathcal{C}$ to $\Delta(C)$, which we write as η. These ingredients allow us to form a monad (Δ, μ, η), obeying the following diagrams:

$$\begin{tikzcd} C \arrow[r, "\eta_C"] \arrow[dr, "id"'] & \Delta C \arrow[d, "\mu_C"] \\ & C \end{tikzcd} \qquad \begin{tikzcd} \Delta\Delta\Delta \arrow[r, "\mu\circ\Delta"] \arrow[d, "\Delta\circ\mu"'] & \Delta\Delta \arrow[d, "\mu"] \\ \Delta\Delta \arrow[r, "\mu"'] & \Delta \end{tikzcd}$$

2.5.5 We can define, for any given Δ, another endofunctor which maps each commodity C to the product of all commodities which would be sent to C under Δ. Formally, $\triangledown(C) : \{\prod D \mid \Delta(D) = C\}$. Again, there exists a monad for this endofunctor, although this time it is not idempotent. Let η_C be the component of the natural transformation at C, which takes it to $\triangledown(C)$. Let μ be the same as η from 2.5.4.

2.6 We defined the set $\mathcal{O}(A_1, A_2, ...A_n; B)$ containing the commodity-preserving operations which have commodities $A_1, A_2, ...A_n$ as the input and B as the output (2.3). For a given subset of the use set $V \subseteq \bigcup_i A_{i_U}$, we can restrict to only those operations which preserve elements in V. We denote this as $\mathcal{O}_V$ and call these operations V-preserving. For example, to make a chair out of wood, we care that certain properties of the wood will exist in the chair. An element corresponding to "sturdiness" may exist in both $\mathtt{wood}_U$ and $\mathtt{chair}_U$ and therefore may have corresponding elements in $\mathcal{O}_V(\mathtt{wood}, ...; \mathtt{chair})$. It is possible however that no process yet exists to preserve such a property, in which case $\mathcal{O}_V(A; B)$ is empty.

2.7 Let us define a special "commodity" 1 in $\mathcal{C}$, called the *terminal commodity*, whose use-set and exchange-set is the singleton set $\{*\}$. While any two commodities can generally have multiple maps, there is a unique map from any commodity C to 1. In other words, $\mathcal{O}(*;1)$ contains one map for every commodity in existence. There also exists in our category an *initial commodity* which we call 0 and has the opposite uniqueness property: $\mathcal{O}(0;*)$ contains one map for each commodity. We will use this formalism later to describe instances where operations that go from inside the operational space to the outside, or vice versa.

2.8 There exists an endofunctor $- \times A$ which takes any commodity C to the product $C \times A$ and takes any commodity map $\psi : C \to B$ to $\langle \psi, \mathrm{id}_A \rangle : C \times A \to B \times A$. Now, this functor uniquely determines [9] another functor $-^A$ which takes every C to a special *commodity operation space* C^A, which stands for all the operations between C and A. The unique determination can be formulated as $\mathcal{O}(A \times B, C) \cong \mathcal{O}(B, C^A)$. This fact, along with 2.3.3 and 2.3.4, means that our category is *cartesian closed*.

Bargaining spaces and the sheaf of exchanges

3. An important instance of a commodity preserving operation is exchange. When I exchange A for B, another property owner is exchanging B for A. There is a crucial dimension of this process that would require us to consider operations that are not intrinsic to the commodity-space, such as property alienation and property rights, which will be briefly discussed in point 8.. From the perspective of commodity-preserving operations alone, however, exchange can be defined as a map $f : A \to B$ such that there exists an inverse $f^{-1} : B \to A$ where $f^{-1} \circ f = \mathrm{id}_A$ and $f \circ f^{-1} = \mathrm{id}_B$. Such a map only exists if $A_E = B_E$, that is, both commodities have the same exchange sets. Therefore, the exchange set of a commodity are the set of all commodities which are *isomorphic* to it. Note that isomorphic commodities do not necessarily have the same use-sets.

3.1 Two commodities can be more or less exchangeable. We can express this as an index of their similarity. We denote this index as a binary function e called the *exchangeability function*. The values of e lie in a Heyting algebra T, and we call the values of T *degrees of exchangeability*. Formally, $e : \mathcal{C} \times \mathcal{C} \to T$

[9]This is known as the product-hom adjunction. Given an object B and an operation from C to A, I can uniquely give you an operation from $A \times B$ to C, and vice versa. For our purposes, C^A is similar to the set $\mathcal{O}(A, C)$ of operations from A to C, except it is "inside" the world of commodities. This should be taken in a literal sense that the operations on commodities are themselves commodifiable (e.g. ownership of the means of production or intellectual property).

3.1.1 A Heyting algebra is a lattice with a join $\vee$ and meet $\wedge$ operation, which we call *envelope* and *conjunction*, and that obey distributive laws:

a) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$.

b) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$.

This lattice is also bounded by a smallest and largest value, which we denote as $\mathcal{M}$ and μ respectively. A *complete Heyting algebra* is one in which the envelope of any set of degrees exists[10]. We sometimes write an envelope of multiple degrees as $\bigvee_n x_n$ which is the same as $x_1 \vee x_2 \vee ... \vee x_n$.

3.1.2 Degrees of exchangeability are partially ordered. For any two degrees p and q, either $p \leqslant q$, $q \leqslant p$ or they are incomparable. A set of comparable degrees is called a *chain* and a set of incomparable degrees is called an *antichain*. Any subset of degrees from T can be partitioned into a set of chains (likewise antichains).

3.1.3 Likewise, a set of exchanges can be ranked from less to more likely if they are on the same chain, since chains are totally, rather than partially, ordered. However, since there can be multiple chains within a single lattice of T, there may be several distinct "measures" of what is valuable (4.). In general, any set of exchanges can be partitioned into one or more chains (likewise antichains).

3.2 e is to be evaluated at a local level - that is, given a particular time and place and set of circumstances. We will call this "virtual" or counterfactual space of possible alternative exchanges the *bargaining space* of e. One can imagine commodities assuming various relations with each other depending on which space they currently occupy. A commodity which is worth very little in one space can be suddenly "promoted" in another space, or vice versa. Contrary to the marginal utility approach[11], we don't suppose that the properties of a bargaining space are to be derived from the preferences of individuals

[10]We refrain from explicitly stating how this Heyting algebra fits into the operational space in section 2. Although throughout this text we implicitly "adjoin" the set of commodities with a foreign object (a lattice) and describe the exchange function as having values in this object, a more ideal approach is to conceive of a special commodity which would embody the lattice. We partially achieve this in 7. when discussing the money-form. For now, let us assume that T and e are additional "equipment" of our operational space, but not necessarily within it.

[11]It seems to us an unnecessary prejudice that value should originate in individuals. In some sense, yes, exchange requires individuals to partake in the act, and the sum of these acts may have generative effects. However, it seems just as coherent to say that, once a collection of individual exchange acts becomes a *network* of exchanges, the individual is compelled to exchange according to the relations established in that network. We can go even further to say that each space induces the preferences of individuals, a hypothesis explored in [46].

exchanging within that space. Rather, we take these spaces as objective logical contexts [33].

3.2.1 When $e(a, b) = \mathcal{M}$, this means a and b are guaranteed to be exchangeable, whereas $e(a, b) = \mu$ means it is effectively impossible to exchange the two. For any two assessments of e, say $e(a, b)$ and $e(b, c)$, either they are the same degree, the two degrees are comparable (one is less than the other), or incomparable. These degrees reveal information about a particular bargaining space.

3.2.2 Let us define an operation q^* on pairs of commodities called *completing the exchange* that returns the set of commodities x which, if x were added to one of the pair of commodities, would guarantee maximal exchangeability. Formally:

$$q^*(a, b) : \{x \in \mathcal{C} \mid e(x \cup a, b) = \mathcal{M} \vee e(a, x \cup b) = \mathcal{M}\}$$

3.2.3 Accordingly, we can define q_* on pairs of commodities to be the operation that returns those *slices* of a commodity which would make exchange maximal. We call this *slicing the exchange*. Formally:

$$q_*(a, b) : \{x \in \mathcal{C} \mid e(x \cap a, b) = \mathcal{M} \vee e(a, x \cap b) = \mathcal{M}\}$$

3.2.4 If two exchanges $e(a, b)$ and $e(c, d)$ are of the same degree, their slice sets and completion sets are the same:

$$e(a, b) = e(c, d) \Rightarrow q_*(a, b) = q_*(c, d) \wedge q^*(a, b) = q^*(c, d)$$

3.2.4.1 When two exchanges are of the same degree and they share a common commodity, i.e. $e(a, b) = e(b, c)$, we say that a and c are *congruent modulo* b. We can denote this formally as $a \equiv c \bmod b$. One should intuitively think of this condition as saying that there exists a common "remainder" which, if added to or subtracted from a and c, would make them exchangeable with b. Therefore, a and c look "congruent" from the perspective of b[12].

3.2.5 If $e(a, b) = p$, and there exists some $r \geqslant p$, then there is a (possibly non-empty) set $q^r(a, b)$ which contains an element that, if added to either a or b, will advance the exchange to degree r. If r is not $\mathcal{M}$ then this is *partially*

[12]Notably, we use this definition again in 3.2.10 and 9.4

completing the exchange. The same holds for $q_r(a, b)$ which contains which is called *partially slicing the exchange*.

3.2.6 Given any set of exchanges, we can order them based on the subset relation of their q^* sets. That is, for two exchanges s and t, $q^*(s) \subseteq q^*(t) \Rightarrow s \geqslant t$. If s has a maximal exchangeability, then its completion would be an empty set. This is called *ordering by completions*. There is also an *ordering by slicings* w.r.t. q_*.

3.2.7 Conversely, whenever $e(a, b) \geqslant e(c, d)$, there is a way to advance from one to the other via the series of q moves. We can intuitively visualize all possible exchanges as distributed in a space by the rule "at least p-degree of exchangeability":

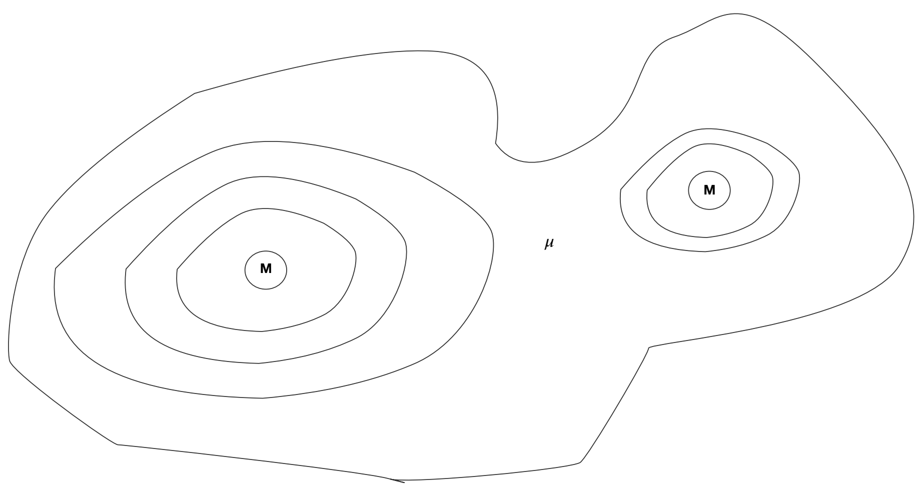


Figure 1: A topographical map of a bargaining space

Each contour line separates degrees of exchangeability, with $\mathcal{M}$ the most restricted and μ the most general. Each degree therefore specifies an open set, or region, of the space. In other words, the bargaining space is a representation of our Heyting algebra T, and the function e assigns exchanges to each region. The partial q operations describes paths moving closer to $\mathcal{M}$ in this space.

3.2.8 The assignment of individual exchanges to the space has an obvious correlative operation[13]: to assign to each region of the exchange space a set of

[13]Technically we refrain from calling it an inverse, which would be to assign a single exchange to each degree-region of the space. Instead, what we are interested in is called a *pre-image* of our original assignment, and the individual sets assigned are called the sections.

exchanges. Given any two regions, there exists a unique set of exchanges which fall within the union of these two regions. This is the minimal formulation for a *sheaf of exchanges*[14]. We have a commutative square where U_x is the open region associated to degree x, $\mathcal{F}$ is the assignment of exchanges and q^p completes an exchange to degree p[15]:

$$\begin{tikzcd}
\mathcal{F}(U_{a\vee b}) \arrow[r, "q^a"] \arrow[d, "q^b"'] & \mathcal{F}(U_a) \arrow[d] \\
\mathcal{F}(U_b) \arrow[r] & \mathcal{F}(U_{a\wedge b})
\end{tikzcd}$$

3.2.9 If two exchanges are incomparable, i.e. neither $e(a,b) \geqslant e(b,c)$ or $e(a,b) \leqslant e(b,c)$, then we can look to their envelope (join) or conjunction (meet). The envelope of two exchanges denotes a third exchange that is reachable via partial completions. The conjunction of two exchanges denotes a third exchange reachable through partial slicings.

3.2.10 A given bargaining space defined over (D, q_*, q^*) where $D \subseteq \mathcal{C}$ has the condition that the congruence relation $\equiv$ induces an equivalence relation on D. This entails some canonical set $M \subseteq D$ of commodities which represent the various *exchange classes* of the bargaining space. For each $m \in M$, there exists a class $[m]$ consisting of all elements $a, b \in D : a \equiv b$ mod m, and all elements in D belong to exactly one such class.

3.3 A Heyting algebra is also characterized by an implication operator $a \Rightarrow b = \bigvee_n x_n \mid x_n \wedge a \leqslant b$ where $x_n, a, b \in T$ and a negation $\neg x = x \Rightarrow \mu$. It should be noted that when a Heyting algebra restricts to a Boolean algebra, then $\neg\neg x = x$, but this is not true in general.

3.3.1 One can evaluate the degree to which one exchange depends on another (called the *degree of exchange-dependence*). Formally, $e(a,b) \Rightarrow e(c,d)$ is the

[14]For an unorthodox discussion of sheaves and their relation to Marxism, see [45].

[15]Two points for those familiar with sheaves or just careful readers of the diagram. First is that the topographical map in figure 1 is order-opposite to the open sets in the sheaf diagram. That is, to ”go up the mountain” is to restrict the set of exchanges, whereas the standard mathematical approach is to ”go down the mountain”, i.e. with inclusion of opens $U_{a\wedge b} \subseteq U_a \subseteq U_{a\vee b}$. We can resolve this simply by assigning our exchanges such that each open set U_p contains exchanges of ”at most degree p” exchangeability. In this view, the degree $\mathcal{M}$ represents the entire mountain. Second, there is a slight abuse of notation here. By definition, q^p actually specifies a set that contains elements that could be added to an exchange to make it more likely, yet we treat it here as a map of sets assigned by the exchange sheaf. Our justification is that specifying the set of ”things one could add” is also to specify how one set of exchanges can be mapped to another. Furthermore, these maps work naturally as restrictions.

degree to which c and d can be exchanged given the degree that a and b can be exchanged.

3.3.2 If $e(a, b) \leqslant e(c, d)$ then the degree of exchange-dependence $e(a, b) \Rightarrow e(c, d)$ is always maximal. If $e(c, d) < e(a, b)$ then it is always $e(c, d)$.

3.4 A commodity is not necessarily exchangeable with itself - commodities can degrade, become useless, etc. and therefore unexchangeable - so let us call $e(a, a)$ the *existence* of a commodity. This existence regulates relations with a and other commodities by the rule $\forall x \in \mathcal{C} : e(a, x) \leqslant e(a, a)$. In other words, the degree of exchangeability with others is limited by the degree of self-exchange.

3.5 A *consistent exchange function* is one which obeys symmetry, $e(a, b) = e(b, a)$, and the triangle inequality, $e(a, b) \wedge e(b, c) \leqslant e(a, c)$. All exchange functions we examine in this text are consistent. [16]

3.6 We have not yet dealt with the question of how the assignment of exchanges to their degrees is actually determined. This is only taken up fully in 11.. However, it is important to note that the labour theory of value (LTV) is not needed for bargaining spaces (3.2) or consistent exchange networks (4.3) to exist. LTV enters the picture as a unified metric of several of such spaces and networks, a collection which we normally call an economy. In that case, labour emerges as a universal basis, insofar as it is part of every commodity's production (modulo the enclosures, which we mention in 10.2). Marx was not content simply to assert that exchangeability was, within capitalism, determined by labour content, which several of his contemporaries had also done. He alone questioned the role that such a determination served in reproducing a particular type of society. Expanding our set of tools to grasp this "second-order" determination is the focus of this work, but especially in sections 8., 16. and 17..

Commodity-measurable space and exchange networks

4. Two degrees are *disjoint* if their conjunction is μ (minimal). Let us define a *measure* $m : T \to [0, \infty]$ that obeys:

a) $m(\mu) = 0$

[16] We are open to the possibility that inconsistent exchange functions may serve as a good alternative to model certain aspects of the commodity world, for example, in cases of asymmetric information.

b) $m(\bigvee_n p_n) = \sum_n m(p_n)$ whenever p_i and p_j are pairwise disjoint for any $i, j \leqslant n$ and $i \neq j$. Equipping our Heyting algebra with a measure makes it into a *measure space*.

4.1 If the underlying objects of a measurable space are commodities, we call it a *commodity-measurable space*. All economic facts are ultimately those about commodity-measureable spaces $(\mathcal{C}, T, m)$. Objective phenomenology is a generalization of measuring, one which takes into account that specific objective means are needed to register phenomena within these particular spaces.

4.2 A commodity's existence, and its exchangeability with others, are degrees in T, and are therefore measurable by functions $\rho : m \circ e$ where m is a measure. We call this the *commodity-measure* function. A commodity-measure is consistent if the exchange function is consistent (3.5) - however, it remains an arbitrary measure.

4.3 We define the *exchange network* as $(\mathcal{C}, \rho)$ where ρ is a commodity-measure. Let us call $\rho(a, a)$ of a commodity a its size. For every pair $a, b \in \mathcal{C}$ there exists a *path* from a to b with *weight* $\frac{\rho(b,b)}{\rho(a,a)}$, i.e. the ratio of sizes. Conversely, the path from b to a is $\frac{\rho(a,a)}{\rho(b,b)}$[17]. A sequence of paths is also a path, and has weight equal to the product of the constituent path weights.

4.3.1 An *exchange cycle* is a sequence of paths which start and end at the same commodity. A network of exchange is *arbitrage-free* if there exist no exchange cycles with weight > 1.

4.3.2 We can derive the *efficient* exchange ratio between two commodities as the "shortest path" (the path with the smallest weight) between them. An efficient exchange network is one where every actual exchange ratio is efficient.

4.4 A map $\phi : A \to B$ between two commodity-measureable spaces with the same underlying algebra is *exchange-preserving* if B is arbitrage-free and efficient whenever A is.

4.4.1 A common kind of *exchange-preserving* map occurs when a network A contains a subset of the commodities of a network B. Namely, each new commodity C added to a society implies a new map ϕ which embeds one network without C into the other containing C. This generalizes to those sudden and dramatic influxes of new commodities as the result of trade, conquest, or tech-

[17]If e is consistent, $\rho(a, b) = \rho(b, a)$ (by symmetry). There can be different interpretations of this value, independent of what we call the path weight. However, by 3.4 we know that this value cannot exceed the lesser of the sizes.

nological improvements. These moments often destabilize the network, so we can model these by a ϕ which is not exchange-preserving.

4.4.2 The dynamics of an exchange network can be modeled as a set of automorphisms $\phi : A \to A$. Take for example the foreign exchange market, which is comprised of exchange ratios between each currency (we define markets and currencies in 8.3). A particular day in this market is described by an indexed set of exchange-preserving maps. At each moment n, we find a map from the start of the day to n which is the composition $\phi_n \circ \phi_{n-1} \circ ... \circ \phi_0$.

4.5 When Marx writes of the "elementary expression of value" as "x commodity A = y commodity B"[26] he is already assuming a certain exchange network has been established, where the path weight between commodity A and B is y/x. But of course there are infinitely many quantities y and x that would fulfill the same ratio, and therefore an infinite number of measures. By specifying exchangeability in an algebraic way, as we've done in 3., we are proposing to view commodities in a measure-independent way (which Marx implicitly does as well). What matters in the commodity world are not the quantities themselves but the logical coherence from which they issue.

Forms of value

5. For some fixed commodity A, denote the set of its degrees of exchangeability with all other commodities E_A. We can also view E_A either as a "fuzzy" set A^*, in which belonging can be many-valued, or alternatively as a predicate $A(x) = e(A, x)$. In the latter case, we can speak of commodities having "A-ness" to the degree p if $A(x) = p$.

5.1 From a logical standpoint, all commodities are predicates, but not all predicates are commodities. To distinguish predicates that are commodifiable, we use the qualification "priceable". This is a crucial feature of the phenomenology we propose. As Marx says, commodities do not have value in themselves, but value arises as a social relation between commodities. In our terminology, the value-form is a predicative relation, and value is nothing other than the obtaining of truth-value under a priceable predicate.

5.1.1 Unlike Marx, however, we treat the quantitative description of commodities as just another predicate: "x amount of C" (4.5) is treated as a special case of "C has the property P to the p-degree". This alternative approach is coherent with the objective phenomenology (1.), which restricts itself to what appears of commodities to each other. The conditions for the emergence of such a special predicate are described in points 6.2.2 and 7.6.

5.1.2 We can perform conjunctions and envelopes (3.1.1) of predicates to form other predicates (i.e. *or* and *and*). This is akin to producing new perspectives on commodities by combining previous perspectives with each other in distinct ways. Moreover, every predicate implies an operation which isolates a particular part of a commodity, the part for which a predicate obtains. This gives us an *algebra of value*, which we can use to "view the inner composition" of commodities. [18].

5.2 In addition to the logical and algebraic viewpoints on value, we also have the spatial one, based on the interpretation of our Heyting algebra as a space (3.2.6). We start again with a distribution within a bargaining space, this time with the unary formula $A(x) = e(A, x) \in T$ ranging over commodities in relation to a fixed commodity A. We then view A as the sheaf over T by assigning to each degree p the set of commodities that p-belong to A. [19].

5.2.1 By considering degrees as open sets of a space, we also get a notion of what is "interior" to the sheaf A, namely, its sub-sheaves. We will expound on an operation that yields a subsheaf given a sheaf in 6.2.

5.2.2 This spatiality does not require points or distances, but only the immanent relation of inclusion.[20]

5.3 To view degrees as probabilities, let us define a *valuation* (a special kind of measure, see 4.2) on our Heyting algebra as $m : T \Rightarrow [0, 1]$ which satisfies:

a) $m(\mu) = 0$
b) $m(\mathcal{M}) = 1$
c) $p \leqslant q \Rightarrow m(p) \leqslant m(q)$
d) $m(p \wedge q) + m(p \vee q) = m(p) + m(q)$.

Then $m \circ e$, which we denote as $e_m(x, y)$, is an *exchange-likelihood*, and A^*_m denotes a set whose belonging is a random variable. Similarly, the predicate $A(x)$ can be viewed as a predicate over probabilities $A_m(x)$.

5.3.1 This allows us to model economic facts in probabilistic terms. A statement of *conditional probability* can be conceived as obeying $m(A|B) = m(A \wedge B)/m(B)$. In other words, the chance of an exchange labeled A occurs given

[18]Note that, unlike a set theoretical approach which imply that some sets are "in" others, this exposition translates the interior into a particular type of operation, called a *monomorphism*[22]. This approach was pioneered by William Lawvere[18].
[19]Badiou calls this the "transcendental functor"[3].
[20]We follow Badiou here in our use of locales. See chapter IX of [24] for more on this. For an interesting critique of Badiou's use of this formalism, see [43].

that an exchange B occurs is the probability of A and B occurring divided by the probability of B occurring.

5.3.2 Bargaining spaces (3.2) are inherently probabilistic, consisting of information from which one can infer the likelihood that a certain exchange may happen. The probability measure is one possible entryway into how the inner logic of commodities appears *to us*, but this form belongs to what we've termed *economic effects*[21](1.).

5.3.3 In 7. we will see how, for any commodity C, there exists another commodity, a certain quantity of money, such that the probability of their exchange is $\mathcal{M}$.

5.4 A predicate $A(x)$ may not evaluate a commodity B as maximal, that is, it is possible that $A(B) = p \leqslant \mathcal{M}$. However, we may find a part of B which would evaluate maximally (3.2.3). We call this part, if it exists, the *A-component of B*. If A is a priceable predicate, and if $A(B) = \mathcal{M}$ then we call A the *equivalent form* and B the *relative form*. If $A(B) = p$ then A is a p-equivalent form of B and B is the p-relative form of A.

5.5 For any fuzzy set A^* let us denote its *core* as the subset whose elements belong to the maximal degree $\mathcal{M}$. Accordingly, the core of A^* is precisely A_E, the exchange set of A (2.), and in this context we call this the *expanded relative form of value* of A. We can relax our condition to include p-relative commodities or higher into our expansion for $p \leqslant \mathcal{M}$.

5.6 Remarkably, we can also view A^* compositionally by looking at each possible value of belonging as defining a part of A. Namely, the collection of all commodities which p-belong to A^* can be considered as itself a predicate $B(x)$ such that $B(A) = p$. This is equivalent to choosing one of the commodities in the p-relative expansion of A and considering the predicative form of that commodity as B. Now, as we stated in 5.4, there may exist a possible "slice" of A whose B value is maximal, the B-component of A. In fact there may be multiple such slices, but they would be indistinguishable from the perspective of B. But we know that B is any general predicate that evaluates A at p. Therefore, for every degree in T, one could obtain a general predicate C and likewise a C-component of A (a set of slices of A indistinguishable to C). We recapitulate this movement in 6.4.

5.7 There is a procedure for converting fuzzy sets into actual sets. Let us

[21] Behavioral economics, which studies how humans make economic decisions in the presence of uncertainty, would fall into the category of a subjective logic. We are not claiming that the objective logic can explain all economic effects, e.g. on human psychology, but we do claim to outline the domain within which these effects originate.

define a *point* ϕ as a map between T and T_2, where the latter is a Heyting algebra containing only M and μ. This point obeys the following rules:

a) $\phi(q) = \mathcal{M} \wedge p \geqslant q \Rightarrow \phi(p) = \mathcal{M}$
b) $\phi(\mathcal{M}) = \mathcal{M}$
c) $\phi(\mu) = \mu$

Each point is a set-theoretical model of our operational space, where every fuzzy set is an actual set, and exchangeability is preserved but with some freedom to "choose" which non-maximal exchanges become maximal (applying the rules above). There are many such points for a given T[22], and they together form a space of extreme generality, more general than what we've described thus far. We plan to further investigate this remarkable space in a future work.

Atomic components

6. We defined predicates over sheaves and fuzzy sets in the previous section as logically equivalent ways that commodities appear in our space (5.). We also asserted that predicates can be combined to form composite predicates (likewise, commodities combine to form a composite commodity). However, we also have the reverse procedure of decomposing a given predicate into its sub-predicates. At the limit of this procedure, we have an *atomic predicate* $a(x)$, the most basic expression of value. Every atomic predicate corresponds to a unique commodity, one which cannot be further sliced to form a different commodity. For example, in the world of commodities, we can evaluate the "pack of cigarette"-ness of any other commodity. Let the set of all atoms be denoted as $\mathcal{V}$.

6.1 Atomicity is regulated not by direct quantities of things, but by the minimal set of commodities such that all other commodities are combinations of this minimal set. For example, a pack of cigarettes may be atomic, but one cigarette may not be (exchangeable), and neither is a commodity comprised of a pack of cigarettes and a lighter (since they could be further decomposed to two separate exchangeable things).

6.2 Even though atoms are in a sense undecomposable, we can derive weaker variations of an atom by limiting the maximum degree of exchangeability that it can possibly measure. This weaker variation is still an atom, and therefore has a commodity counterpart. Just as a commodity's existence can be less than maximal (3.4), these weaker atoms are constrained by their existence. We

[22] To actually study these points, we need the notion of an *ultrafilter*.

define a binary operation $\lceil : \mathcal{V} \times T \Rightarrow \mathcal{V}$ called *localization* such that $a(x) \lceil p = e(a, x) \cap p$. Two atoms are *compatible* if the localization on their respective existences are the same, i.e. $a(x) \lceil e(b, b) = b(x) \lceil e(a, a)$. We can also write compatibility as the relation $a \ddagger b$.

6.2.1 The localization of an atom on a degree yields another atom. If each atom is conceived as a measure of exchangeability with respect to a fixed commodity, then localization is the constraint on the upper bound of this measure. Atoms correspond to commodities, so localization imply various states that a commodity can be in, each affecting its capacity to measure others. These various states are mutually compatible since they are possible destinies for the commodity in question.

6.2.2 We can derive quantities (5.1.1) of a given commodity through localization. In this sense, quantity is just another predicate. For example, if $a(x)$ is the atomic component associated to j units of C, and if it is possible to exchange exactly 1 unit of C, then there must exist a degree p such that the localization of a on p will yield the atom corresponding to a single unit. We express this as $a_j(x) \lceil p = a_1(x)$.

6.2.2.1 In the previous notation, we call j in a_j the rank of the atom. We introduce a notation for the quantity predicate as $|C| \in \mathbb{N}$ which is equivalent to the rank of the atomic component for C. When considering a variable quantity, we also write C^n (where $|C^n| = n$).

6.2.3 If an atom a is derivable from another atom b through localization, then we say that $a \leqslant b$. This order relation on atoms corresponds to an ordering of value.

6.2.4 Two atoms a, b are compatible when either $a \leqslant b$ or $b \leqslant a$, or both.

6.3 Let us define *the general equivalent* to be any atom g that is compatible with all other atoms. In particular, g is the *money-form* and $e(g, g) = \mathcal{M}$.

6.4 We can now translate Marx's development of the value-form into our formalism:

a) A maximal exchange function taken in isolation is *accidental*: $e(A, B) = M$

b) From the standpoint of the atomic predicate $A(x) = e(A, x)$, we have A as equivalent and x as relative.

c) The set of elements $x \in X$ for which $A(x) = M$ is the expanded relative form. This condition can be relaxed to $A(x) \geqslant p$ to obtain the p-relative form.

Here, A embodies the equivalent for a series of other commodities - value takes the predicative form of "x is worth A".

d) For a given commodity C, the set of predicates $\{A(x), B(x), ...\}$ that evaluate C maximally is the expanded equivalent form. There is accordingly a p-equivalent form for each $p \in T$. Each p-equivalent form corresponds to a predicate which obtains for a component of C, namely a priceable part of it. The various forms are ordered according to the rules of localization.

e) The general form of value is found when there exists an "elementary" predicate $g(x)$ that is compatible with every other predicate, that is, there always exists some p for which $g(x)\lceil p = A(x)$ for any predicate A.

6.5 If atoms are considered as predicates, then they are the "generators" of the non-atomic predicates. This allows us to interpret the lattice structure of T as a commodity composed of priceable parts, and to view this composition as an algebraically closed system. This is a radical step which connects our Heyting algebra and what we earlier termed commodity-preserving functions, such as in 2.3.4 and 2.3.5.

6.5.1 A commodity can be seen through the perspective of its collections of atoms. Each atom itself is itself a "narrow" perspective of the commodity. In 2.7, we stated that there exists, for every commodity C, a unique map to the terminal commodity 1. But there can be several operations in the reverse direction, from 1 to C. Each of these operations uniquely identifies a single atom, or more specifically, an *atomic element* of C. Therefore the collection of atoms for C is a collection of all such maps $\mathcal{O}(1; C)$.

6.5.2 The value-form is therefore a logical space within which it makes sense, for example, to talk about the "salt"-like value of every other commodity. More importantly, one can form logical sentences in the particular language of this space, and evaluate the truth of these sentences.

6.5.2.1 Let every commodity C in $\mathcal{C}$ be considered a sentence $\phi_C(x)$ with only one *term* - this corresponds to what we previously called an atomic priceable predicate, with x a *variable*. We evaluate a sentence by assigning a commodity to each of its variables. The truth value of a sentence $\phi_C(x)$ is precisely the term C evaluated at some other commodity D, that is, $\phi_C(D) = e(C, D) = p \in T$.

6.5.2.2 The conjunction (meet), envelope (join), or implication of two sentences is a sentence. The negation of any sentence is a sentence as well (3.3). Every sentence likewise can be evaluated. We call a sentence *valid* if its evaluation is maximal over any possible assignment of its terms. For example, consider the sentence: "If a commodity x is worth C, then it is worth at least D". Its formula would be $\forall x \exists y : C(x) \Rightarrow D(y) \wedge x \geqslant y$. If there is an assignment of commodities to C, D and a range of commodities for x and y such that this formula is valid, we say there exists a model for the statement.

6.5.2.3 Given certain axioms[30], one can deduce sentences from other sentences. Let A be a set of sentences, then $A \vdash \phi$ means that the sentence ϕ is derivable if all the sentences in A are valid (are the axioms). The value-form is a model of *intuitionistic propositional logic* where there is no axiom $\neg\neg\phi \Rightarrow \phi$ in A. [23]

6.5.3 The categorical construction which corresponds to the value-form is the *topos*[24]. Namely, let us define a map for every priceable predicate A, called the *characteristic map*, defined as $\chi_A : \mathcal{C} \to T$. This is interpreted as the map which evaluates to maximal for those "parts" of C exchangeable with A. In addition, there exists, for every degree in T, a singular arrow $1 \to T$ which "picks out" that degree. We can isolate the part of A corresponding to C as the object C_A that fulfills the following commutative square:

$$\begin{tikzcd} C_A \arrow[r] \arrow[d] & C \arrow[d, "\chi_A"] \\ 1 \arrow[r, "\mathcal{M}"'] & T \end{tikzcd}$$

6.5.3.1 The operations of the Heyting algebra $(\neg, \Rightarrow, \wedge, \vee)$ can be considered as particular maps that compose with maps into T. Namely, for the unary operator of negation, we have $\neg : T \to T$ and the others have a signature of $T \times T \to T$.

6.5.3.2 The sentences we construct using the Heyting operators can be formulated with composition. The following diagrams depict this for $\Rightarrow$:

[23] There exist other intriguing logics which might be more suitable for the complete model, which includes more sophisticated forms of interaction between elements of the world, such as *linear logic*. However, this would require technical resources currently beyond us!

[24] A topos is a category which is cartesian closed, has all finite limits and has a subobject classifier. We've discussed the cartesian closed condition in 2.8. We treat T as the subobject classifier, but we leave out details of its precise construction in the world of commodities. For now, let us say it is a particular kind of sheaf over the bargaining spaces of 3.2.7, although in 7. and 8. we try to be more specific. We mention limits in various parts of the text, but in particular we define it in 9.2. A technical treatment can be found in [24] although for non-specialists it may be better to consult the last chapter of [7].

$$\begin{tikzcd}
C_{\psi\Rightarrow\phi} \arrow[r] \arrow[d] & C \arrow[d, "\langle\chi_\psi,\chi_\phi\rangle"] \\
1 \arrow[r, "\langle\mathcal{M},\mathcal{M}\rangle"'] \arrow[d] & T \times T \arrow[d, "\Rightarrow"] \\
1 \arrow[r, "\mathcal{M}"'] & T
\end{tikzcd}$$

The object $C_{\psi\Rightarrow\phi}$ can be interpreted as the part of C where the sentence $\psi \Rightarrow \phi$ holds true, where ψ and ϕ are themselves sentences. As in 6.5.2.2, our interpretation of the statement requires that the commodity we assign to $C_{\psi\Rightarrow\phi}$ is a slice of the commodity assigned to C.

6.5.4 There may in fact be multiple parts which satisfy the above constraints. In the absence of further constraints, we say these parts are equivalent "up to isomorphism".

The money-form

7. The money form unites the various predicates of the world of commodities under a single continuum, the price measure. We denote its atomic predicate as g.

7.1 In practice, g is identified with a specific commodity called the *money-commodity* $M \in \mathcal{C}$, which has to be produced and issued. Each part of money is also a money-commodity, and is therefore also atomic. In other words there exists for any two quantities of money m, n a degree p such that either $g_n\lceil p = g_m$ or $g_m\lceil p = g_n$. This implies a total order on both money atoms and money commodities. We call this property the *formal use value* of the money-form. This property must be reflected in the use set of the commodity itself, e.g. gold, salt, and numbers are good candidates for a money-commodity, but cows, pianos, and essays are not. We will therefore use m_n to denote both a specific (quantified) money commodity and an atomic predicate.

7.2 The formal use-value is part of M_U, the use-set of M. Let us consider any Δ-resolution where Δ contains the formal use-value as one of its elements to be a *money-compatible* resolution. Under this resolution, there exists only a single family of money commodities, which we denote as m_n where $n \in \mathbb{N}$.

7.2.1 The money-compatible resolution is in fact a *homomeric* attribute. It implies that some aspect of the commodity's substrate is organized in such a way that what can be said of the commodity as a whole can also be said of any given part of itself: as noted, while, under a certain Δ-resolution, parts of the use set of gold are also gold, the same being valid for salt or spices, most commodity use sets do not present such property - half of a piano is not a piano, for example - and therefore cannot reflect in their internal composition the external ordering of the commodity space. Homomeric attributes allow us to decouple quantity and quality in a mereological analysis, fixating the latter under transformations of the former.

7.3 Let us call $p(x)$ the *price function* that indexes all commodities, within a money-compatible resolution, by that money-atom which has maximum exchangeability with that commodity, i.e. $p : \mathcal{C} \to M$. There exists exchange operations b and s where $s = b^{-1}$ and $b = s^{-1}$ between C and m_n, the quantity of money corresponding to the price of C. We call b ”buying” and s ”selling”.

7.3.1 Given that two money-atoms which localize commodities to the same degree of T are said to be logically the same, and given that a money-resolution allows us to extract parts of a money-commodity such that they are only quantitatively distinguished, we can now associate to parts of M_U *of the same cardinality* a number that orders these parts. This allows us to turn the statement ”Commodity C has the predicate P to the p-degree” into ”The value of C is x amount of money”.

7.4 Price yields a total order on a bargaining space (3.).

7.5 The consistency of a given network of exchanges is the degree to which arbitrage is impossible (arbitrage being a sequence of exchanges that yield a profit, see 4). Let us call $(\mathcal{C}, p)$, where $\mathcal{C}$ are some set of commodities and p a price function, a *consistent pricing* of $\mathcal{C}$ if the chance of arbitrage is close to zero.

7.6 The money atom obtains for parts of a given commodity which possess a certain price. Formally, if there exists some S which is part of a commodity C such that $m_n(S) = \mathcal{M}$, which is the same as saying $p(S) = m_n$, then we say S is the *pullback* of $C \to M$ along m_n. Let all the cores of $m_n(C)$ for each price (indexed by n) be called the *priceable parts* of C. In particular, C can be divided into some numerical units of another commodity D if there exist a set of priceable parts, whose union is C, which are each the same price as D (5.1.1).

7.6.1 We can relax the condition of priceable parts to include degrees less than M, i.e. $m_n(s) \geqslant q$ contains the q-priceable parts.

7.7 If the atomic structure of the money-commodity allows it to express the value of any commodity in terms of a definite part of itself (price), we say the money-commodity guarantees the *universal exposition* of the exchange relation (3.) - it offers a privileged perspective from which to "see" the consistency of the value structure.

7.7.1 The commutative square defined in 6.5.3 can be re-defined in terms of money, in which the characteristic map becomes that of a specific price, and the parts are priceable in terms of money. This comes with a certain loss of generality, given the totally ordered structure of M:

$$\begin{tikzcd} C_p \arrow[r] \arrow[d] & C \arrow[d, "\chi_p"] \\ 1 \arrow[r, "m_n"'] & M \end{tikzcd}$$

The multilayered transcendental structure of the capitalist world

8. Following Alain Badiou, we call a space of operations closed under composition of its parts and equipped with a complete Heyting algebra (CHA) (3.1.1) a *world* and its *transcendental*, respectively. The commodity-space (2.) is therefore a world and its money-commodity, which allows for a price structure (7.), embodies its transcendental.

8.1 However, the definition of worlds and transcendentals does not require the exclusivity of a single operational closure. We call a logical space composed of multiple irreducible transcendental operations a *multilayered transcendental* and index each transcendental as T_i. We call the *dominant transcendental logic*, and write T^i, the CHA which guarantees an universal exposition of relations (7.7) which is *maximally reaching* within the multilayered space. We call *sub-dominant* transcendentals, or T_j, those operational spaces which, though irreducible to T^i, "see" less of the total space of the multilayered world. Taken together, the multilayered transcendental is notated as ${T^i}_{jk}$ indicating that T^i is dominant and T_k and T_j are sub-dominant.

8.1.1 A complete transcendental analysis of a multilayered world $(W, {T^i}_{jk})$, which in this context we call a *social formation*, requires the concurrent description of each intra-layer operational space as well as their inter-layer dependencies. Each operational space maintains its own relative independence from the others, that is, constitute their own categories 2.3. One way to articulate the interconnection of two categories is by assigning the operations in one category to the objects of another. For example, the operation of ”exchanging labour power” in $\mathcal{C}$ becomes assigned to ”an employment contract” in a different layer of W. There can be operations that modify a given employment contract which are again closed under composition. In this case we say that $\mathcal{C}$ has been *enriched* in another category K. [25] However, this is not the approach we are taking in the present text. For practical purposes, we treat these additional layers as simply adjoining new maps in our operational space.

8.1.2 Each indexed transcendental has its own identity function: $\mathrm{Id}_i : W \times W \to T_i$. There exists identity functions for various combinations of layers in the multi-layered transcendental, i.e. ${T^i}_{jk}$ may have Id_{ijk}.

8.2 Following Kojin Karatani, we call a *capitalist social formation* a world (W, ${T^C}_{BA}$) where the universal exposition of value relations is the maximally reaching transcendental, T^C, but where there are two additional sub-dominant operational spaces: the logic of *social contracts* T_B and the logic of *reciprocal relations*, or gift economy, T_A.

8.2.1 We introduce *communal elements* $a \in \mathcal{A}$ and *legal elements* $b \in \mathcal{B}$. In addition, we have two operators Id_A and Id_B which describe relations within $\mathcal{A}$ and $\mathcal{B}$. That is, given $x, y \in \mathcal{A}$, $\mathrm{Id}_A(x, y) \in T_A$ and the same for $\mathcal{B}$. These layers correspond to different levels of description. An individual person can be considered a member of their community (T_A), a citizen and property owner in the eyes of the State (T_B), and also commodified labour power (T_C). And there are legal entities and communal elements which are not people at all, but nevertheless play an essential role in the functioning of layers. The full examination of these sub-dominant logics and their interdependence is left for another installment of this research.

8.2.2 We further introduce some basic inter-layer operations:

a) We call *layer-dependence* when an operation in a given layer entails an operation on another. For example, formal employment is the case where exchange of labour for money implies a formal commitment validated by the State.

b) We call *layer-conformance* the special case where this operation concerns existence: for example, a plot of land can only become a commodity if it first is recognized as private property, as an existing legal entity.

[25] In fact one could view the category of commodities we've introduced as enriched over the category of use-values, where ”uses” are just sets of certain operations on commodities. Enrichment takes a broader view of attaching any kind of ”extra data” to the operational space, such that this extra data itself can be viewed in compositional terms. For a good introduction to this idea, see [7].

c) *Layer-exclusion* is the inverse case, where existence in one layer implies inexistence in another: as social reproduction theory demonstrates, the value of reproduction of labour-power relies on unpaid care and house work, which mediates the consumption of means of subsistence - here, existence of labour-commodity as an individualized commodity implies that reproductive work does not acquire commodity-existence.

d) Finally, we introduce *layer-collapse* as the case where operations that are possible in one layer induce otherwise impossible relations on others - as we will see, in section 9, this is the case when monetary exchange is deformed by gift-giving relations, such that the debt incurred by money lending is larger than the sum advanced, producing a non-equivalent exchange in an otherwise equivalent exchange networks.

8.2.3 For convenience, we write T_j/T_i as the class of inter-layer operations that modify the appearance of an object in T_{ij} solely in terms of its appearance in T_j. The layer in the denominator of the expression, in this case T_i, is said to be "bracketed" for the object. This conditioning can happen between any two layers or between sets of layers. Once a layer is bracketed for a given object, it can be un-bracketed with an inverse operation. This imposes a condition on a diagram composed of several inter-layer movements, namely, that for it to commute, every bracketing process must have a corresponding un-bracketing one.

8.2.3.1 For example, given a transcendental T_{ij}, the following are possible inter-layer operations:

a) T_j/T_i : sends the object to layer j, bracketing i

b) T_i/T_j : sends the object to layer i, bracketing j

In this case, a and b are inverses, so applying a and then b will yield an object in the same layer as applying b and then a. In general, however, the composition of inter-layer operators is not commutative.

8.2.4 The proof of the dominant status of T^C is only indirectly provided here, as a consequence of the reach of valorization processes (9.) across the limits of state and national boundaries.

8.3 Let us call a *market* a set of consistent exchange networks which are indexed by transcendental layers T_A and T_B in the same way. That is, even though the material substrate of a commodity equivalence structure might change over time or space, its elements remain indexed as the same legal entities and as part of the same communities and territories.

8.3.1 We further define a *currency* as a money-commodity indexed by T_A and T_B, thereby constraining it to specific state regulations and national boundaries.

Capital

9. So far we have defined commodities within simple circulation - that is, commodity-preserving operations that remain closed under a consistent exchange network - which corresponds to the cycle "C - M - C′" proposed by Marx. But it is possible, under certain conditions, to embed these operations within larger structures that function across exchange networks and which correspond to Marx's formula "M - C - M′".

9.1 The world of commodities under a consistent exchange structure is stratified. An operation which takes as input a commodity (or commodity bundle) to another of a higher strata is *valorizing*. Increase in value through exchange alone is not possible in a consistent market (7.5). But it is possible to define an operation which maps commodities of a determinate monetary value m_1 in one exchange network to another consistent network where they are evaluated at m_2. We denote an object K_ϕ , called *capital*, which embodies this property. Note that, unlike arbitrage, K_ϕ subsumes consistent exchange networks, rather than only inconsistent ones. Accordingly, K_ϕ is universal among relations which increase value across consistent exchange networks.

9.2 We can denote this universal property of K_ϕ categorically. First, we recall the definition of diagrams, then cones, then limits. A *diagram* is a functor from some indexing category $\mathcal{J}$ to our target category $\mathcal{C}$. It "selects" certain arrows and objects and indexes them on $\mathcal{J}$. A *cone* for a functor $F : \mathcal{J} \to \mathcal{C}$ is defined as a natural transformation π from the constant functor to F:

$$\begin{tikzcd} & C \arrow[dl, "\pi_j"'] \arrow[dr, "\pi_k"] & \\ F(j) \arrow[rr, "F(u)"'] & & F(k) \end{tikzcd}$$

This implies that π has components $j \in J$. A *limit* is a cone such that all other cones factor through it uniquely. If we designate the limit as $g : C \to L$ where σ is a cone for the functor F, then we have the following commutative diagram:

$$
\begin{tikzcd}
C \arrow[rr, dashed, "g"] \arrow[dr, "\sigma_j"] \arrow[ddr, "\sigma_k"'] & & L \arrow[dl, "\pi_j"'] \arrow[ddl, "\pi_k"] \\
& F(j) \arrow[d, "F(u)"] & \\
& F(k) &
\end{tikzcd}
$$

9.2.2 Under the regime of equal exchange, money is a limit for the simple diagram J:

$$
\begin{tikzcd}
\bullet \arrow[r] & \bullet
\end{tikzcd}
$$

A functor from this diagram to the category of commodities would amount to selecting two commodities and an arrow between them. This arrow denotes a simple exchange.

To be a limit for this diagram implies that any ”general equivalent” will factor uniquely (or in earlier terms, localized into) the money commodity. For example, if we take cheese as our equivalent, and our two commodities being exchanged are bread and fish, then the following commutes:

$$
\begin{tikzcd}
\text{cheese} \arrow[rr, dashed, "g"] \arrow[dr] \arrow[ddr] & & \text{money} \arrow[dl] \arrow[ddl] \\
& \text{bread} \arrow[d] & \\
& \text{fish} &
\end{tikzcd}
$$

The limit is also the mathematical name for Badiou’s concept of ”universal exposition”.

9.2.3 The above characterization only goes as far as to define money as isomorphic to every commodity, which is a simplification. In actuality, we have different quantities and qualities, market conditions, and so on which would differentiate our category. This differentiation is performed in sections 2 through 7.

9.2.4 What we need however, is to demonstrate this universality for a particular sum of money. We rely here on the price function defined in 7.3 - two things are exchangeable when their prices are the same - this is a "living proof" of the universality of the money commodity for exchange.

9.2.5 However, capital operates at a higher level of abstraction than money. Essentially, where money "sees" equality, capital "sees" surplus. When Marx says that money is "always already potentially capital", he is calling our attention to the fact that money crosses these levels of abstraction - it operates differently in simple exchange and as capital.

9.2.6 Logically, it may be possible to exchange a lesser amount of money for a greater amount, even if their exchangeability is of a low degree. In actuality, there must be a concrete path to valorization, a process which exploits the multiple existences of commodities in different layers. Let us define a functor again from our simple diagram J to $\mathcal{C}$, except now it chooses only valorizing exchanges (where the target object is priced higher than the source object). We then have a diagrammatic definition of K_ϕ, as the limit of our new functor:

$$\begin{tikzcd}
C \arrow[rr, dashed, "g"] \arrow[dr] \arrow[ddr] & & K_\phi \arrow[dl] \arrow[ddl] \\
& m_s \arrow[d, "\phi"] & \\
& m_{t>s} &
\end{tikzcd}$$

9.3 We can distinguish between three valorization structures depending on the path they take between these exchange networks and their respective indexes on multiple transcendental layers. We call *interest capital* the path $\phi_i : m_1 \to m_2$ that exploits the layer-collapse (8.2.2) of m_1 on T_A and T_B - constraining the money-commodity to the debt-structure of gift giving - in order to guarantee, through a contract, the return of a higher sum m_2.

$$\begin{tikzcd}
m_1 \arrow[r, "\phi_i"] \arrow[d, "T_A/T^C{}_B"'] & m_2 \\
m_{1_A} \arrow[r, "T_B/T_A"'] & m_{1_B} \arrow[u, "T^C/T_{AB}"']
\end{tikzcd}$$

Here m_{1_A} obeys the condition $e(m_{1_A}, m_{1_A}) = \mathrm{Id}_A(m_{1_A}, m_{1_A})$ and m_{1_B} obeys $\mathrm{Id}_A(m_{1_B}, m_2) = \mathrm{Id}_B(m_{1_B}, m_2)$.

9.4 While interest capital relies on the internal properties of a gift-community, the alternative path $m_1 \to C \to m_2$ explores the possibility of a new accidental form of exchange at the frontier between two spaces distinctively indexed by the layered transcendental. We call *merchant capital* the path ϕ_m where the transformation of m_1 into m_2 is mediated by a reindexing of a particular commodity C on T_A and T_B, so that $e(m_1, C) = \mathcal{M}$ and $e(C, m_2) = \mathcal{M}$, such that $m_1 \equiv m_2 \bmod C$ 3.2.4.1. Therefore, these two quantities of money now belong to the same bargaining space 3.2.10.

$$\begin{tikzcd}
m_1 \arrow[r, "\phi_M"] \arrow[d, "b"'] & m_2 \\
C \arrow[r, "T_{AB}/T^C"'] & m_{1_{AB}} \arrow[u, "T^C/T_{AB}"']
\end{tikzcd}$$

where C costs m_1 ($e(C, m_1) = M$) and $m_{1_{AB}}$ obeys $(Id_A(m_1, m_1) = \mu) \Rightarrow (e(C, m_2) = M)$.

9.5 A third possible type of capital is *industrial capital*. Rather than exploiting inter-layer transcendental relocalizations - as when money is submitted to reciprocal forms of social bond (9.3) or commodities are re-indexed across market boundaries (9.4) - industrial capital constructs a path through *compositional* transformations. This is expressed, in Marx, in the expanded formula of capital, where "M - C - M'" becomes "M - (MP+LP - C) - M'" - where LP stands for labour power and MP for means of production - requiring us to consider the composition of commodities within the sphere of capitalist production. In this case, let ϕ_P be the class of productive relations, which are each self-relations where the source and target markets are the same. However, a productive relation assigns a particular quantity of money, the money advanced for production, to a new quantity of money, the value of commodity produced. Formally, $\phi_P(m_i) = m_j$ with $i \leqslant j$ defining the line between *valorizing* and *trivial* ventures.

9.5.1 We can form a commutative square that decomposes ϕ_P into acts of buying b and selling s, along with the productive process itself ν. While the arrows b and s are basic commodity-preserving arrows, ν is commodity-producing - which implies that C_j's use set is at least partially composed out of C_i (10.) - and value-transferring - meaning its exchange set is at least the value of the exchange sets of C_i (11.).

$$\begin{tikzcd} m_1 \arrow[r, "\phi_P"] \arrow[d, "b"'] & m_2 \\ C_i = MP \otimes LP \arrow[r, "v"] & C_j \arrow[u, "s"'] \end{tikzcd}$$

The above is Marx's expanded formula for industrial capital in category-theoretical terms.

The commodity-production point of view

10. In order to present how v enables the previous diagram (9.5) to commute, we must elaborate on our initial definition of commodity-producing operations (2.3.5 and 2.3.6). The set C_U of a given commodity C defines all the ways in which a commodity C is the input or output of a production process. We can distinguish these two possibilities with a division of C_U into $\mathcal{C}_U(*; C)$ and $\mathcal{C}_U(C; *)$, where the former are production maps into C and the latter are those out of C. In figure 2 and figure 3 we abbreviate these sets to $(*, C)$ and $(C, *)$ respectively. In 10.3.1 we show that this division is not a strict partition, as there are maps which are common to both. We can further restrict these sets by specifying their resolutions, or more importantly, by specifying constraints based on the exchangeability function.

10.1 Recall 2.3.6, where we defined a set $\mathcal{O}(A_1, A_2, ...A_n; B)$ containing the commodity-preserving operations which have $A_1, A_2, ...A_n$ as the input and B as the output. For a given operation, we write its input as a *monoidal product* $A_1 \otimes A_2 \otimes ... \otimes A_n$. This notation implicitly relies on the fact that $\mathcal{C}$ is closed under $\otimes$ - indeed, it is the operation of counting multiple commodities together as a single commodity (2.3.3). In addition, we say that this operation is *symmetric*, such that $A \otimes B = B \otimes A$. Therefore the category of commodities is a *symmetric monoidal category*. [26]

10.2 Every commodity is either (1) an output of a previous commodity-producing map, a *product* or P, for convenience, or (2) introduced into the commodity space through an appropriate layer-conforming operation in T_A and T_B (see 8.2.2)- what is called a *fictive commodity*. Land enclosures and intellectual property rights are examples of a previous localization of an invaluable object on T_B which allows it to be alienated through contracts and used in a

[26] We leave out several technical details here. To fully define the SMC structure, we need to first define a functor $\otimes : C \times C \to C$, and natural transformations which categorify the notions of associativity and commutativity.

productive endeavour so that it affects the magnitude of value of the product, while unpaid housework is an example of an activity that remains outside of the commodity space, even though it mediates the private consumption of commodities, due to being localized on T_A as a "family affair".

$$
\begin{tikzcd}
& \begin{array}{c}\text{commodity}\\ (*, C)\end{array} \arrow[dl, "(1)"'] \arrow[dr, "(2)"] & \\
\begin{array}{c}\text{product (P)}\\ (C_i \otimes C_j) \to C\end{array} & & \begin{array}{c}\text{fictive commodity}\\ 0 \to (C)_{AB} \to C\end{array}
\end{tikzcd}
$$

Figure 2: Possible inputs to commodities

10.2.1 We call the *commodity-production point of view* the perspective that treats every $C \in \mathcal{C}$ either as a fictive commodity or as a product.

10.3 Every commodity is either (1) consumed as an input in the production process of other commodities, as a *productive consumable*, (2) consumed outside the commodity production process, as a *simple consumable*, that is, as the input of a process that is not commodity-preserving - like eating - or (3) removed from commodity space through a transcendental re-indexing that does not affect its use set. The set of productive consumables bought with the same anticipated capital for the sake of production of the same product belongs to the same *production process*.

$$
\begin{tikzcd}
& \begin{array}{c}\text{commodity}\\ (C, *)\end{array} \arrow[dl, "(1)"'] \arrow[d, "(2)"'] \arrow[dr, "(3)"] & \\
\begin{array}{c}\text{productive consumable}\\ (C \otimes C_i) \to P\end{array} & \begin{array}{c}\text{simple consumable}\\ C \to 1\end{array} & \begin{array}{c}\text{layer-excluded}\\ C \to (C)_{AB}\end{array}
\end{tikzcd}
$$

Figure 3: Possible outputs from commodities

10.3.1 We introduce now the notion of *reproduction structure* of a commodity. That is, given a commodity C, we can consider the set of commodities C_{MS}

such that $r : (C \otimes C_{MS}) \to C$. The map r functions as a sort of "indirect" identity map for use-sets, with C_{MS} returning the consumed part of C_U, such that its use and exchange sets are preserved[27]. We call r the reproduction structure of C and C_{MS} the *reproductive consumables*, or means of subsistence, of C - the last can be either productive or simple consumables. Notice that because r has C in both its domain and codomain, it actually belongs to both $(C, *)$ and $(*, C)$.

10.3.2 Considering both the input and reproduction structures of commodities, we can now further discern three different classes of productive consumables:

a) There are commodities which are exclusively composed of productive consumables - that is, they were previously the output of commodity production and their reproductive consumables are part of the same production process as they are, forming an additional cost to the buyer of the commodity in question. These we call *private means of production*, or MP^P.

b) There are commodities which are not exclusively the output of commodity production, but whose reproduction costs are part of the same production process which employs the commodity - most natural resources fall under this category: they are not exclusively the product of commodity-composition (10.2), though transforming them into a commodity usually requires labour, while their reproduction takes place inside the productive sphere: if one buys land for farming, one must also buy fertilizers and water to replenish the land. These are called *natural means of production*, or MP^N.

Unless where otherwise specified, we denote the set of both of these types of commodities by MP. Finally:

c) There are productive consumables that are neither the output of previous commodity production nor reproduced inside a production process - this is the case of the *labour-power*, or LP: like MP^N, labour-power becomes a commodity through the layer-conformance of human labouring-capacity as a private property, in T_B, but the commodities it needs to reproduce this capacity come from the set of simple consumables and are therefore exchanged for money outside the production process.

[27]Though we don't properly develop it here, we can further define *reproductive labour* as the employment of labour use in the preservation of another commodity's use set - such that $(LP \otimes C) \to C$

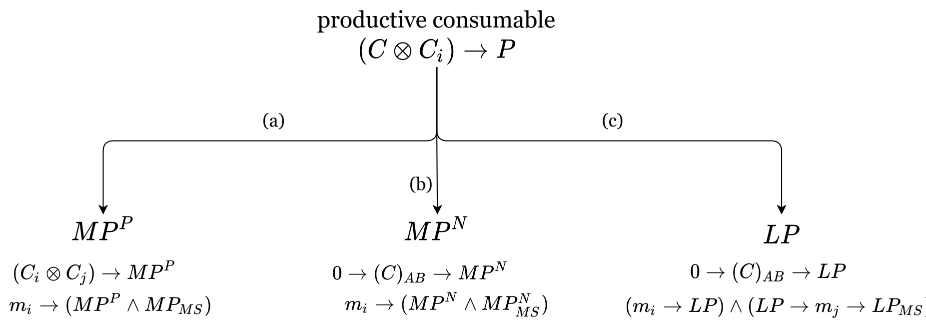


Figure 4: Distinctions amongst productive consumables

The labour-determination of the magnitude of value

11. The labour-commodity LP, like any commodity, has sets LP_U and LP_E (2.) which cannot be empty (2.1). As with other commodities, the first defines the set of ways one might use labour power, the latter the set of other commodities it can be exchanged for. Among the productive consumables, labour power is unique, in that its reproduction process lies outside the production process that employs it. By looking at the production process from the standpoint of this commodity, we are able to not only measure magnitudes of value, but find the sufficient conditions for valorization.

11.1 From the set of commodity-preserving operations $\mathcal{O}(A_1, A_2, ...A_n; B)$ (2.2.2.2), we defined in 2.3.6 a subset called the commodity-producing operations $C_U(A_1, A_2, ...A_n; B)$ which are operations that produce an output which can be differentiated from its inputs on the basis of use-sets. Now, we define *concrete labour* as the subset of these commodity-producing maps which combine a given LP with other commodities, including potentially other labour power, in the production process - written as $\mathsf{CL}(\mathsf{LP}, A_1, A_2...A_n; B)$ for a fixed LP - which can be abbreviated in the form $\mathsf{CL} : (\mathsf{LP} \otimes \mathsf{MP}) \to \mathsf{P}$. For now we can ignore the other LP_n in this abbreviated notation.

11.2 We call *abstract labour*, AL, the restricted subset of such maps which are, additionally, *value-preserving* - that is, $\mathsf{AL}(\mathsf{LP}, A_1, ...A_n; B) \subset \mathsf{CL}(\mathsf{LP}, A_1, ...A_n; B)$ where $\psi \in \mathsf{AL} \Rightarrow (p \circ \psi)(\mathsf{LP} \otimes \mathsf{MP}) \geqslant p(\mathsf{LP} \otimes \mathsf{MP})$ where p is the price operator from 7.3. Abstract labour is therefore not a mental abstraction from different concrete activities, but a class of real activities where effort is not spent solely in view of material transformation of things - albeit this remains its necessary medium - but is guided by the concern with at least preserving the value of productive consumables in the output. It is, in other words, real labour that "sees" value, rather than only use.

11.3 Given that abstract labour conserves value and that, because of this, it is a privileged form of productive consumption of LP, which allows the diagram of ϕ_p to commute, it also offers *an alternative metric for value*, one which, like capital (9.), works across consistent exchange networks. To construct it (11.5), we must first establish a particular property of abstract labour which allows for the induction of a value relation between labours.

11.3.1 Recall that we define a Δ-resolution in 2.5 which allows us to treat commodities as equal when their use-sets have some common property in Δ. We have defined AL as a subset of commodity-producing maps, but this restriction warrants the name "abstract" because it introduces an *indistinction* between types of labour which produce an equivalent exchange value, say LP_1 and LP_2, from the perspective of value-preservation, such that if both of these labour-commodities satisfy, for the same P, MP and initial money m_1, $(LP \otimes MP) \to P$ such that $P_E \geqslant m_1$, then $LP_1 = LP_2$ under this resolution.

11.3.2 We can alternatively write maps $AL(LP;*) \subset CL(LP;*)$, as LP_A - the set of value-transferring-uses of a given LP_U - as a convenient way to single out a restriction of commodity-preserving uses of labour to those which are also at least value-preserving. To re-formulate the indistinction of abstract labour, we can say that $(LP_A \subset LP_{1U} \wedge LP_A \subset LP_{2U}) \to (LP_{1U} = LP_{2U})$. If AL is abstract labour seen as a differentiation of CL, LP_A highlights its internal indifferentiation.

11.3.3 The constraints outlined in 11.3.1 and 11.3.2 can be formalized diagrammatically. We can then express LP_A as a limit of this diagram:

$$
\begin{tikzcd}
C \arrow[d, dashed] \arrow[dr] & & m_1 \arrow[r, "\phi"'] \arrow[d] & m_2 \geqslant m_1 \\
LP_A \arrow[r] & LP \arrow[r, shift left, "f_{LP_1}"] \arrow[r, shift right, "f_{LP_2}"'] & LP \otimes MP \arrow[r, "\psi \in AL(LP;P)"] & P \arrow[u]
\end{tikzcd}
$$

We say that LP_A is an *equalizer* for any two arrows f and g from LP to $LP \otimes MP$.

11.4 We have said that the commodity-production point of view considers all commodities to be fictive or produced (10.2.1). This implies that means of production are themselves the products of previous labour (or they are fictive, as in the case of land enclosures). Let us consider the view that all private means of production can be entirely translated into past labour expended. This is called the *labour-value point of view*. We will continue to treat natural means of production and labour itself as "black boxes" in the sense that their internal composition requires operations that are not commodity-preserving and therefore not immanently expressible in the category $\mathcal{C}$. Formally, any commodity C which is of type MP^P can be written as *embodied abstract labour* C_{LP_A} which is a set consisting of past expended abstract labour $LP_{A1} \otimes LP_{A2} \otimes ... \otimes LP_{An}$ where $1..n$ belongs to some indexing set of input commodities for C.

11.5 The labour-value point of view allows us to view commodities as products of labour alone. Because of this, just as commodities express value in their relations with one another, we are able to express value as relations between different forms of labour-use. Under this condition, we can construct our labour-valuation structure through a series of steps remarkably similar what was outlined in 6.4.

a) Let us fix a commodity C and consider a particular commodity-producing operation ψ whose output is C. There can be different inputs to this operation, that is, different combinations of MP and LP which satisfy $\psi(MP \otimes LP) = C$. Then, let us restrict to only combinations such that the exchangeability of C is at least that of $MP \otimes LP$. Under this constraint of value-transfer, and considering the indexed set of LP^n (6.2.2.1) - each an individual labour being employed within ψ - we can construct an "average" for social labour.

b) Let us call *accidental labour use* the case where, under the previous conditions, two different labours $LP_1, LP_2 \in LP^n$ are compared with regards to their value-transfer uses, that is, LP_{1u} and LP_{2u}. We can employ the exchange function here, even though we are assessing use-sets, because we are comparing labour-uses in term of their specific capacity for value-preservation: $e(LP_{1u}, LP_{2u}) = p$. This allows us to state that they are equivalent to the p-degree in their contribution to the transfer of value to commodity C.

c) However, we can expand these accidental comparisons by fixating one of the compared labour-uses, say LP_{1u}, and treating the given labour-use as an atomic component of the form $LP_{1u}(x)$, where x is any other labour-use under the same general constraints (item a). Note that these comparisons, at this point, are not *quantitative* comparisons. All we can say is that several individual labours are "more or less" equivalent with regards to value-transfer to C - and, in the specific case of the atomic component $LP_{1u}(x)$, we can only say other labours are "more or less" "LP_{1u}-like", which in fact is a common exemplary measure of labour-use performance in the workplace[28]. The set of all x such that $\mathsf{LP_U}(x) = \mathcal{M}$ is the *expanded equivalent form of labour use* of $\mathsf{LP_U}$, while the list of predicates which obtain for a fixed $\mathsf{LP_U}$ forms the *expanded relative form of labour use*.

d) We now define a predicate SNLU, or *socially necessary labour use* for the commodity C such that for every component $\mathsf{LP_U}(x)$ in LP^n, we have a possible localization on a degree p, i.e. $(\mathsf{SNLU}\lceil p)(x) = \mathsf{LP_U}(x)$, allowing us to translate any comparison of other labours to one in particular into a comparison of those labours to some socialized form of labour. This means that all expanded relative comparisons of labour-use find a common expression in this new predicate, which is able to consistently assess the contribution of individual labour-uses to the total value-transfer process to C. Still, comparisons to a necessary *form* of labour use give us a standard evaluation of individual labours in terms of their similarity to the p-degree with a certain "way of working" - evaluations which, albeit unified in a single list, remain partially ordered.

e) So far, $\mathsf{SNLU}(x)$ expresses a socially average form of labour use under the constraint of value-transfer. However, because we view the end product of labour within a money-compatible resolution, we can also view this virtual predicate in terms of its measurable slices. Recall that a measure (4.) allows us to translate the partial order of our Heyting algebra into a total order - which would require us to find a predicate within the set of labour-uses which can also allow for a total ordering of values. We define *socially necessary labour time*, or $\mathsf{SNLT_C}$, as a certain quantity (which we will generally write in units of hours). Intuitively, this is "how long it takes" for the average worker (who incarnates a way of working like $\mathsf{SNLU}(x)$) to produce C. $\mathsf{SNLT}(x)$ then gives us the portion (from 0 to $\mathsf{SNLT_C}$) that a given $\mathsf{LP_U}$ transfers within ψ. The sum of all contributions of every $\mathsf{LP_U}$ in the process cannot exceed the $\mathsf{SNLT_C}$, that is, $\sum_i \mathsf{SNLT}(\mathsf{LP}_{\mathsf{U}i}) \leqslant \mathsf{SNLT_C}$. This means that any time taken by $\mathsf{LP_U}$ above what is "socially necessary" is lost in terms of value-transfer. The crucial aspect of this part of labour's use-set is that it is measured in units of chronological time[29] and is therefore totally-ordered and homomeric - in other words, it is also itself a "money-compatible resolution", making it possible to assign to parts of $\mathsf{LP_U}$ equally ordered parts of the *magnitude* of value that were transferred to the product.

11.5.1 Once equipped with a consistent valuation of the socially necessary

[28]One needs only to recall the "Stakhanovite movement" in the USSR to find an example of such a measure by example: `https://en.wikipedia.org/wiki/Stakhanovite_movement`

[29]It is crucial to note that "chronological time" means here a linear, ordered and marked passage of time that is socializable beyond a more subjective experience of durations - it is therefore already at stake in the pulsations of work songs, much before the adoption of mechanical clocks which further objectivate this and add more granularity to this measure. A similar argument is made in [35] - for a beautiful study of music and labour, please see [25]

labour time that goes into a given product such that the value of the composed productive consumables is at least preserved, it is possible to express value in terms of a given quantity of labour hours - once more, a quantitative information has been derived through a predicative logic. If we now return to our value-preserving maps AL, we can use $\mathsf{SNLU}(\mathsf{LP}_U) = \mathcal{M}$ as a positive content for the indistinct labour-use LP_A, thus assigning to "value-transferring abstract labour" an equivalent quantity x of labour hours h: $\mathsf{LP}_A = \mathsf{SNLT}(\mathsf{LP}_U) = xh$.

11.5.2 It is worth noting that, while we defined $\mathsf{SNLT}(x)$ under constraints of value-transfer of a specific commodity C and a specific time-quantity of total labour-use ${\mathsf{LP}_U}^n$, alternative and more generalized measures of the same form are possible as long as we are able to meaningfully construct the object - in our case C_E - that constrains the value-transfer, or even the *type* of price-structure, such that it can serve as a new index of the productive sphere.

11.5.3 We introduce a *rewrite rule* of past labour of a given commodity as follows: given the set C_{LP_A} consisting of various embodied abstract labours for C, which is the commodity under the labour-value point of view (11.4), we can now sum all of these to yield a measurement of the total value of the commodity. We can therefore say that "C is worth x hours" just as previously we said that "C is worth y units of money". Excluding labour-power, the value-transferring properties of any commodity are limited by its total embodied value, so we say that SNLT exclusively determines the magnitude of value of a commodity.

11.6 The value of commodities which are products of previous production processes is thus measurable by the amount of socially necessary labour hours that went into their production, while the value of commodities which are not produced, but only *reproduced* (MP^N and LP, see 10.3), is measured by the social necessary labour time that went into the production of the commodities C_{MS} needed for their maintenance, or their reproduction cost.

11.7 Finally, returning to our definition of labour-power in 10.3.2, consider that while LP_E is determined by $\sum S_E \mid S \in C_{MS}$, LP_U, not being a product of commodity-production, is constrained into value-preserving labour solely by its output maps. This introduces an important asymmetry between the labour-measure and the monetary measure of value: while every other commodity can be analysed in terms of the commodity-producing maps that go into it, that is in terms of labour-compositions, labour itself cannot be further decomposed in the same terms. In this sense, though $\mathsf{SNLT}(x)$ was generated through further localizations of the exchange function, it emerges as an undecomposable atom of value, therefore acquiring logical precedence over the magnitude of value of products.

The operad of capitalist production

12. We have described the conditions which allow for the diagram of industrial capital to commute (9.5.1) - that is, the distinction of types of commodities from the standpoint of commodity-production (section 10.) and the induction of a value-preserving relation within the production process, through the concept of abstract labour and socially necessary labour time (section 11.). These conditions, however, establish the definite possibility of a *trivial* venture, that is, the case of ϕ_P where ν guarantees that $\phi_P : m_i \to m_j$ such that $i = j$. To understand how a properly valorizing process is possible, where $\nu : (LP \otimes MP) \to P$ such that $p(P) = m_j > m_i$, we must have at hand formal tools to explore the different ways that labour-power and means of production can be combined in a production process.

12.1 We now define an *operad* for commodity production. First we take the usual definition of a *category*[22] with classes of objects and morphisms with the appropriate composition and identity axioms. Then we allow the domain of each morphism to be a finite list of objects (the defining feature of a *multicategory*) [23]. Composition proceeds by attaching the output of one operation to one of the inputs of the next. An operad is such a category with only one object O, meaning that the domain of any morphism is a list of O and the output is a single O. Formally, let $P(k)$ be the operad that gives for each $k \in \mathbb{N}$, a set of k-ary operations (morphisms) on O. Note that the order of the inputs to a k-ary operation does not matter. Composing an m-ary operation with a n-ary operation gives us a $(m + n - 1)$-ary operation, and is associative. There also exists a unit $I \in P(1)$ which behaves as an identity.

12.1.1 If we envision each element of $P(k)$ as a "box" with input and output "wires", then we can define an *algebra* of these boxes [36]. Formally, let $q, p_1, p_2, ..., p_n$ be elements of $P(k)$ and let $\mathcal{W}(p_1, p_2, ..., p_n; q)$ be the set of ways of "connecting the wires" of $p_1, p_2, ..., p_n$ such that they form q.

For example, an element of $\mathcal{W}(p_1, p_2, p_3, p_4; q)$ may look like this:

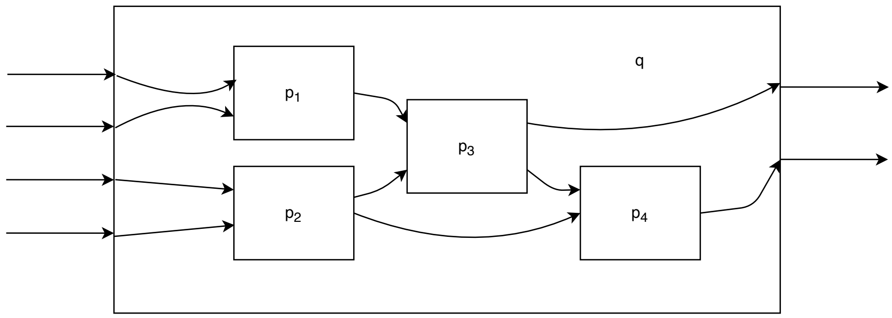


Figure 5: A morphism in the operad

We could also rewire this such that p_1 and p_4 are swapped, and that would also belong to $\mathcal{W}(p_1, p_2, p_3, p_4; q)$. But if we added or removed boxes, then it would belong to a different set still within the operad. Furthermore, each inner box can be "revealed" to contain further boxes (12.1.3).

12.1.2 In order to support multiple input and output wires, we say the inputs and outputs of each box q are sets $\mathrm{In}(q)$ and $\mathrm{Out}(q)$. A successful wiring requires:

a) that every wire $w \in \mathrm{Out}(x)$ is either identified with the wire $u \in \mathrm{In}(y)$ where x and y are (not necessarily distinct) boxes inside of q, or is identified with an element of $\mathrm{Out}(q)$.

b) that every $w \in \mathrm{In}(q)$ is identified with a wire $u \in \mathrm{In}(x)$ for some box x in q or is identified with an element of $\mathrm{Out}(q)$.

12.1.3 The composition of morphisms of the operad can be interpreted as the operation of "encapsulating" parts of the wiring diagram. Conversely, factoring morphisms is the operation of "unpacking" the boxes. One can, for example, view p_3 and p_4 above as a single box r with $|\mathrm{In}(r)| = 3$ and $|\mathrm{Out}(r)| = 2$. This amounts to the composing $\mathcal{W}(p_1, p_2, p_3, p_4; q)$ with $\mathcal{W}(p_3, p_4; r)$ to form $\mathcal{W}(p_1, p_2, r; q)$.

12.1.4 Finally, there exists a way of assigning values to each wire, such that we can add further conditions onto the boxes, which is called V or the *value-set*. This is the function $V(q) : \mathrm{In}(q) \times \mathrm{Out}(q) \to \mathrm{Set}$ where q is a box.

12.2 Every commodity B is the result of some production process that can be modeled as an operad. Let us call the elements of $P(k)$ *production processes* which have k inputs. The inputs themselves are assigned elements of $\mathcal{C}$, namely, commodities. Therefore, $\mathrm{Out}(i)$ and $\mathrm{In}(i)$, where i is a production process, are finite lists of elements from $\mathcal{C}$. We can compose production processes together to form other production processes as detailed in 12.1.

12.3 Value transferring operations, and commodity preserving operations more broadly, can require certain types as inputs. These types are defined by being indistinguishable under a particular Δ-resolution (2.5). Throughout this text, we use a resolution that separates MP and LP as two distinct "types" of commodities (10.3.2), but there are certainly other useful resolutions.

12.4 The algebra of operads allows us to conceive of "connecting" separate processes of commodity production together to form a larger process. Conversely, we can "look into" a given process to see its constituent processes. We can add, rewire, substitute, or eliminate processes within a given process, while maintaining its "outward shape".

12.5 A commodity-producing process is a restricted type of operad, where inputs for boxes are not only constrained by the use set of objects, but also by their exchange set - which implies all inputs are commodities - and where boxes are not only commodity-preserving - with $P_E \neq \emptyset$ - but in fact value-transferring - such that $p(P) \geqslant p(LP \otimes MP)$ - which implies that the product of the operad itself, considered as box, is at least as valuable as the productive consumables.

12.5.1 The input and output wires of a box within the commodity-producing operad accepts only use-sets. We require this to emphasize that, from within commodity production, the commodities themselves are no longer primarily exchangeable objects, but rather objects whose sole purpose is to be combined to generate new value. This requires re-organizing them in novel ways, naturally involving their use-values. While the goal of our production operad is not to investigate the manifold possible re-compositions of commodities themselves and instead focus on the question of value transferred, there is certainly a question of the extent to which "usefulness" is restricted under these conditions.

12.5.2 The above conditions can be translated as the statement that there always is some particular granularity of a commodity-producing operad in which every box x has $\mathrm{In}(x) = \{LP_U, MP^1_U, MP^2_U ..., MP^N_U\}$, given that LP_U functions as a regulator for the preservation of value of itself and other commodities (11.5). Nothing prevents, however, that other resolutions might decompose these boxes into a set of relevant "smaller" boxes amongst which there are fully "automated" processes $\mathcal{C}(MP^1_U, MP^2_U, ..., MP^N_U; MP_U)$, - allowing us to discern, for example, how a machine processes its inputs in a series of discrete steps.

12.6 The operad of commodity production becomes fully coherent once we develop a temporal interpretation of the boxes and wires. Namely, production processes (the boxes) do not occur instantaneously, but rather over a duration specific to each of them. Also, since one process may require the completion of a certain previous process in order to have the inputs needed to continue, the durations of an outer box depends on those of the ones inside of it. We define a *propagation time* $n \in \mathbb{N}$ to be the number of hours[30] required to produce one output unit given the inputs, and a *value propagator* $\delta_n : \mathrm{In}(x) \to \mathrm{Out}(x)$ for every box which maps the elements on the input wires to the output wires. We also consider the elements of our sets $\mathrm{In}(x)$ and $\mathrm{Out}(x)$ as being lists of inputs and outputs produced in each cycle, where lists can contain a special element $*$ to denote delays[31].

12.6.1 A propagation time for process x is how long x takes to produce one unit given that it has its necessary inputs[32]. Following the developments from 11.5, 11.6 and 11.7, we denote another quantity, the *average value transfer rate* dV, to represent the amount of labour-hours transferred, or propagated, from the input list(s) to the output list(s). For example, if a labour process involving ten people and a machine produces a component in two hours of chronological time, this does not mean only two hours of abstract labour were transferred. Although the propagation time n is 2 hours, the average value transfer rate dV[33] is determined by the $\mathrm{SNLT}(x)$ predicate defined previously. Let us say that dV is 15. Then, if this same process has been running for 8 hours, then the total value transferred would be $15 * 8/2 = 60$ (socially necessary) hours.

12.6.2 A given production process x may require at least n units of a given input on one of its wires before it can begin its cycle. Accordingly, if x has multiple input wires, each may need require different amounts $[m, n, o, ...]$ which would arrive at different times from other processes. In the meantime, x will output the special element $*$ (12.6) to denote a delay (no production) in its output wire(s). We therefore associate to every δ_n a vector of *unit requirements* $\vec{\delta_n}$ consisting of a number of units for every input wire necessary to a start a cycle. While a cycle is underway, x will also emit $*$ tokens, which are not counted towards the unit requirements.

[30]We choose hours for a purely didactic purpose, since it is the time-unit most commonly used in Marx's work. But it is just as viable to use $\mathbb{R}$ as the domain, or to measure in months or nanoseconds. The point is that this time should index the propagator.

[31]Here we assume one element of a list signifies one cycle, either of the previous process (if the element is part of $\mathrm{In}(x)$) or of the current process (if the element is part of $\mathrm{Out}(x)$). This is a highly simplified account of the notion of *historical propagators* expounded in [36].

[32]For practical purposes, we use hours to measure propagation time, but we could consider the base unit to be the total propagation time of the enclosing box. Then all interior boxes would have propagation times which are fractions of this base unit.

[33]A more accurate model may involve replacing an average rate with a differential equation or system of such equations, which can be solved for certain intervals of time to yield a function rather than a quantity.

Constant and variable capital

13. Equipped with the means to analyse different types of processes and composition structures, we can finally explore the ways surplus value, expressible as $m_s > 0$, can satisfy the valorization process $\phi_p : m_i \to (m_i + m_s)$.

13.1 Given the structure of MP^P and MP^N presented in section 10, there is a *constraint* on the value of the parts of MP_U: since the value of a commodity is determined by the socially necessary labour time that goes into its production or reproduction (11.5.2) and since means of production are themselves composed of productive consumables, the value of their decomposed parts is limited by the value of MP itself: $[(MP^1{}_U, MP^2{}_U, ..MP^n{}_U) \to MP_U]$ where $[MP_E \leqslant \bigcup(MP^1{}_E, MP^2{}_E, ..MP^n{}_E)]$. In the operad structure, this means that the value set V of constituent MP_U's remains bounded by MP_E, even under changes of granularity. Means of production can, at most, transfer their existing value to a new product[34].

13.2 LP, on the other hand, is *not* the product of a commodity production process - being rather the product of a layer-conformance operation. Its value is determined by its *reproduction cost* (10.3.2), that is, the labour-valuation of the *simple consumables* LP_{MS} it needs for its subsistence (11.5.2), a determinant that is *not* directly co-variant with changes in the production process that employs LP_U, since simple consumables are bought by the worker in the market. In other words, the value-transferring potential of LP_U is not necessarily bounded by LP_E: not all parts and use-resolutions of of LP_U are bound by the value of LP_{MS} and it is possible that there might be maps that mobilize parts of LP_U which, combined with MP, exceed the value boundary of labour itself.

13.2.1 Once it is established that the compositional properties of MP are constrained by MP_E while the same is not the case for LP, we can return to our valorizing diagram for industrial capital:

$$\begin{tikzcd} m_1 \arrow[r, "\phi"] \arrow[d, "b"'] & m_2 \\ (LP \otimes MP) \arrow[r, "v"] & P \arrow[u, "s"'] \end{tikzcd}$$

[34]We bypass in this reconstruction the differences of rates of value-transfer between distinct types of means of production: materials that are fully consumed in the production process pass their total value to the product, while tools, workspace and machines, for example, transfer that value in a longer duration, relative to the time it takes to fully consume them, etc. This difference in value-transfer rates is however crucial to understand several features of the reproduction cycle of individual and complex capitals.

And propose two restricted diagrams. The first is the one that ”carves out” a part of m_1 which, being invested in MP, can only pass on to product P a value bounded by $\mathsf{MP_E}$ - that is, the *constant capital* $m_c \subset m_1$:

$$\begin{tikzcd}
m_c \subset m_1 \arrow[r, "\phi_c"] \arrow[d, "b"'] & m_2 \leqslant m_c \\
\mathsf{MP} \arrow[r, "\nu_c"] & \mathsf{P}_c \arrow[u, "s"']
\end{tikzcd}$$

The second is a restriction to consider only the part $m_\nu \subset m_1$ which has been transformed into LP and which, therefore can be used in such a way as to transfer to P more value than $\mathsf{LP_E} = m_\nu$ - that is, the *variable capital* part of m_1, which alone can satisfy the case where $m_2 > m_1$:

$$\begin{tikzcd}
m_\nu \subset m_1 \arrow[r, "\phi_\nu"] \arrow[d, "b"'] & m_2 \leqslant (m_\nu + m_s) \\
\mathsf{LP} \arrow[r, "\nu_\nu"] & \mathsf{P}_\nu \arrow[u, "s"']
\end{tikzcd}$$

13.3 Given a restriction to variable capital $m_\nu \to m_2 \leqslant m_\nu + m_s$ we can construct a correlative restriction on the commodity-producing and value-preserving maps $\nu : (\mathsf{LP_U} \otimes \mathsf{MP_U}) \to \mathsf{P_U}$. Consider, first, the case where $m_s = 0$. This means that what the value transferred to the product is at most the worth of the employed productive consumables $\mathsf{LP_E}, \mathsf{MP_E}$, that is: $m_2 \leqslant (\mathsf{P_E} = \mathsf{LP_E} + \mathsf{MP_E})$, where $\mathsf{LP_E}$ is determined by its reproduction cost, which itself defined by the value of certain simple consumables $\mathsf{LP_{MS}}$. We call this restriction of value-transferring maps of labour use the *equivalent labour* [35] $\mathsf{AL_{Eq}} \subseteq \mathsf{AL}$. Furthermore, for a given P, we can establish the socially necessary labour time $\mathsf{SNLT}(x)$ (11.5) which defines a particular amount of labour hours which measure $\mathsf{AL_{Eq}}$ for a given commodity production, which we express as labour hours $\mathsf{LP_{Eq}}$.

13.3.1 Alternatively, consider the case of ”non-trivial ventures” - where $m_s > 0$. Having recourse to $\mathsf{AL_{Eq}}$ we can now define that, in such cases, the

[35] We changed Marx’s term from ”necessary” labour to ”equivalent” for convenience and coherence with other concepts in the *Primer*.

corresponding commodity composition maps $\nu : (LP \otimes MP) \to P$ will be such that $LP_{Eq} < LP_A$. That is, we will have: $LP_A = LP_{Eq} + LP_S$, where the *surplus labour* LP_S corresponds to the additional value that satisfies non-negatives m_s in $m_2 > m_1$.

13.3.2 Furthermore, given the total product P, we can also define its partition into *equivalent product* P_{Eq} and surplus product P_S such that the total exchange value of the product is $P_E = P_S + P_{Eq}$, with two maximal identities $e(P_S, LP_S) = \mathcal{M}$ and $e(P_S, m_s) = \mathcal{M}$ establishing that surplus labour defines the magnitude of surplus value expressed as m_s.

13.3.3 We define a *working contract* as the indexing of labour hours $LP_A = LP_{Eq} + LP_S$ on the transcendental layer T^B: $(LP_A)_B$ - defining a contract between the seller and the buyer of labour-power for the employment of labour-power for a certain journey[36].

13.4 Once we have the concepts of constant and variable capital - m_c, m_ν and m_s - and of equivalent and surplus labour time, We can define the *surplus production rate* s_k for a given $K_{\phi P}$ in terms of the relation between the invariant and variant parts of variable capital itself:

$$s_k = \frac{m_s}{m_\nu}$$

And a corresponding *rate of exploitation* e_k in terms of the surplus labour time that exceeds equivalent labour hours:

$$e_k = \frac{LP_S}{LP_{Eq}}$$

It is clear that $\delta e_K \to \delta m_s$. We therefore call a commodity production process a *capitalist production process* when $AL_{Eq} \subset AL$, that is, when the use of value-conserving labour exceeds the boundary of the exchange value of labour.

13.4.1 We have shown that labour-power is seen by K as *variable* capital because, unlike MP_U, the value of the set of priceable parts of LP is not bounded by LP_E (13.2). For example, once LP_U is decomposed in terms of labour hours (11.5) we actually have a possible inequality between the value of the set of labour hours employed in production, namely, LP_A and the value of labour-power itself, determined by LP_{MS}, and expressible in terms of a part $LP_{Eq} \subset LP_A$.

[36]Here we simply highlight the connection between commodity and contracts, leaving the wage contract to be defined in 15.

13.5 Following 6.2.2.1, we write as LP^n and MP^n the quantity n of commodities bought with m_v and m_c respectively. This allows us to further determine a distinction between the *value composition* v_k of a productive venture - that is, the proportion of constant to variable capital:

$$v_k = \frac{m_c}{m_v}$$

And the *technical composition* t_k, which gives us the proportion between the *mass* of means of production bought by m_c to the *mass* of labour bought by m_v:

$$t_k = \frac{MP^n}{LP^m}$$

It is clear that v_k and t_k can vary independently from one another to some degree - as the same amount of constant and variable capital can be exchanged for different quantities of means of production and labour respectively.

13.5.1 We can, however, express v_k in terms of the capacity for value-transfer of MP^n and LP^n, showing how changes to technical composition might affect valorization. We have already established that, due to the boundary that MP_E establishes for the consumption of MP, direct increases in MP^n cannot lead to an increase in surplus value (13.2.1). Instead, changes to the mass of means of production can only affect the valorization process on condition (a) that more investment is made to LP^n - as more labour-power, under a constant e_k, will consume more MP - or (b) that the productivity rate for the same labour-force increases - that is, the rate p_k changes:

$$p_k = \frac{MP^n}{LP_A}$$

In both of these cases, the increase in surplus-labour leads to an increase in $m_s \subset m_2$. We call this projection of value composition v_k into the technical configuration of production t_k the *organic composition* of capital.

Surplus extraction and the organic composition

14. Now that we have an intrinsic description of the different roles of MP and LP in the process of value-transfer in commodity-production (See 13.1 and 13.2), as well as a way to project value composition v_k into the technical composition t_k, we can discern different ways in which MP and LP might be combined in a given operad structure (12.) and consider why certain forms of production are more conductive to surplus extraction than others.

14.1 As established in the previous section, the surplus extraction rate s_k is not conditioned by the total advanced capital m_1, but only by its variable part m_v. Furthermore, the magnitude of the additional value, expressible as $m_s \subset m_2$, is itself determined by the exploitation rate e_k, which is quantitatively measured as the number of surplus labour hours LP_S that exceed the labour hours LP_{Eq} whose value is equivalent to LP_E. Having established the dependence of non-trival capitalist ventures on the exploitation of labour-power - on $e_k > 0$ - we now consider the two ways in which the total number of value-transferring labour time LP_A employed in a productive endeavour can vary such that surplus labour time increases.

14.1.1 We call *absolute surplus extraction* any change to e_k that does not alter the set of labour hours that is equivalent to the reproduction cost of labour itself, namely, LP_{Eq}:

$$e_A = \frac{\delta LP_S}{LP_{Eq}}$$

For example, the extension of the work day for an individual LP, raising LP_A, affects δLP_S while keeping the equivalent labour time invariant. Since this individual extension of LP_A is constrained by physical and legal boundaries to some number below 24h, it is also possible to increase the absolute number of surplus hours LP_S by extension of the number of workers: LP^n will multiply ${LP^n}_S$ while keeping individual LP_{Eq} invariant.

14.1.2 We call *relative surplus extraction* e_R any change to the e_k which affects the amount of labour time LP_{Eq} needed to purchase LP_{MS}:

$$e_R = \frac{\delta LP_S}{\delta LP_{Eq}}$$

For example, if due to some reason the reproductive cost of labour-power becomes cheaper, even though LP_A remains invariant, the distribution of these hours between surplus and equivalent will change, with a decrease in LP_{Eq} - conditioned by the change in the value of LP_{MS} - and correlative increase in LP_S. Alternatively, if p_k increases - that is, the productive rate of labour-power - less time might be needed to produce value equivalent to LP_{Eq}, again shifting the frontier between surplus and necessary labour without the need to extend the length of the work day. We can, therefore, distinguish between *consumptive* and *productive* changes to e_R: the former affect LP_{Eq} in another production process, the latter affect their own surplus extraction rate.

14.2 One of the most important forms of consumptive changes to e_R is the effect of the *co-operation of constant capital* in the production of a simple consumable. Its most basic form is the adoption of a single workspace - which we write distinctively as MP^0 - to be shared by several workers at a time. For the sake of brevity, let us simply exemplify this change as two different wire diagrams W and W' which capture different compositions of LP and MP. Note that we utilize a special box with one input and two outputs called *resource* *sharing* which takes a given MP and allows it to be used as inputs to multiple other boxes, provided that its output wires are eventually recombined in a *resource returning* box with two inputs and one output:

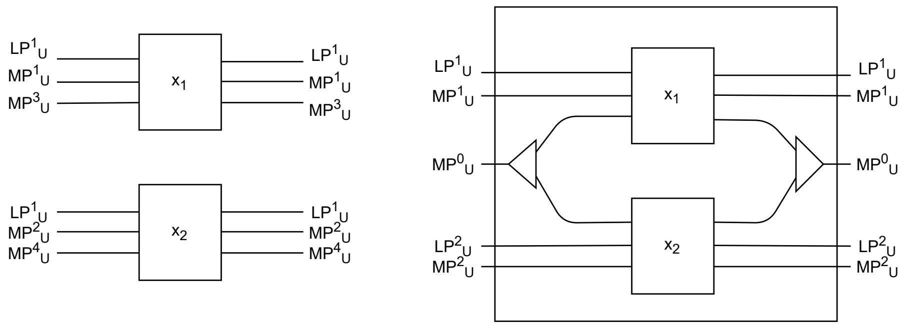


Figure 6: Co-operation of means of production

It is clear that, on condition that $V(MP^0) \leqslant V(MP^1) + V(MP^2)$, each being the respective value of constant capital employed in W and W', the aggregate value of product P in W' will be proportionally smaller, though no variations in surplus extraction were effected. However, if P is a simple consumable that composes the subsistence cost of LP, i.e. $P \in LP_{MS}$, the reduction of value of P will also affect future use-maps LP_{Eq}, therefore also affecting e_R (14.1.2).

14.3 Let us now consider some of the basic forms of productive change to e_R. If changes to the consumption of constant capital - for example, the adoption of common workspaces - can propagate a decrease in the reproduction cost of labour-power, it is only in changes to in the consumption of variable capital which, altering the ratio between LP_Eq and LP_S, can lead to immediate increases in surplus extraction rates. We begin, therefore, by considering the *co-operation of variable capital*.

14.3.1 We call *parallel* cooperation the transformation of W into W', such that a box composing $In(x) = LP^1{}_U, MP^1{}_U$ and $Out(x) = P_U$ where $LP_A = 4h$ is decomposed into some boxes allowing for their simultaneous composition. Here, consider that $MP^2, MP^3 \subset MP^1$, just as the Δ-resolution (2.5) of LP^1 remains the same as that of LP^2 - that is, that the decomposition of W in W' did not affect the quality of the commodities. We have then boxes x_1 and x_2 with the following structure:

$$In(x_1) = LP^1{}_U, MP^2{}_U2$$
$$Out(x_1) = P_U$$

$$\mathrm{In}(x_2) = LP^2{}_U, MP^3{}_U$$

$$\mathrm{Out}(x_2) = P_U$$

And the following wire diagrams:

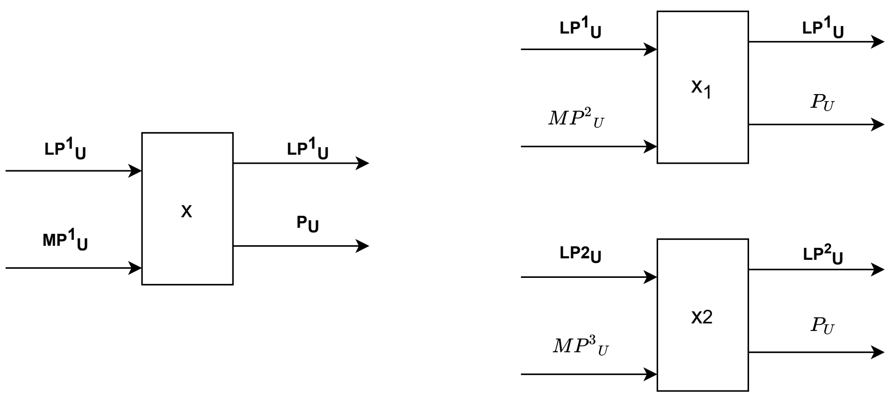


Figure 7: Parallel cooperation of LP^n

The possibility of converting $MP^1{}_U$ into P in half the time keeps individual p_k invariant, but doubles the total productivity, as twice as much product can be sold in half a productive cycle. This change in productivity affects LP_{Eq} and therefore e_R.

14.3.2 In order to consider other changes to e_R we need introduce a new feature into our operad structure, allowing us to speak of decompositions of labour-power which do *not* preserve its form of use - as we will see in the case of serial cooperation. To account for this, we add the powerful concept of a *resolution box* to our operad, as a development on the idea of Δ-resolution of use sets, previously introduced in 2.5. Recall that a Δ-resolution is an endofunctor (2.5.1) that renders different C_u homogeneous from the standpoint of some property - as if finding the appropriate aspect of a given object such that, from the perspective of that aspect, several objects can be said to be the same. We used this same operation to establish "money-compatible" resolutions of objects, allowing them to be divided in such a way that the quality of their parts remained invariant, as their quantities vary (7.2). A resolution box allows us to make explicit the specificity of labour-use in a given process. We add it to every process that consumes LP_u, through the following structure: $\mathrm{In}(\Delta) = LP_U$ and $\mathrm{Out}(\Delta) = LP_{\Delta U}$ where $LP_{\Delta U} \subset LP_U$. Furthermore, given a box which takes LP

as an input, we include a feedback structure $\mathrm{Out}(x) = (\Delta)$, which ties together the refinement of labour use to its effect on the compositional process:

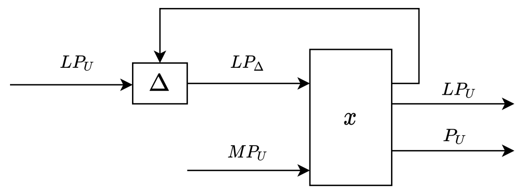


Figure 8: Use-resolution boxes

Given two labour-resolutions LP_{Δ_1} and LP_{Δ_2} we say these are two *specialized labours*. The same labour-commodity can be employed under several different use-resolutions in the same production process, but if the indexing of a working day on T_B already considers a specific Δ or Δ-range, we say $(\mathsf{LP}_\Delta)_B$ is a *professional contract*.

14.3.3 The simplest example of resolution changes affecting e_R is the increase in *labour intensity*, which we can define as a transformation from W to W' where LP_{Δ_1} is substituted for a refinement LP_{Δ_2} such that it now takes less time to output $\mathsf{P} = \mathsf{LP}_\mathsf{E}$.

14.3.4 We call *serial* cooperation the transformation of W into W', such that a box in W with $\mathrm{In}(x) = (\mathsf{LP}^1{}_\mathsf{U}, \mathsf{MP}^1{}_\mathsf{U})$ and $\mathrm{Out}(x) = \mathsf{P}_\mathsf{U}$ is decomposed in W' into several smaller steps, transforming the use of labour such that the new resolutions $(\mathsf{LP}^1{}_{\Delta_1}, \mathsf{LP}^2{}_{\Delta_2}) \subset \mathsf{LP}_\Delta$ and labour hours $\mathsf{LP}_{\mathsf{Eq}} > (\mathsf{LP}^1_{\mathsf{Eq}} + \mathsf{LP}^2_{\mathsf{Eq}})$:

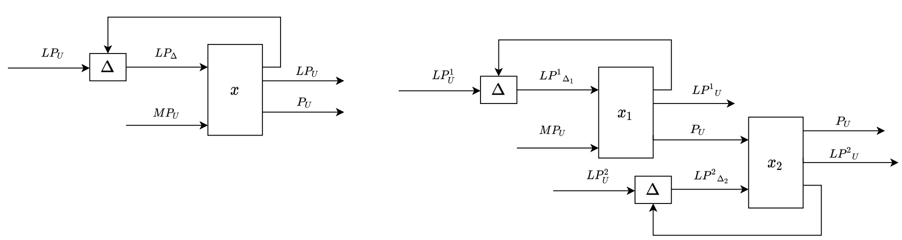


Figure 9: Serial cooperation of LP^n

14.3.5 Resolution boxes are relevant not only in making labour-use commensurate with some means of production, but also in making different labours commensurate amongst each other. We call *coordinated* cooperation the transformation of W into W' such that the resolution box of each labour employed in the production process is "synced" together, increasing the average refinement of labour-use, thereby affecting productivity and, consequentially, e_R:

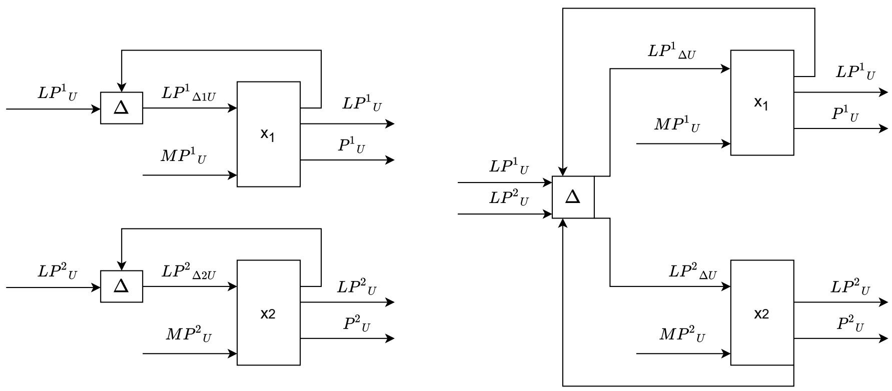


Figure 10: Coordinated cooperation of LP^w

In the case of coordinated cooperation, labour refines its own form not through the exclusive mediation of the means of production and their material resistances, but rather through an overlapping of different labour-uses themselves.

14.4 The case of coordinated cooperation is the first instance we have seen thus far of composition of labour-uses. While socially necessary labour time establishes a homogeneous and homomeric predicate that allows to compare labour uses to priceable parts of themselves and to one another, resolution boxes allow us to further describe how, within the confines of a valorizing venture - that is, within the constrains of abstract labour and SNLT - there is still some space for further refinement of the forms of labour and, specifically, for the *collective composition* of different labouring activities. We call a *collective worker* LP^W the composition of LP^n_U such that $\cap(LP^1_U ... LP^n_U) \neq 0$ - that is, the case where different labour-uses share actions amongst each other.

14.4.1 A crucial addendum must be made here to the status of this collective within the capitalist world, since the very definition of labour-power (10.3.2) implies that m_v buys *bundles* of labour, never a collective - as the layer-conformance operation $0 \rightarrow (LP)_{AB}$ indexes labour capacity to individual citizens. If we now recuperate the definition of existence from 3.4, we can say that $e(LP^w, LP^w) \leqslant \sum(LP^1{}_E, LP^2{}_E, ..LP^n{}_E)$ - a collective of workers only exists as much as the sum of the individual workers [37]. This generative effect of *surplus-*

[37] A trade union, for example, can be defined as an attempt to give a new status to this collective,

coordination enters the productive sphere not merely as free individual labor, but as a labour that is freed from the individual, and can only be expressed within production through its effects on p_k and e_R.

14.5 We offer now two crucial examples of relevant changes to the organic composition of capital which mobilize all the previous forms of coordination into complex organizational structures.

14.5.1 The *manufacture* structure of production employs simultaneously many types of labour, under a single place, for the production of a single product-output. It can be defined as an operad $W(x_1, x_2, ..x_n : p)$ with objects $LP^n, MP^n \in \mathcal{C}$ such that there are at least two labour-commodities with different use sets, that is, $LP^1{}_{\Delta 1U} \neq LP^2{}_{\Delta 2U}$ and one MP^0 such that $MP^0 \in In(x_n)$ - that is, a common workspace.

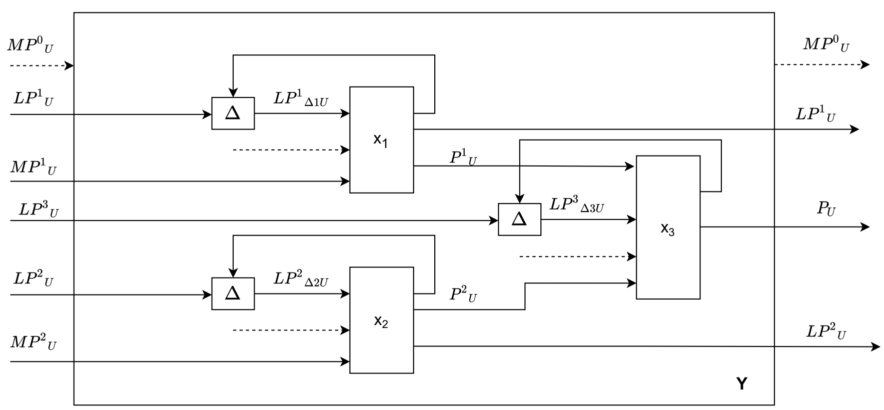


Figure 11: General structure of manufacture

14.5.1.1 Still, we can discern two ways in which such general arrangement can be structured. We call an *extrinsic* manufacturing system the case where the operad W can be decomposed back into some of its constituent parts W^n, which employ Δ-resolutions that are somewhat unaffected by this decoupling and which have outputs $P, P_E > 0$. The production of a clock, for example, requires the combination of several partial products, but most of these parts could themselves be bought separately on the market and only assembled together by a final producer - the transformation into a manufacture occurs, here, when the same capitalist employs all these specialized workers and buys all the

by giving legal existence to a set of workers bound by a common use-resolution or skill: $(LP^w{}_\Delta)_B$ - though many other strategies are possible requires means of production, adapting them and setting them to work within the same production process.

14.5.1.2 We call an *intrinsic* manufacturing system one where rather than combining and adapting together autonomous processes, a single production process is decomposed into several partial ones. Here, both Δ-resolutions that define specialized labour-use and the partial products of the process are meaningful only in the context of the larger productive unit, having little or no exchange value on their own. This is the case of production processes that either decompose previously autonomous productive units into smaller parts or develop new productive processes which require the combination of previously inexistent forms of labour. As an example, consider the decomposition of a glass blower's complete work process into a series of fragmented and concatenated activities, performed by different workers: each activity has very little or no value and no recognizable meaning outside the chain of labours. We can think of the transformation of labour under intrinsic manufacturing processes as the decomposition of $\mathsf{LP_U}$ under an *partial resolution* Δ^p, such that the use and the value of LP_{Δ^p} is partially dependent on its composition with other forms of labour in a given production process, not existing on its own. We call a *partial worker* a labourer indexed as $(\mathsf{LP}_{\Delta^p})_B$, where the usefulness of its partial resolution is dependent on the usefulness of other forms of labour in the same process, that is, $[(\mathsf{LP}_{\Delta^p} \otimes \mathsf{MP_U}) \to \mathsf{P^1}_\mathsf{U}] \to \exists[(\mathsf{LP^n}_\Delta \otimes \mathsf{MP^n}_\mathsf{U}) \to \mathsf{P^n}_\mathsf{U}]$, and its value is also conditioned: $\mathsf{LP}_{\Delta^p{}_E} > 0 \to \mathsf{LP}_{\Delta^p} \in W \wedge \exists \mathsf{LP^n} \in W, \mathsf{LP^n}_E > 0$.

14.5.2 In order to introduce a second form of complex organic composition, we must first consider another special type of means of production: *machines*. And, to introduce machinery, we must first expand our treatment of MP to include resolution boxes MP_Δ. While the specific use of a certain material in a production process is determined by the form and type of transformation it undergoes within it, this is not the case with means of production which mediate and aid the composition of materials and labour: we call *tools* those means of production tailored to certain use-ranges to be further regulated by the uses of labour. Considering an operad W, we say $\mathsf{MP^1}$ is a tool if $\mathsf{MP^1}_{\Delta\mathsf{U}}$ depends on $\mathsf{LP^1}_{\Delta\mathsf{U}}$:

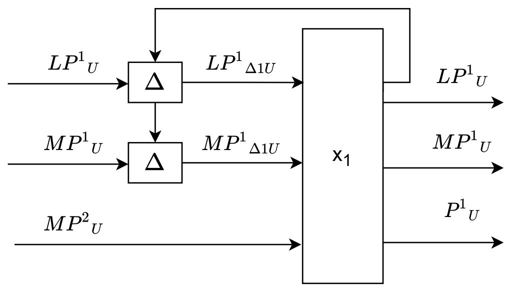


Figure 12: Tool structure

A machine, written MP^{M}, on the other hand, is defined as a complex system M composed of at least three nested boxes (x_1, x_2, x_3) which "internalize" three processes: $\mathsf{In}(x_1)$ takes on MP^1 as fuel, $\mathsf{Out}(x_1)$ gives out energy, $\mathsf{In}(x_2)$ takes in energy and $\mathsf{Out}(x_2)$ gives out workable energy - it is a transmission box - while $\mathsf{In}(x_3)$ takes in a material MP^2 and workable energy and $\mathsf{Out}(x_3)$ gives out the transformed material into a product. Though further specification can be added, decomposing each box into further processes and specifying the particular interventions of labour-power in the process, generally, we write $\mathsf{LP}^1 \otimes \mathsf{MP}^{\mathsf{M}}$ to highlight that there is only an *indirect* relation here between labour use and the consumption of materials: though LP still partially determined the resolution box of the machine-tool, not only is its own use-resolution not directly determined by the processing of the material, but mediated by the internal process x_3, but the further determination of the use of the machine by labour is also actively constrained by the machine's own process, unlike the more passive refinements of use with tools:

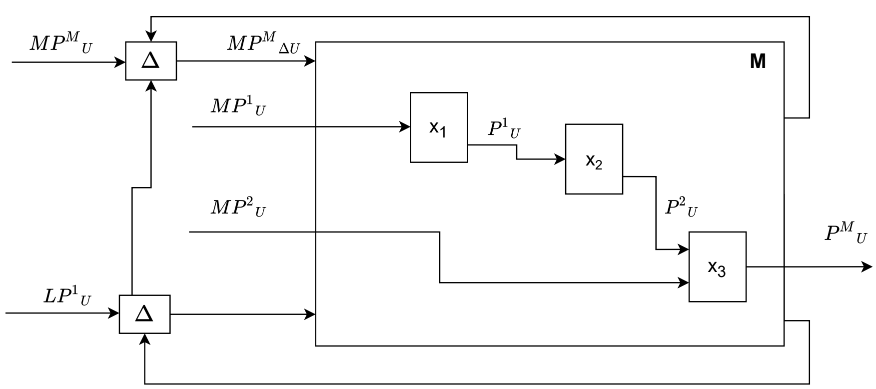


Figure 13: Machine structure

14.5.2.1 we call a *factory* a productive unit defined by an operad W with the following complex properties: gathering the appropriate means of production under a common workspace MP^0, it connects the machinery MP^M in such a way that the output of one machine is taken as the input of another, just as their refinements Δ are themselves sync'd together, displacing the main determinant of production coordination from the coordinated cooperation of LP^w (see 14.3.5) to the machine system itself:

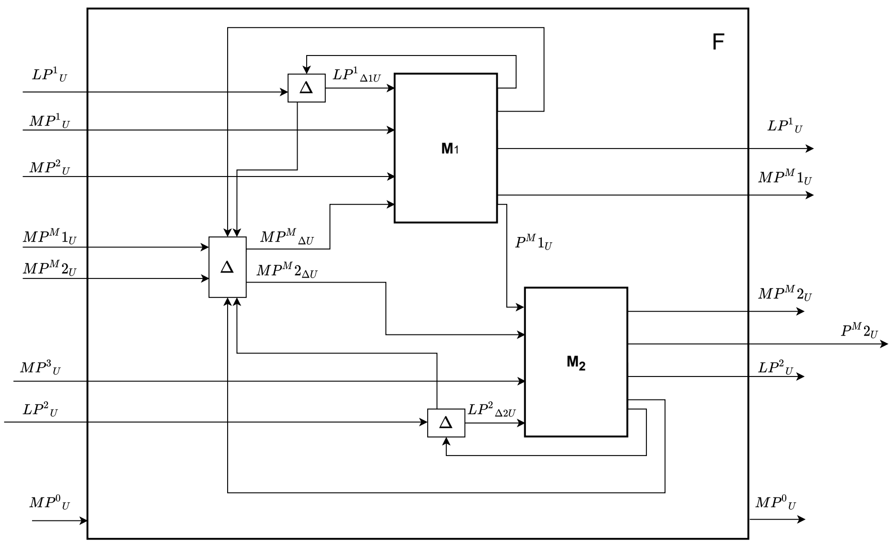


Figure 14: Factory structure and machine coordination

14.6 In proposition 14.4.1 we established the inexistent status of the collective worker in a capitalist production process. This property is particularly important when we consider the differences and similarities between the factory structure and the coordinated cooperation one (14.3.5). If the intrinsic manufacture system (14.5.1) already places the coordination of labour-uses under the primacy of the commodity-output - since Δ-resolutions of specialized labours are determined by the fragmentation of an autonomous production process through what we called Δ^A - the factory system transfers the main feedback circuit that determines labour coordination from LP back to MP^M. In a sense, the collective worker, as an organizational and coordinating power, is objectivated as part of constant capital, through the machine-system: it gains existence if and only if it appears as a property of the means of production themselves.

14.7 Finally, we introduce the difference between *technical* and *technological* changes to production by comparing two possible relations between Δ-

resolutions of LP and MP^{M}. We call a technical transformation one that transfers to a machine system a form of use $\mathsf{MP}^{\mathsf{M}}{}_{\Delta}\mathsf{U}$ that could otherwise be performed as $\mathsf{LP}_{\Delta}\mathsf{U}$. We call a technological transformation one where the resolution of $\mathsf{MP}^{\mathsf{M}}{}_{\Delta}\mathsf{U}$ could never be directly performed by LP_{U}, even when mediated by tools - these are the cases where the appropriate use-resolution of the material is inaccessible from within the range of use-resolutions of labour-power. Given that technological changes are embodied in means of production which are themselves the products of labour, we again find here a situation where something that does not exist as a function of labour - intellectual innovation - becomes visible only as a function of objective machinery.

Wages and the representation of labour

15. Having established some of the logical features of production operads, and the special role variable capital plays in valorization, let us now consider another aspect of the exchange of money and labour-power, that is, the wage-form. Wages are the money exchanged for labour-power, but we have already established that the magnitude of the value of LP is determined by the reproduction cost $\mathsf{LP}_{\mathsf{MS}}$ (11.6) of the labourer. Recall that labour-power, a commodity originally owned by the worker and sold on the market, is not the same as labour itself LP_{U}, which are a set of operations employed to transfer value to a particular product. From the standpoint of the production process, the combination of various LP_{U} are together seen through a certain resolution such that the internal operations (which may consist of the work of multiple individuals to varying degrees of specialization) are treated as indistinct. From the manifold of productive labour, we derived a time-measurable quantity of labour in 11.3.1, denoted as LP_{A}, and the way wages represent this employed labour-time leads to what Marx calls a "mystification"[38]. Namely, although the *magnitude* of wages are determined by the means of subsistence of labour-power, which measured by the socially necessary labour time $\mathsf{LP}_{\mathsf{Eq}}$ required to "reproduce" the body and mind of the worker (the true substrate of LP), the wage-form appears to represent the value of the *entirety* of employed labour-time LP_{A}. This mystification is, as we will show, not a subjective illusion, but part of an objective logic.

[38] "Hence, we may understand the decisive importance of the transformation of value and price of labour-power into the form of wages, or into the value and price of labour itself. This phenomenal form, which makes the actual relation invisible, and, indeed, shows the direct opposite of that relation, forms the basis of all the juridical notions of both labourer and capitalist, of all the mystifications of the capitalistic mode of production, of all its illusions as to liberty, of all the apologetic shifts of the vulgar economists."[26]

15.1 In 10.3.1 we introduced the reproduction set of a given commodity as C_{MS}. Let LP_{MS} be the *subsistence set* of labour-power LP. This is a set containing all the simple consumables required to sustain the labourer during the duration of LP, measurable in the form of LP_A. The ratio between the duration of work employed and the means of subsistence required to reproduce it is determined by the *reproduction cycle* of LP. For example, employing a worker for 10 hours does not necessarily imply that the means of subsistence will be consumed by the worker in 10 hours, but may include many days worth of recuperation. The *total subsistence wage* is the sum of prices of all commodities $C \in LP_{MS}$, also written as the formula $m_w = p(LP_{MS}) = \sum p(C) \mid C \in LP_{MS}$. For a given valorizing process from m_1 to m_2, we find $m_w \leqslant m_1$, that is, a portion of the capital put forward for production.

15.1.1 We can derive hourly, daily, weekly, monthly subsistence wages by division $p(LP_{MS})/u$ where u is the number of working hours, days, weeks, or months in LP_A. These are called *time-wages*.

15.1.2 We can derive *piece-wages* by interpreting the denominator u as units of the product, so $p(LP_{MS})/u$ where u is the number of units produced by the worker in time LP_A.

15.1.3 The unit of the denominator is generally decided on the contract and community layers, namely T_{AB}. This arrangement may be more or less formal, depending on the legal status of the work and worker. For instance, migrant workers who may not have legal existence are limited in the types of contractual powers that other workers are bestowed by the State. In certain circumstances it is advantageous to pay workers in time-wages since the capitalist can then increase intensity of work (more units produced per hour) while keeping the wage the same. On the other hand, piece-wages act as an "incentive" to increase intensity - take for example, the modern "gig economy" in which workers are treated as permanent contractors who are only paid based on the "amount" of services rendered.

15.2 The wage, m_w, corresponds to a specific quantity of labour. For the capitalist, this is payment for the time quantity LP_A, the entire time required to produce a product P. However, in 13.3 we derived a decomposition of LP_A into two parts, LP_{Eq} and LP_S. The first represents the portion of time required to transfer enough value (to the product) to equal its portion of m_v (13.2.1), but this is exactly the same amount as the wage. In other words, m_v (as total variable capital) and m_w (as total wages) are the same quantities, but only the former is capable of generating surplus (recall that $\phi_v(m_v) = m_j \leqslant m_v + m_s$). In the case of wages, it is simply money which purchases LP, without the constraint that it must generate at least as much value as that of its inputs.

15.2.1 Compare the following diagram which depicts the constraint on m_w:

$$\begin{tikzcd}
LP_{MS} \otimes LP \arrow[r, "r"] & LP \arrow[r, "AL"] \arrow[d, "s"'] & LP_A \arrow[d] \\
LP_{MS} \arrow[u] & m_w \arrow[l, "b"] \arrow[r] & LP_U
\end{tikzcd}$$

with that of the m_v:

$$\begin{tikzcd}
m_v \arrow[rr, "\phi_v"] \arrow[d] & & m_v + m_s \arrow[r] \arrow[d] & m_1 \arrow[r] \arrow[d] & m_2 \arrow[d] \\
LP_{Eq} \arrow[r] & LP_{Eq} \otimes LP_S \arrow[r] & LP_A \arrow[r] \arrow[l, bend left] & LP \otimes MP \arrow[r] & P
\end{tikzcd}$$

15.2.2 Just as money can only see exchange while capital is able to see valorization, wages only view LP_{MS} and LP as commodities to be bought, whereas variable capital can see the process by which surplus labour LP_S, beyond a given LP_{Eq}, is extracted from LP. This process is subject to all of the scale-relative forces that we've outlined in sections 13 and 14. Just as exchangeability is information which is "glued together" locally starting from bargaining spaces (3.) to consistent exchange networks (4.) and finally to markets with a price mechanism (7.), valorization consists of local information which are "glued" as components of the production operad (12.), and at the limit, determines the ratio between LP_{Eq} and LP_S.

15.3 The working contract, introduced in 13.3.3, specifies the terms under which labour-power will be employed. The *wage contract* includes those constraints on how the worker will be paid, including the frequency or duration of work required, and magnitude of the wage. These constraints allow the capitalist to have full ownership of the full labour journey LP_A. Contracts are not required for labour power to be sold, but dictate terms which are recognized by the State (T_B). Commonly, these contracts contain provisions for protecting the capitalist - although he has the power over how LP_U should be deployed, in many cases, he is absolved of what happens during the production process.

15.3.1 On the part of the State, these contracts allow the possibility of taxation, which is a primary channel of value-transfer from labour power to the State via Capital.

15.3.2 Nothing prevents a wage contract from specifying wages above the level of subsistence. However, since surplus extraction is predicated on the difference between LP_{Eq} and LP_A, wages must on average tend towards the value of LP_{MS}.

Simple and expanded reproduction of capital

16. Thus far we have analysed the first two arrows which allow the $K_{\phi p}$ diagram (9.5.1) to commute:

$$\begin{tikzcd}
m_1 \arrow[r, "\phi_P"] \arrow[d, "b"'] & m_2 \\
C_i = MP \otimes LP \arrow[r, "\nu"] & P \arrow[u, "s"']
\end{tikzcd}$$

These are the exchange-functions that allow m_1 to purchase a certain set of productive consumables C_i (points 7., 10., 11. and 15.) and the commodity-producing operations which turn it into a product P such that $C_{iE} \subset P_E$ - which implies some surplus-value was added to the output of the production process (points 12. to 14.), a surplus-product such that $P_E = P_{Eq} + P_S$ (13.3.2). We must now analyse the third arrow of the diagram, the selling of product, which effectively guarantees ϕ_p through the selling of the product and the return of "commodity-capital" into money-form m_2.

16.1 We introduce now the idea of a *cycle* which we implicitly relied on in 12.6. The valorizing diagram ϕ_p can be viewed as a single cycle which contains several sub-cycles of various durations. However, it is possible to describe the structure of the cycles without reference to a single chronological measure (16.1.1). We now define different types of cycles of production and valorization[39] with reference to a wine-making process:

a) We call the cycle determined by wage-relation $m_w = LP_{MS}/u$ (15.1) the *labour cycle*. Within this cycle, LP in $LP_U \otimes MP_U$ is fully consumed, but that doesn't imply all available MP was consumed[40]. As an example, to produce wine, workers are paid every two weeks to do various tasks, such as picking

[39]We anticipate here some ideas that are only fully developed in parts I and II of [27].

[40]We call all commodities employed by capital and whose value is fully transferred to the product during a labour cycle, through their full consumption, *circulating capital*, while means of production which require multiple cycles to be consumed and have their value fully transferred to the product are called *fixed capital*.

grapes off the vine, crushing and storing them, etc. Regardless of the duration of the tasks for each worker, the labour cycle is two weeks.

b) We call the cycle determined by the production of a finished product P_U the *commodity cycle*. Here it is the cadence of the output in $(LP_U \otimes MP_U) \rightarrow P_U$ that determines a complete cycle, regardless of how much fixed or circulating capital have been consumed, and in how many labour cycles: $m_i \subset m_1 \rightarrow C_i \rightarrow P_U$. In vinification, P_U is a bottle of wine, and the commodity cycle is therefore the time required to produce one bottle given the raw inputs.

c) We call the *productive cycle* the extended production time defined by the use of means of production rather than labour - so, for example, if grapes need to ferment in order to become wine, this extended duration, which is not fully covered by labour cycles, is part of a productive cycle. Different means of production have different cycle times. The productive cycle of machinery in a factory includes a machine's standby time, where it is not directly transferring further value to an output. An example would be a machine for crushing grapes, which would have a cycle ending when it needs to be replaced. All labour cycles are productive cycles, but the inverse is not true - and often it is the case that idle time in productive cycles leads to the further expansion of labour cycles into surplus labour time.

d) We call a *minimal valorizing cycle* the period it takes for a portion of C_i purchased by m_1 to become a product that is sold for more than the value of the consumed inputs: $(LP_U \otimes MP_U) \rightarrow P_U$ with $P_E > P_E q$. A single bottle of wine may not be enough to repay the cost of purchasing the materials and labour required - but a batch of wine bottles could.

e) A *complete valorizing cycle*, on the other hand, is defined as the full circuit $m_1 \rightarrow C_i \rightarrow P \rightarrow m_2$ where $m_2 > m_1$ [41]. As an example, several batches of wine can be produced given the initial outlay for production, and the complete cycle ends once all of the inputs have been used up.

f) An *empty cycle* is duration-less and is included in all other cycles.

16.1.1 Valorization has an inherent temporal-topological structure, given by the complete lattice of sub-cycles, where 1 is the complete valorizing cycle and 0 is the empty cycle. We call this the *cycle frame*. When one cycle is included in another, we use the subset notation $a \subseteq b$. Two cycles are said to be overlapping if their intersection is not the empty cycle.

[41] In Volume 2, Marx further distinguishes between the cycle that goes $C_i \rightarrow P \rightarrow m_2 \rightarrow C_j$, which he calls the "commodity-capital cycle", the period that goes $P \rightarrow m2 \rightarrow C_j \rightarrow P'$, which he calls the "productive capital cycle" and the cycle $m_1 \rightarrow C_i \rightarrow P \rightarrow m_2$ which he calls the "monetary-capital cycle" - each covering the same process from different fixed perspectives.

16.1.2 Each valorizing diagram is an operad which describes the productive process as a series of nested boxes and wires (14.). The outermost box, representing the totality of the process, has a cycle 1 (12.6). Each inner box has a productive cycle, therefore are included in 1, whereas 0 is included in every cycle.

16.1.3 A map $f : A \to B$ sending one valorizing diagram to another is said to be *cycle-preserving* if, for any two sub-cycles a, b of A, $a \subseteq b \Rightarrow f(a) \subseteq f(b)$.

16.1.4 Recall that we formalized the notion of production delays in 12.6. With our duration-less description of cycles (in terms of their cycle frames), we can view delays as originating in a sub-cycle and "bubbling up" the lattice, eventually impacting the maximal cycle 1.

16.1.5 Every commodity that is produced within a capitalist process (discounting the enclosing operation we introduced in 10.2) can be viewed as the result of a cycle of production. The labour-value point of view introduced in 11.4 amounts to a choice of atomic cycles for the cycle-frame of each commodity, namely, the labour-cycles.

16.2 We call a *simple reproduction cycle of capital* k the case of a complete valorizing cycle with an additional pair of operations rv_p and rv_i which "quotients" a portion of m_2 into the form of *revenue* m_p to be spent on simple consumables[42] and the original m_1 respectively. We therefore write k as the 3-tuple (ϕ_P, rv_i, rv_p) where $rv_i \circ \phi_P = id_{m_1}$ and $b \circ rv_p$ is map from m_2 to the corresponding private consumable C_{m_p}:

$$\begin{tikzcd}
m_1 \arrow[r, "\phi_P"'] \arrow[d, "b"'] & m_2 \arrow[l, bend right, two heads, "rv_i"'] \arrow[r, two heads, "rv_p"] & m_p \arrow[r, "b"] & C_{m_p} \arrow[r] & 1 \\
LP \otimes MP \arrow[r, "v"'] & P \arrow[u, "s"'] & & &
\end{tikzcd}$$

16.2.1 The constant revenue m_p that is removed from the reproduction cycle k, and therefore spent by the capitalist with simple consumables ($C \to 1$) (2.7), is the first "substantial" form of existence of the difference between money - which only "sees" equivalences - and capital - which "sees" valorizing ventures.

[42]"If this revenue serve the capitalist only as a fund to provide for his consumption, and be spent as periodically as it is gained, then, *caeteris paribus*, simple reproduction will take place."[26] Marx uses "revenue" in the chapter on simple reproduction to denote the portion of sales which go to funding the consumption of the capitalist, in other words, net revenue.

16.2.2 Further note that, unlike with wage-labour, where $\delta LP_{MS} \to \delta m_w$, here the relation is inverted, as the capitalist can afford more simple consumables the more he profits: $\delta m_p \to \delta C \to 1$.

16.3 A crucial transformation happens, however, when we move from $\phi_P : m_1 \to m_2$ to $k : \phi_P \circ rv$. As we consider anticipated capital as a *moment* of its reproductive cycle, it becomes clear that original quantity m_1 is slowly substituted, throughout the cycles and the consumption of commodity-capital, by an equal quantity of money that is the product of the valorization v carried out by labour. In other words, the purchase $m_v \to LP$ is revealed, from the perspective of reproductive cycles, to be determined by $(LP_U \otimes MP_U) \to P \to m_v \subset m_2$, that is, to be itself a function of surplus extraction from labour. The value of $LP \otimes MP$ "stored" in m_2 is called the *labour fund* [43].

16.4 In parallel with the cycle of simple reproduction of capital, we can also define the cycle of labour-power's reproduction:

$$\begin{tikzcd}
LP \arrow[r, "id"] \arrow[d, "s"] & LP \\
m_w \arrow[r, "b"] & LP \otimes LP_{MS} \arrow[u, "r"]
\end{tikzcd}$$

This diagram highlights the reproduction structure of labour-power, already suggested in 10.3.2, where reproduction costs take place outside of the productive sphere, through the purchase of simple consumables LP_{MS} through simple circulation (9.). We see, thus, that the *complete labour cycle* would require the composition of productive and reproductive consumption of labour. It further highlights that the identity of labour with itself is mediated by the money-form.

16.4.1 Though this would deserve a much more detailed presentation, it is crucial to note that the reproduction process $(LP \otimes LP_{MS}) \to LP$ is but a small part of the actual *social reproduction* structure at stake here. In fact, a full exposition of the process of labour-power reproduction would require us to

[43] "Variable capital is therefore only a particular historical form of appearance of the fund for providing the necessaries of life, or the labour-fund which the labourer requires for the maintenance of himself and family, and which, whatever be the system of social production, he must himself produce and reproduce. If the labour-fund constantly flows to him in the form of money that pays for his labour, it is because the product he has created moves constantly away from him in the form of capital. But all this does not alter the fact, that it is the labourer's own labour, realised in a product, which is advanced to him by the capitalist"

consider, for example, the further composition of non-commodified *care work*, which we notate as CW, in the actual transformation of means of subsistence into a reusable LP:

$$\begin{tikzcd}
LP \arrow[rr, "id"] \arrow[d, "s"] & & LP \\
m_w \arrow[r, "b"] & LP_{MS} \arrow[r, "T_{AB}/T^C"] & (CW \otimes LP_{MS} \otimes LP) \arrow[u, "T^C/T_{AB}"'] \\
& & 0 \arrow[u, "T_{AB}"']
\end{tikzcd}$$

In this sense, although labour-power is indexed in ${T^C}_B$ as an individualized private property, it is often the case that it truly refers to a family unit, in which case social reproduction can be written as $r : (CW \otimes LP \otimes LP_{MS}) \to (F_A)$, with F_A standing for a set family members that depend on these means of subsistence and the family's indexing on T^C appearing as a single labourer: $(F_A)_C = LP$[44].

16.5 If we now consider not single reproduction cycles of individual capitals and labour-powers, but the totality of such cycles, and therefore the large mass of their outputs, we can mobilize our previous definitions (10.) to highlight some crucial interconnections: while capitalists buy back, through constant capital, the output of MP-producing ventures, and through profit, the output that corresponds to their share of simple consumables, labour-power buys with wages the mass of means of subsistence whose value is then split into profit (used, again, to buy simple consumables) and capital (used to buy labour and means of production). This simplified systemic view already ties together the reproduction cycle of different capitalist ventures with the reproduction cycle of labour[45].

16.5.1 The re-indexing of the consumption structures back onto T_A give us a first, very coarse-grained, definition of two different community-forms that

[44]Further considerations into social reproduction would require us to engage in a more detailed analysis of layers T_A and T_B. It is worth noticing, nevertheless, that the dependence on unpaid care and housework lowers the cost of the social reproduction of labour - as it also lowers the value of necessary means of subsistence - and this has several crucial effects both on δAL^R as well as on other fundamental structures of capitalist society, consolidating a dynamic that reproduces the sexual and gendered divisions of labour at the same time that it remains layer-excluded from T^C.

[45]In other words: "Capitalist production, therefore, under its aspect of a continuous connected process, of a process of reproduction, produces not only commodities, not only surplus-value, but it also produces and reproduces the capitalist relation; on the one side the capitalist, on the other the wage labourer" - see chapter 23 in[26]

rely on T^C to differentiate themselves: the *working class* - those whose livelihood is constrained by the value of labour-power - and the *capitalist class* - whose whose reproduction relies on profit.

16.6 We call the *expanded reproduction cycle* of capital k^+ the case where, rather than $rv_i \circ \phi_P = id_{m_1}$ - that is where surplus is accumulated *as money*, as m_p not to be reused in production, we actually have $k^+ : (\phi_P, rv_i, rv_p)$ such that $rv_i \circ \phi_P = id_{m_2}$ where $m_2 > m_1$. Here, part of the profits are reinvested into a new commodity-production cycle and surplus-value is accumulated *as capital*.

$$\begin{tikzcd}
 & C_{m_p} & & \\
m_1 \arrow[r, "\phi_{P1}"] \arrow[d, "b"] & m_1 + m_{s_1} \arrow[u, "b \circ rv_p"] \arrow[r, "rv_i"] & m_2 \arrow[r, "\phi_{P2}"] \arrow[d, "b"] & m_2 + m_{s_2} \\
C_i \arrow[r, "v_1"'] & P_i \arrow[u, "s"] & C_j \arrow[r, "v_2"'] & P_j \arrow[u, "s"]
\end{tikzcd}$$

16.6.1 The expanded cycle of capital's reproduction further enriches our description of K as a universal exposition that is capable of expressing relations between consistent exchange spaces (9.), allowing us to define an individual capital K not as a particular valorization process, but as a valorizing trajectory:

$$\begin{tikzcd}
 & K_\phi \arrow[rr, dashed, "k^+"] \arrow[dl] \arrow[dr] & & K^+{}_\phi \arrow[dl] \arrow[dr] & \\
m_1 \arrow[rr, "rv_i \circ \phi_p"] & & m_2 \arrow[rr, "rv_i^+ \circ \phi^+{}_p"] & & m_3
\end{tikzcd}$$

16.7 However, while the condition for the cycle k are already in place if regular commodity-production outputs for its relevant productive consumables remain stable, the expanded cycle k^+, in which a higher investment is made in C_j than in C_i, can only happen if accompanied by an increase in the output of the relevant commodities C_j - either through expanded commodity-production or expanded layer-conformance of LP and MP^N. This means that the expanded cycle k^+ is not really describable solely from the standpoint of individual capitals - even when we focus on a single expanded cycle, we have already changed the scale of analysis, to consider a *network* of productive processes.

16.7.1 We introduce therefore the concept of *total social capital* $\mathcal{K} \subset \mathcal{C}$. Objects in $\mathcal{K}$ are individual capitals K^n plus their respective reproductive cycles $k(\phi_P, rv_i, rv_p)$. The general network of dependencies discussed in 16.5 endows this point of view with its first intrinsic determination, allowing us to discern three types of capitalist ventures in $\mathcal{K}$ whose total inputs and outputs are connected together in different ways: $\mathcal{K}_{MP}$, responsible for the production of further means of production, $\mathcal{K}_{LP}$, responsible for the production of the means of subsistence of wage-labour and $\mathcal{K}_{SC}$, responsible for the production of simple consumables that are not used in the reproduction of labour-power, but bought by capitalists through revenue. Though we will not explore the interrelation between sectors of capitalist economy here [46], some aspects of this basic systemic structure for capitalist reproduction will become relevant in section 17 for understanding how the process of accumulation k^+ for individual capitals K is dependent on transformations to larger regions of the capitalist world.

16.7.2 For now, we can expand our diagram for k^+ to include arrows that localize it within $\mathcal{K}$ and which constrain the possibilities of further purchase of means of production and labour depending on the average growth of other capitalist ventures:

$$
\begin{tikzcd}
\mathcal{K} \arrow[drr] \arrow[drrrr, bend left] \arrow[dddr, bend right] & & & & & \\
& & K_\phi \arrow[rr, dashed, "k^+"] \arrow[dl] \arrow[dr] & & K_{\phi^+} \arrow[dl] \arrow[dr] & \\
& m_1 \arrow[rr, "rv_i \circ \phi_p"] \arrow[d] & & m_2 \arrow[rr, "rv_i^+ \circ \phi^+{}_p"] \arrow[d] & & m_3 \arrow[d] \\
& C_i \arrow[rr] & & C_j \arrow[rr] & & C_k
\end{tikzcd}
$$

As we will see, however, this is only a partial determination of the process - one where the increased demand in k^+ for commodities $C_j > C_i$ does not yet distinguish between increased consumption of means of production - that is, products of previous commodity-production - and increased consumption of labour-power. To properly conceptualize the constraints of capitalist growth - and, specially, its consequences for the labouring class - we need to further expand our investigation.

[46] The proper theory of capitalist circulation, which considers the interaction between departments of production, is developed by Marx in [27]

The Greater Logic of capitalist accumulation

17. The passage from valorizing ventures ϕ_p (9.5) to reproduction cycles k (16.2) and finally to total social capital $\mathcal{K}$ (16.7.2) operates two crucial changes to our formalism. On the one hand, once multiple capitalist ventures are considered, this enriches our understanding of inputs and outputs of particular capitalist operads (12. and 14.), allowing us to connect them in a feed-forward structure where, for example, the output of some capitalist process is consumed by another. On the other, the same large-scale perspective which "sees" these interactions also starts to unfold the connection between the individual feedback structure of capital accumulation k^+ (16.6) and the growth-dynamic of total social capital itself.

17.1 We begin by recalling here the basic structure of a commodity-operad W, with inputs MP and LP and a product P (12.5) - and, being commodity-preserving, the additional constraint that $|P_E| > 0$ (2.2.2):

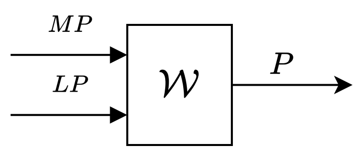


Figure 15: Basic commodity-operad

As we have seen (14.), through the analysis of nested boxes and wire diagrams, the operad structure allows us to investigate in detail the different forms of cooperation and organization which are otherwise condensed in our formalism as $(LP \otimes MP)$. It is useful, then, to also translate our expanded reproduction cycle k^+ (16.6) in a similar way. However, because we are interested in how individual capitals are "seen" by the total social capital - the way their structures make a difference to "self-valorizing value" - we define initially only the four terms which constrain the value-composition ν_k of W's inputs:

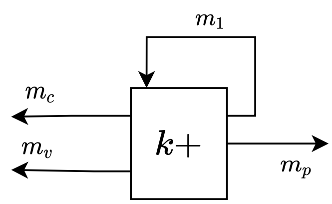


Figure 16: Reproduction cycle as operad

Here, following 16.6, we have $(rv_i \circ \phi_P)(m_1) > m_1$, which implies that k^+ reverts increasingly larger sums of money m_c and m_v into commodity-capital, while also removing a portion of revenue as profit m_p. Note that the quantity assigned to the feedback arrow fluctuates as function of the number of cycles (16.1) - we write it as m_1 to denote its initial state, but may write as m_n to denote an arbitrary state n in the future. In this alternative notation, our original term for $\phi(m_1)$, that is, "m_2", is substituted for "$m_1 + m_s$", with m_n being used to define the initial capital of any given cycle n.

17.1.1 Given that W corresponds to the technical composition of a productive process - the masses of MP and LP employed by it - and k^+ assigns its value-composition - the value m_c and m_v invested in productive consumables - we can now combine them into W^{k+}, which makes intelligible the organic composition of K and how growth in capital accumulation affects the inputs and outputs of W:

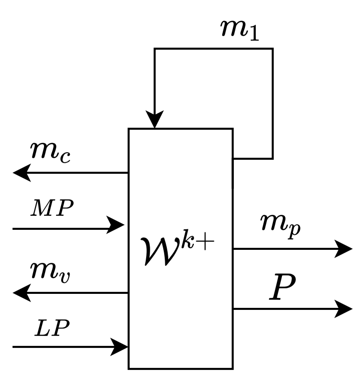


Figure 17: Organic composition of K

This enriched operad gives us the necessary inputs and outputs to treat the individual accumulation of capital in a reproductive cycle k^+ as a minimal dynamical unit of $\mathcal{K}$.

17.1.2 Following these input and output structures, we could start connecting different productive units, thereby locally expanding our view of the interactions between these dynamic nodes:

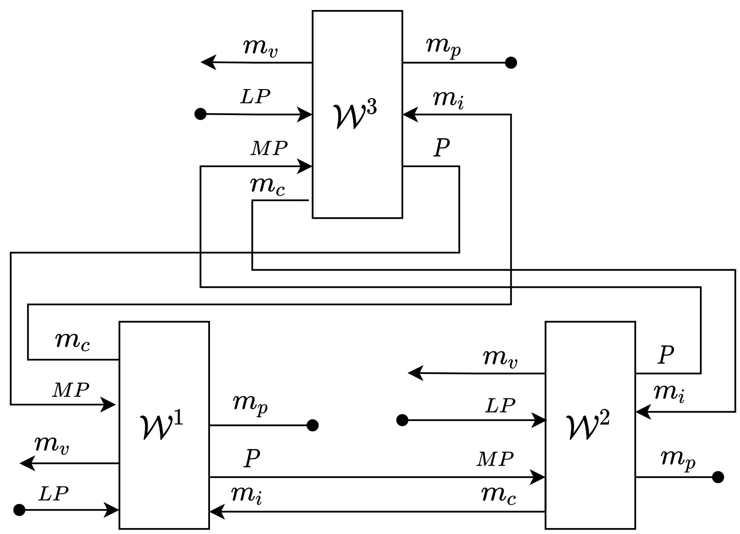


Figure 18: Local dynamic view of multiple k^+

It is clear, however, that labour-inputs, simple consumption and immediate profit-revenues remain unmotivated at this local level - and the feedback structure between products, means of production and accumulated money m_i are conditioned by independent variables, the availability of LP and its cost m_v.

17.2 Let us move, then, from the localized description to the perspective of total social capital $\mathcal{K}$. In order to do this, we must disregard, for now, the specificity of natural means of production (10.3), and therefore treat all MP-input of a given k^+ as the previous output of some other capitalist production process. This leaves us to define the structure of the other productive input of W^{k+}, labour-power. We have already seen (10.3.2) that LP, unlike other commodities, is intrinsically defined by the fact it goes through *two* separate compositional cycles: a productive one, inside W^{k+}, and a reproductive one, written $r : (LP \otimes LP_{MS}) \to LP$. With this in mind, we define the *active labour-force* $\mathcal{L}_{\mathcal{A}} \subset \mathcal{C}$ as the subset of all labour-commodities structured by two consumptive cycles, ν and r.

17.2.1 The nesting of all W^{k+} give us the overall wiring structure of $\mathcal{K}$, while the remaining input and output wires that go in and out of total social capital can be presented - assuming the low granularity of this initial description - as (1) connections from the initial object 0 to both $\mathcal{K}$ and $\mathcal{L}_{\mathcal{A}}$, corresponding to layer-conforming operations that output fictive commodities (10.2); (2) as connections from both capital and total active labour-force to the terminal object 1, corresponding to layer-excluding operations and non-productive consumption; and, finally, (3) as connections between $\mathcal{K}$ and $\mathcal{L}_{\mathcal{A}}$, defined by the dual cycle $r \circ v$ of LP.

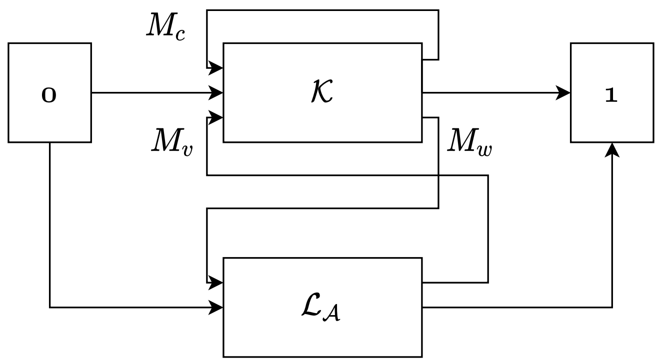


Figure 19: Large-scale structure of capital accumulation

We highlight here, for the moment, only the value-composition constraints to this open system, with M_c, M_v and M_w standing for the total value that can be invested in each productive and reproductive inputs - also note that the feedback structure of individual k^+ is redescribed here as the circuits M_c and M_v, just as profit rates are only expressible as differences between advanced capital $M_c + M_v$ at each cycle.

17.2.2 In order to consider relations between sectors $\mathcal{K}_{\mathcal{MP}}, \mathcal{K}_{\mathcal{MS}}, \mathcal{K}_{\mathcal{SC}} \subset \mathcal{K}$ distinguished in 16.7.1, thus revealing arrows constrained by average profit rates as well as other forms of interaction between capitalist departments, we would have to investigate the following system:

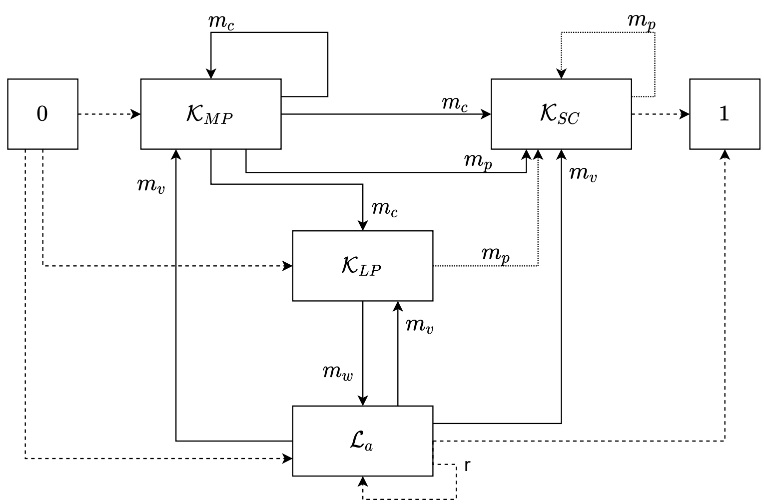


Figure 20: Large-scale structure of capitalist sectors

However, this analysis remains beyond the scope of the *Primer*, as it requires the intrinsic construction of a series of intermediate structures - from our current standpoint, no determinate object "sees" these different interacting relations. We can, however, establish some crucial implicative structures from an analysis of Figure 19.

17.3 Let us introduce a measure for the growth rate between cycles of expanded reproduction of a given capital - the compositional process we previously called k^+ (16.6). Let a_k be the *accumulation rate* for a given capital K:

$$a_k = \frac{m_{n+1}}{m_n} = \frac{(m_c + m_v)_2}{(m_c + m_v)_1}$$

Following our dynamical presentation, we rewrite the share of surplus m_s that becomes accumulated capital in terms of the difference between two different cycles.

17.4 Given that, as established in 13.4, only the variable part of commodity-capital contributes to a_k, even if it is reinvested in m_{n+1} as increased constant capital, we can formulate the first *dynamic law of capitalist production* [47] as the statement that, even though, from a *static* perspective, a_k is conditioned on the

[47] "The law of capitalist production, that is at the bottom of the pretended "natural law of population," reduces itself simply to this: the correlation between accumulation of capital and rate of wages is nothing else than the correlation between the unpaid labour transformed into capital, and the additional paid labour necessary for the setting in motion of this additional capital" in chapter 25 [26].

further employment of labour-power - a demand which, under certain conditions, can lead to increase in wages - *dynamically*, it is the inverse relation of dependence [48] that holds:

$$(\delta^{+}LP_{S} \implies \delta^{+}m_{s})_{n} \implies \delta^{+}m_{v_{n+1}} \wedge (\delta^{+}LP_{Eq} < LP_{A})_{n+1}$$

Note that we use δ^{+} to denote positive increase. In other words, the value available for investment in wages at cycle 2 is conditioned by the transformation of surplus labour time LP_S at cycle 1 into available capital $m_n + m_s$ and the further investment in variable capital at cycle 2 is also bounded by the condition of not decreasing exploitation rate in subsequent production. Marx's summarizes the way the dynamic picture inverts the static relation between labour and accumulation by saying that "the rate of accumulation is the independent, not the dependent, variable; the rate of wages, the dependent, not the independent, variable" [26] - or: wages will never grow to the point of compromising the surplus extraction rate, for these increases are paid for with unpaid labour previously transformed into available additional capital for the purpose of further valorization.

17.5 This dynamic constraint establishes two conditions for the growth of total social capital. On the one hand, $\delta^{+}m_{v_{n+1}}$ implies a higher demand for labour, but - given the necessity that this investment in variable capital remains such that $\delta^{+}LPEq < LP_A$ - the increased share of m_v must, as a general tendency, be spend on *more* labourers, and not more *expensive* labour-power. On the other, since $\delta^{+}LP_{S_n}$ conditions the increase in available capital at cycle $n+1$, the more resources for m_v are needed, the more δLP_S must increase, which corresponds to an increase in e_k and p_k: so the mass of labour-power diminishes relative to the mass of employed means of production. Ultimately, capital must mobilize more and more labour-power in *absolute terms* while mobilizing less and less labour *relative* to total capital invested.

17.5.1 If we call A_k the general or aggregate rate of accumulation for $\mathcal{K}$, we can define a *second dynamic law of capitalist production*:

$$(\delta^{+}|\mathcal{L}_{\mathcal{A}}| \wedge \delta^{+}p_k) \implies \delta^{+}A_k$$

In other words, total growth depends on the increase in the mass of active labour-force at the same time that the productive capacity of units of labour

[48] A careful comparison of the theory of capitalist production and Alain Badiou's theory of change (see [2]) would help us to establish the following: that the labour-power commodity is a *site* which contains "sub-atomic" structure that is not localized by the value form, and that the existence of a capitalist "point" k^{+}, assigning higher value of existence to m_{n+1} than to m_n, depends on the capacity of the "organs" of production - that is the organic composition of capital - first giving form to this surplus-use inside the labour atom. In the next steps of this research, we will examine more carefully this issue.

increase - leading to a relative decrease in the need for labour-power as accumulation expands.

17.5.2 This second formulation can be translated in terms of an underlying tendency in the organic composition of capital, that is, in terms of a relation between v_k and t_k (13.5). If the absolute increase of mobilized labour-power $|\mathcal{L}_{\mathcal{A}}|$ is necessarily accompanied by an even further increase in the consumption of MP, on the other hand, due to the general tendency for increase in p_k, the necessary means of production for expanded reproduction of a given K will be available now for cheaper than before, leading to an asymmetric increase in $\delta m_{c_{n+1}}$ and $\delta MP^n{}_{n+1}$:

$$\delta^+ A_k \implies \delta^+ \frac{m_c}{m_v} \leqslant \delta^+ \frac{MP^n}{LP^n}$$

In other words, the higher the productivity and, with it, the rate of exploitation of labour, the vaster the amounts of constant capital needed to be bought in order to keep increasing the total growth rate A_k, but, at the same time, less money is needed to buy these increased quantities.

17.6 Having established some structural traits of the global accumulation rate A_k for $\mathcal{K}$, we can now determine a basic theory of *competition*, defined as the theory of the dynamic partition of total social capital amongst individual $K \in K^n$. Competition is the struggle for the share of total capital viewed in time. For one capital to gain, another one must lose. This basic fact underpins all *capitalist decisions*, which mobilize the logic we've outlined but in an obfuscated form. The information of any given partition is held in a self-morphism of $\mathcal{K}$. Since self-morphisms are closed under composition, we obtain an inner operational space of total social capital itself. To temporalize it, we need to order this space in terms of successive moments (or cycles). Finally, from a temporal view of the moments of total capital, we can "see" how competition plays out among individual capitals as "factorizations" of maps between capitals.

17.6.1 We can view the dynamic partition of total social capital as a moment in a "path" within the space of possible partitions of $\mathcal{K}$. Let $\mathcal{K}^{\circlearrowleft}$ be all partitions (all automorphisms of $\mathcal{K}$). We use the commodity operation space (exponentiation) from 2.8 to define the object ${\mathcal{K}^{\circlearrowleft}}^{\mathbb{N}}$[49] where $\mathbb{N}$ is a natural number object[50]. This is the space of all possible maps from $\mathbb{N}$ to the object of all partitions on $\mathcal{K}$. Since any map from $\mathbb{N}$ obeys the logic of succession (commutes with a certain "successor" map on $\mathbb{N}$), ${\mathcal{K}^{\circlearrowleft}}^{\mathbb{N}}$ represents "all possible paths" within the partition space.

[49] $\mathcal{K}^{\circlearrowleft}$ could also be written as $\mathcal{K}^{\mathcal{K}}$.

[50] See chapter 9 of [20]. We owe a debt to Lawvere for his demonstration of the dynamical view of categories.

17.6.2 A moment in the history for an individual K can be captured through a factorization with $\mathcal{K}^{\circlearrowleft\mathbb{N}}$. We need to utilize the map $1 \to \mathcal{K}^{\circlearrowleft\mathbb{N}}$ to choose a path in the space of possible histories of partitions. Within that single history, K is identified with one of the partitions. Diagrammatically:

$$
\begin{tikzcd}
 & 1 \arrow[d] \\
K \arrow[ur] \arrow[r] \arrow[dr] & \mathcal{K}^{\circlearrowleft\mathbb{N}} \arrow[d] \\
 & K
\end{tikzcd}
$$

17.6.3 We call *concentration* of capital the intrinsic movement through which k^+ expands its own share of total capital via expanded accumulation a_k, such that $m_n \to m_{n+1}$ with $m_{n+1} > m_n$.

17.6.4 We call *division* of capital the movement by which a given capital K is partitioned into two or more parts, $K_1 \times K_2 \times ... \times K_n$ (2.3.4):

$$
\begin{tikzcd}
 & 1 \arrow[d] \\
K \arrow[ur] \arrow[r] \arrow[dr] & \mathcal{K}^{\circlearrowleft\mathbb{N}} \arrow[d] \\
 & K_1 \times K_2 \times ... \times K_n
\end{tikzcd}
$$

17.6.5 We call *centralization* of capital the movement by which two or more capitals $K_1, K_2, ...K_n$ are joined into one $K_1 + K_2 + ... + K_n$ (2.3.3):

$$\begin{tikzcd}
 & 1 \arrow[d] & \\
K \arrow[ur] \arrow[r] \arrow[dr] & \mathcal{K}^{\circlearrowleft\mathbb{N}} \arrow[d] & ...K_n \arrow[ul] \arrow[l] \arrow[dl] \\
 & K_1 + ... + K_n &
\end{tikzcd}$$

17.7 We have previously defined $\mathcal{L}_{\mathcal{A}}$ as the class of all LP which go through the complete double cycle of production and reproduction. But, having established in 17.5 that a general tendency in the organic composition of capital leads each K to employ less labour-power relative to its growth rate, we must now consider the destiny of the *inactive* labour-force, what Marx calls the "reserve army of labour".

17.7.1 We call $\mathcal{L}_{\mathcal{U}}$ the *unemployed* labour-force the class of all labour-commodities that only go through the reproductive cycle r. Though it is not consumed in a productive capacity - therefore receiving payment in the form of wages[51] - LP in $\mathcal{L}_{\mathcal{U}}$ still reproduces itself through he consumption of commodities, contributing in this way to the definition of the average set of *socially necessary means of subsistence* LP_{MS}. If we recall that, according to 11.6 and 11.7, the value of labour-power LP_E is determined by the value of LP_{MS}, we see that the reproductive cycle of $\mathsf{LP} \subset \mathcal{L}_{\mathcal{U}}$ contributes to the decrease in the value of labour in general - and, through this decrease in the value of labour-power, it also contributes to a decrease in the number of equivalent labour hours LP_{Eq}, thereby raising the exploitation rate. It is clear, then, that the transformation of active in inactive labour-force, and vice-versa, should be understood as a movement that is internal to the dynamic of capitalist accumulation, rather than as an extrinsic process of exclusion from the world of commodities.

17.7.2 We call $\mathcal{L}_{\mathcal{P}}$ the *pauperized* labour-force, that is, the set of all LP which is not only excluded from the productive cycle ϕ, but whose reproductive cycle r is not exclusively, or mostly, composed of simple consumables LP_{MS}. Rather than acquiring necessary means of subsistence in the form of commodities - bought with money - pauperized consumption implies that a large share of goods needed for survival are acquired outside of exchange process, either through charity, familial relations, "dumpster diving", theft, or other forms

[51] We do not analyse here the alternative forms of income that allow for unemployed workers to continue to consume: from social security, through familial and communitarian support, to credit and meager savings - what is crucial, at this point, is that the reproduction of $\mathcal{L}_{\mathcal{U}}$ is essentially mediated through the purchase of commodities

that rely on other transcendental layers, a dependence on non-commodities that preserves the commodity-form of labour-power itself. [52]

17.7.3 Finally, we have $\mathcal{L}_{\mathcal{A}}, \mathcal{L}_{\mathcal{U}}, \mathcal{L}_{\mathcal{P}} \subseteq \mathcal{L}$, which defines the total labouring class. And we are now in condition to consider some of the interactions between parts of $\mathcal{L}$, which we now include as necessary parts of our diagram for total social capital and global capitalist accumulation:

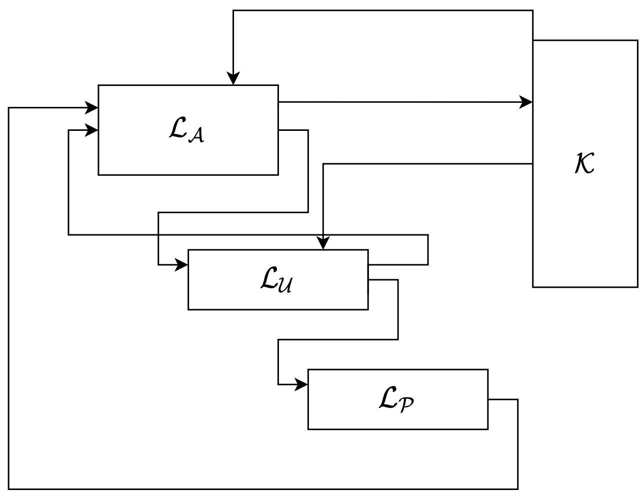


Figure 21: Dynamic between active and inactive labor-force

Here we see that $\mathcal{K}$ (1) outputs $\mathsf{LP}_{\mathsf{MS}}$ to $\mathcal{L}_{\mathcal{A}}$ and $\mathcal{L}_{\mathcal{U}}$ and (2) receives inputs only from $\mathcal{L}_{\mathcal{A}}$, we also see that parts of the active labour-force $\mathcal{L}_{\mathcal{A}}$ can (1) become the input of commodity-reproduction without production, that is, $\mathcal{L}_{\mathcal{U}}$, that is, workers become unemployed, and (2) can become the input for non-commodified reproduction of labour-power itself, as part of $\mathcal{L}_{\mathcal{P}}$. Alternatively, we can see that the larger $\mathcal{L}_{\mathcal{U}}$ and $\mathcal{L}_{\mathcal{P}}$, (1) the more the total wages received by $\mathcal{L}_{\mathcal{A}}$ are shared amongst more non-productive reproductive cycles and (2) the smaller the set of socially necessary $\mathsf{LP}_{\mathsf{MS}}$ consumed by inactive workers, hence smaller the average wage for the labouring class as a whole.

[52]Here is Marx's description: "Pauperism is the hospital of the active labour-army and the dead weight of the industrial reserve army. Its production is included in that of the relative surplus population, its necessity in theirs; along with the surplus population, pauperism forms a condition of capitalist production, and of the capitalist development of wealth. It enters into the faux frais of capitalist production; but capital knows how to throw these, for the most part, from its own shoulders on to those of the working class and the lower middle class"

17.8 Given the second dynamic law of capitalist production introduced in 17.5.1, we know that, while there is an absolute increase in the mass of labour-power employed by $\mathcal{K}$ - that is, in $|\mathcal{L}_{\mathcal{A}}|$ - there is also a relative increase in the proportion of inactive to active workers, as less workers are needed to consume larger masses of means of production. We can write this corollary to the global tendency in the organic composition of capital as a corresponding tendency to the composition of $\mathcal{L}$ such that:

$$\delta^+ A_k \implies \delta^+ |\mathcal{L}_{\mathcal{A}}| < \delta^+(|\mathcal{L}_{\mathcal{U}}| + |\mathcal{L}_{\mathcal{P}}|)$$

This statement, which connects global dynamics of capitalist accumulation to the increased immiseration of the labouring class, is what Marx calls *the fundamental law of capitalist accumulation* [53].

17.9 One final remark is in order with regards to the structure of $\mathcal{L}$. From this construction, it is clear that, just as an inversion was operated when we considered the dynamic perspective of accumulation, revealing that wages are constrained by unpaid labour, here too a similar inversion takes place, once we realize that $\mathcal{L}_{\mathcal{A}}$ is a *constrained* subset of $\mathcal{L}$, one that goes through two cycles - productive and reproductive - while the underlying determination of $\mathcal{L}$ as such is rather the reproductive cycle. We can interpret this to mean that, in capitalism, the "essence" of the working class is to be *unemployed*.

Conclusion

A reader of Marx should recognize that, by the end of proposition 17., we have offered a reconstruction of the first volume of *Capital* up to the chapter on the law of accumulation [54]. As anticipated in our introduction, our objective in providing this reconstruction is, however, neither exegetical nor aimed at the

[53] The greater the social wealth, the functioning capital, the extent and energy of its growth, and, therefore, also the absolute mass of the proletariat and the productiveness of its labour, the greater is the industrial reserve army. The same causes which develop the expansive power of capital, develop also the labour power at its disposal. The relative mass of the industrial reserve army increases therefore with the potential energy of wealth. But the greater this reserve army in proportion to the active labour army, the greater is the mass of a consolidated surplus population, whose misery is in inverse ratio to its torment of labour. The more extensive, finally, the lazarus layers of the working class, and the industrial reserve army, the greater is official pauperism. This is the absolute general law of capitalist accumulation.

[54] Since the theory of enclosures plays a crucial part in our concept of "organizational commons", to be explored in our next essay, we decided to leave for this second installment the discussion of the chapters on primitive accumulation and the modern theory of colonization

scientific validation of Marx's thinking: instead, we invite the reader to situate it within a broader political project which also allows for an alternative evaluation of its merits and deficits. In these final remarks, we intend to recuperate the results of this highly condensed exercise with these ampler objectives in mind, hopefully clarifying some of the conceptual consequences of the *Primer* and preparing the ground for further developments.

On objective phenomenology and fetishism

The starting point of this essay is the notion of an "objective phenomenology" (1.) and, since a lot hinges on this heterodox idea, a few words about it are in order. Alain Badiou proposes this concept in his book *Logics of Worlds* [2] where he offers an alternative treatment of several classical concepts of continental philosophy such as transcendental, world, appearance, existence, perspective, object-constitution and objectivity. Badiou's main objective is to propose a general theory of what it means for something to exist in a world, one that does not presuppose a subject as the synthetic point of view to which everything appears and exists. His theory, in other words, is not a theory of how the world appears *to us* - for example, how I look at a painting - but a phenomenology of the way a painting is inserted in the world of other works of art, a world that is first defined by the internal relations between paintings themselves - for example, between artistic movements, or different works by the same artist - such that only later, in an accessory way, do we consider how they appear to us in particular, given these pre-existing logical constraints on what counts as part of the picture.

Even though this idea might look terribly strange to most philosophers - what could a statement like "an object appears to other objects" possibly mean? - Badiou's basic wager is that other fields of thought might actually not have such a hard time welcoming it: scientists for whom information about phenomena is only accessible through experimental mediations, artists who feel that what orients them in their works is something intrinsic to the art works themselves - and even lovers who believe that their romantic decisions cannot be reduced to the sum of two people's personal preferences. Marxist militants, on the other hand, might receive this idea in a more ambiguous way, as Badiou's proposal seems strikingly close to Marx's own definition of *commodity fetishism* in section 4 of the first chapter of *Capital*, where we find the famous statement that, in capitalism, "relations between people that appear as relations between things" [55]. That is, in a sense, Marx already works with the possibility that certain social phenomena require a certain *social form* to make themselves intelligible, but this is usually understood exclusively as an ideo-

[55] See section 4, Chp.1, Capital, volume 1 [26]

logical obstacle[56].

It is likely that many Marxists would therefore interrupt us here and claim that, for that very reason, "objective phenomenology" is nothing but the greatest fetishistic project a philosopher could hope to accomplish - specially since a theory of how "paintings appear to one another" seems to willingly erase any trace of the painter or of artistic reception, thus de-contextualizing these objects to the extreme. However, before such a (rather unfounded) criticism of Badiou's project is even addressed, one should return to that famous definition of fetishism and look at it more closely.

Two things are worth noting here. First, that fetishism is discussed in section 4, while the *value form* is presented in section 3 of the same chapter, with no mention to it being a fetishistic operation - and that is because while the theory of value-form is meant to describe the *actual* logic of the expression of value in a singular commodity, money, fetishism is defined as the way this logic is deformed from *our standpoint* as private individuals. That is, implicit in the first chapter of *Capital* there is already a distinction between a logic of how commodities come to appear to one another - the theory of the value-form, with its successive stages of generality and consistency - and the theory of how this relatively autonomous commodity system appears to us. The reproach that objective phenomenology is a fetishistic endeavour would thus not hold, as it is used here precisely as a natural grammar for presenting the commodity world in its most intrinsic determinations. Hence the employment of the metaphor, throughout the *Primer*, of what commodities "see" of their world depending on their composition and structure.[57]

But there is yet a second point worth mentioning, which also concerns these first famous pages of *Capital*, as the very first sentence of the book can also help us better understand the nature of the fetishistic reduction of the commodity world. The first chapter begins with the famous lines:

> The wealth of those societies in which the capitalist mode of production prevails, presents itself as 'an immense accumulation of commodities', its unit being a single commodity.[58]

[56]A crucial exception in this regard is Alfred Sohn-Rethel and his analysis of the commodity-form and its relation to the Kantian transcendental subject in [38], which deeply inspired us. However, Sohn-Rethel opts for an analysis of value that is still trapped in the dualism between circulation or production and a Kantian understanding of subjectivity which prevents him from asking about *other* possible forms of real abstraction.

[57]The connection between the categorial approach and the "cinematic" or "visual" metaphor is explored by Badiou himself in [3]

[58]See section 1, Chp.1, Capital, volume 1 [26]

We begin, therefore, with a sort of "zooming in" movement: from an "immense" or "monstrous" [in German, *ungeheure*] accumulation of commodities to the single unit - and we should not forget this when analyzing the end of the chapter and the definition of fetishism. When Marx introduces the definition in section 4, he begins by talking about the way the social reality of producers does not appear directly to them as such, since consumption and production are private businesses in capitalism and each person is concerned only with their immediate needs, as are companies. Because of this, the vast network tying together a myriad of supply chains is rather invisible in itself, appearing only as the social relation between commodities themselves, where these private uses of labour are then compared through the produced commodities. In other words, we could augment the definition of fetishism to include, there as well, a mention to such a "zooming" effect, the scalar issue that, in fetishism, the relation between *millions* of people appear as the relation between *two* commodities. In other words, a complex social system is reduced to a measure that is compatible with our apprehension - a systemic dimension that would otherwise simply remain inaccessible to us as private people.

In this sense, while fetishism might be understood as an approach to the world of commodities that uses the value-form as means to reduce a large scale system to the measure of private individuals - losing all sorts of information in the process - objective phenomenology can be defined as an approach that takes the value-form as a means to "lift" our private apprehension to a measure that is intrinsic to the commodity world itself [59].

From Logics of Worlds to a theory of possible social worlds

The recognition that the value form defines specific conditions for the intelligibility of certain social phenomena - phenomena constrained by functional and scalar properties - does not mean, however, that it is *the only* form capable of doing so. In fact, the first strategic benefit of the reconstruction of *Capital* provided here, from the standpoint of our larger project, is precisely to offer an analysis of the value-form that treats it as a particular instance of a general theory of *social measures*. In the restricted world of commodities, money, and then capital, are both well-formed objects that relate to and affect - better than we as individuals ever could - the structural and dynamical properties of the commodity space. However, they cannot capture more information than what

[59] In fact, when Marx claims that he treats capitalists as "personifications of economic categories", we can understand this as the entry door to an additional field of study: the application of the method of objective phenomenology to the study of how individuals must actually behave in order to "incorporate" themselves into the objective perspective of capital, a process which implies a change in sensibility and forms of reasoning - the sociology of finance has achieved interesting results in this direction [14], while a more aesthetic take on the "cognitive mapping" of capitalism can be found in [39]

pertains to this specific ”transcendental” regime: other objects, composed under different operational restrictions, might present a totally different picture of a complex social system. As anticipated in our introduction, our current hypothesis is that a theory of political organization is possible where alternative forms of social measure can be designed and experimented with. By reconstructing value-theory with the appropriate grammar - that of objective phenomenology, with its own formal machinery - we hope to open the space for such a novel line of enquiry: how well, and how differently, can forms of collective organization, under specific and complex constraints, *see* and therefore change the social world?[60]

As we already said, this new approach to the theory of political organization is also indebted to Alain Badiou's work: being a militant philosopher, his proposal of an objective phenomenology in *Logics of Worlds* is accompanied by a surprising theory of ”bodies”[61], understood as material supports for new formal arrangements that are capable of capturing previously invisible information about a given situation. And - perhaps the most important contribution of the book - *both* parts of the theory are developed within a homogeneous formalism, borrowed from category theory, called ”localic topos theory”. What this means is that no conceptual leap is needed to construct the transcendental analysis of a world and to suggest how novel structures might sustain themselves and further affect the specific logic of such a space - in fact, Badiou introduces in this work an alternative approach to novelty, structurally representable as a *regionalization* of a previous horizon, an embedding of a certain system of constraints within a richer environment, where new operators render new possibilities intelligible[62]. All of this is, evidently, accomplished at the highest degree of generality and abstraction, as it should, if the philosopher truly wishes to open the *space* for novel experiments rather than to state what these should look like.

Therefore, to move from this broad philosophical discussion to the specifics of our own project, one should, on the one hand, restrict the domain of study from Badiou's formal analysis of ”worlds in general” to the specific logic of capitalist social formations and, on the other, pass from his general theory of ”subjectivizable bodies” to a much more restricted theory of complex political organizations - which still benefits greatly from preserving the underlying homogeneous formalism of the theory, even if with the necessary additions of some further specifications which are absent in the original framework and that are not necessarily coherent with Badiou's *personal* political taste[63]. It is

[60] We previously explored this idea, in a less developed way, in the texts [41] and [42]

[61] See books V-VII of [2]

[62] A complete interpretation of Badiou's theory of regionalization can be found in the forthcoming book *Subject and Infinity* by Reza Naderi

[63] Unlike some of his critics, we don't find it so hard to separate the core of Badiou's philosophy from his specific militant commitments and wagers - after all, a good part of his project aims at

worth mentioning, for readers already acquainted with his *Logics of Worlds*, that the structure of the present essay not only maps well onto *Capital* but also partially mirrors the structure of Badiou's book: a discussion of transcendentals (objective phenomenology and value-form), of objects (atomic components and money form), of relations (circulation and exchange) and of points and organs (capital and production) - with the proviso, which we will address more fully later on, that the theory of points and organs (of, quite literally, "the organic composition" of capital) is here impoverished by the limits of what remains coherent with the transcendental of the capitalist world.

This long preamble hopefully already provides the reader with some justification as to why so much of our effort here went into the reconstruction of the money-form. Our hypothesis is that, in discussing the logical relations between commodities, labour and money, Marx sometimes takes shortcuts and accepts inconsistent formulations[64] which might have very well been enough for the sake of his particular project, but which require further conceptual precision if one wishes to embed them in a larger political framework, as we do.

As we already noted, our goal is to offer not only a "political reading" of Marx - or to participate in the scholastic debate about what the text "really means" - but to translate his ideas into a more robust environment where new questions and new ideas can be posed. As a particular example, consider the benefits for the crucial debate around the "socialist calculation problem" if we are no longer stuck between having to side either with those who claim we should substitute the price-system for another central mechanism for resource allocation or with those who claim that the social coordination problem should be entirely dismissed. If we can show what are the *particular* properties of the money-form, then we can investigate if new forms of coordination that could substitute it are truly breaking with its logic or not, and in which way - without having to simply turn away from the problem because we suspect the alternatives to the market-mechanism reproduce its underlying problems. This is one of the contributions we would like to make with this project.

The method of logical restrictions

In our introductory remarks, we stated that one of the main objectives of the *Primer* is to reconstruct some of the crucial concepts of Marx's critique of political economy within a new *grammar*, imposed on us not only by the principles of objective phenomenology, but also by the inner workings of category theory. Let us now consider how this change might affect our conceptual method.

allowing us to distinguish between thinking and its representation

[64] Krause discusses some of these inconsistencies in the first chapters of *Capital* in [16]

Returning once more to the first lines of *Capital*, we can already spot an important methodological difference. For example, Marx's original distinction between the ”immense accumulation” and the ”single unit” of commodities, which suggests a simple relation between substantial units and their collection, is recast here in terms of an *operational closure* (2.) under ”commodity-preserving functions” (2.2). That is, commodity space is defined as the space of everything that can be done to commodities such that the output of this transformation remains a commodity: the ”single unit” is therefore a variable element that is determined by the space of operations, not the other way around. The decision to start our analysis not from a subjective impression - an *immense* accumulation... but compared to what? - but from the limits of a space, its closure under a series of operations, is certainly in line with the objective phenomenological approach, which privileges ”differences that make a difference” to commodities themselves, but it is also a strategy that is more natural within the grammar of category theory. A common path for conceptual construction in this mathematical field is to introduce a very general type of morphism or map - capable only of delimiting a broad space, without much determination - and then to find meaningful *restrictions* of these operations which correspond to specific structural differentiations. Just as, in category theory, information is often a function of constraints on these operations, here too we look for specific restrictions that allow us to split these general commodity-preserving functions into *exchange* and *production*-operations, each taking up around half of the propositions of the text to construct.

Now, one might conclude that, because we rely on mathematical formalisms and we explicitly propose a change to Marx's method, our approach would have to be termed ”un-dialectical” or at least foreign to Marx himself. We believe neither is the case[65]. In fact, it might be useful to consider it as a way to further develop Marx's *thinking* beyond the means of his *conceptual language*. For example, we believe our ”method of restrictions” is clearly implied in this famous passage on method from the *Grundrisse*:

> the concrete is concrete because it is the concentration of many determinations, hence unity of the diverse. It appears in the process of thinking, therefore, as a process of concentration, as a result, not as a point of departure, even though it is the point of departure in reality and hence also the point of departure for observation and conception[66]

[65] Interesting connections could in fact be developed between our approach and Marx's own mathematical intuitions concerning differential calculus. In fact, Lawvere once wrote: ”The dialectical philosophy, developed 150 years ago by Hegel, Schleiermacher, Grassmann, Marx, and others, may provide significant insights to guide the learning and development of mathematics, while categorical precision may dispel some of the mystery in that philosophy.” in [19], where he also deals with Marx's interest in mathematics and proposes some possible paths for reconstructing dialectical concepts in categorial terms.

[66] See Introduction in [29]

The change in method or "grammar" might look contingent, but it deeply affects what and how we are able to think. Since, to arrive at the "unity of the diverse", Marx is constantly moving between concepts like "essence" and "appearance", or "substance" and "form", it is not uncommon for interpreters to accept that the logic of commodities relies on a rather classical ontological partition - which some even interpret as a sign of Marx's materialism - between real and concrete "use values" and formal and abstract "values" and "exchange values". And once this division is accepted, then several other conceptual distinctions will eventually be assigned one of these two poles: quality against quantity, concrete against abstract labour, production against circulation, etc. If our analysis is committed to the eerie perspective of what a commodity sees - of differences that make a difference within the commodity world - then we should also try to avoid the recourse to such a dualism. And, from our perspective, it really is not necessary to accept this presupposition, as the method of restriction we mentioned before is very well suited to distinguishing and relating regions based on the different constraints each impose on possible transformations. As a passing remark, it should be noted that there is a precursor within Marxist political thinking who also read Marx in this way - Alexander Bogdanov - for whom the distinction between nature and society, manual and intellectual labour, or between economic base and ideological superstructure was more one of *types* of organization than one of essentially ontological nature[6]. Nevertheless, in adopting this approach we were led to a slightly different conception of how to think the "use sets" of commodities and their "exchange sets" (2.-2.1.1).

Most readers of *Capital* recall the statement that "as use values, commodities are, above all, of different qualities, but as exchange values they are merely different quantities, and consequently do not contain an atom of use value"[67]. Do we disagree with this? Not really, the issue is rather if distinguishing use and exchange in terms of the "point of view of quality" and "of quantity" is truly the best way to arrive at the two perspectives Marx wants to highlight. In our approach, commodity-exchange operations are defined by having an "inverse" - such that A becomes B and B becomes A - if the exchange sets agree - if the value of A is equivalent to B - while commodity-production operations are those that take existing use sets and compose them into a new one, on condition that the new use set is featured in the exchange set of other commodities (2.1). The distinction between circulation and production is therefore based on how transformations are restricted - you participate in circulation if you preserve certain properties, you participate in production or consumption if you preserve others - but both are logically defined and distinguished. What changes is rather what "counts" of a commodity - and, in fact, the reference to "atoms" remains quite relevant for us: the "atomic components" of *both* use sets and exchange sets are logically contrived by the value-form - novel or strange forms

[67] See Section 1, Chp.1, [26]

of use that we might find for an object do not appear to commodities as part of the use of commodities until these uses can be localized on a ”priceable” part of the commodity (7.6).

This brings us to a related matter, which is the introduction of a *resolution* variable [68] to the ”atom of use value” (2.5). It further develops our idea that one should treat the set of possible uses of a commodity as a logically-constrained assemblage of maps, rather than as an ontological given. The so-called ”Delta-resolution” of commodity use sets already allows us to talk about quality in logical rather than ontological terms: commodities which can be gathered by the same ”compositional resolution” can be said to be homogeneous with regards to certain operations - and this becomes, later on, a fundamental piece in our understanding of labour-power and the use of labour in production, as ”resolution boxes” (14.3.2) quite naturally model how work can both refine its use and also be substituted where technological changes allow for uses that diverge from our human scale and capacities (14.5.2.3).

The derivation of quantity through the structure of money

However, this change might not even be the most polemical one amongst our first seven propositions - at least when compared to the way this same method led us to reconstruct the famous equation of ”simple exchange” - ”xA=yB” - *without* any reference to quantities. At first, it seems like there is a lot to lose if we assume the perspective that two commodities of the same quality, but of different quantities, should be originally treated as two different commodities altogether. Is it not self-evident that we see the world in such a way that we can distinguish between ”what” and ”how many” or ”how much” of something is in front of us? Is it not obvious that if I own a car I own *one* commodity?

Several reasons motivated us in taking a more counter-intuitive approach to this issue. First of all, within Marx’s own use of the notions of quality and quantity there is sometimes a strange short-circuit - popularized in Engels’ famous statement that, in dialectics, ”quantity becomes quality” - and which is particularly important in special passages of *Capital* where we find changes to the form of economic phenomena due to their scale. If we simply assume some basic division between quantity and quality, it becomes more difficult to formally account for these transformations - which, as we said in the introduction, are of great interest to us, as they might help us to also think about scale-dependent issues in collective organization and social coordination.

[68] Our approach to scale-dependency was greatly influenced by Laurent Nottale’s ”scale relativity theory” - even though we do not use any ”fractal geometry”, there is an interesting development to be made to our theory of use-resolutions in terms of divergent resolutive scales. For an introduction to Nottale, see [34]

At the same time, though it is manifestly true that when *we* see a car, we see that it is "1 car", it is not hard to recognize that this is highly dependent on the appropriate "resolution" at stake there: if one was suddenly really short on cash and the car is old or broken, we might very well start looking at it as a bunch of auto parts to sell. Not only this, but the *number* of auto-parts we would start to see once I put these "commodity-goggles" on is actually itself constrained by the set of "priceable" parts of the car that have non-empty use and exchange sets! So even from the phenomenological side, it is only partially true - and mostly true only for us as phenomenological individual subjects - that the quantity of commodities is so self-evident to be taken as a primitive.

Finally, there is a third reason for us to propose that the *intrinsic* description of commodities in terms of quantities is mediated by the money-form, rather than its condition. That is, the fact that we are committed to the hypothesis that Marx's theory of the value-form is a particular case of a broader theory of social measure, or social intelligibility, and since not all social information is quantitative - and even when it is, it is not always assignable to a totally ordered index - the fact that the price-system sees the world in terms of "how many" and "how much" should be ideally treated as the consequence of some other factor. For us, it is the consequence of the logic of the value-form being, first, a predicate-logic (5.1.1), and, second, a consequence of the fact that the use set of a money-commodity can itself be structured in such a way as to totally order the assignments of price in terms of parts of one single substrate (7.3.1). Because it has specific pre-conditions, nothing in principle prevents there existing partial orders of value, or multidimensional indexes, or forms of social coherence which are composed without the recourse to a "principal ultrafilter" for that value space [69].

Multilayered transcendentals and the limits of capitalist novelty

This brings us to a crucial section of the text, which is admittedly underdeveloped: "The multilayered transcendental of the capitalist world" (8.). In part, this section introduces some of Badiou's terminology - such as "world" and "transcendental" - but it is also one in which we venture into uncharted territory for us. It is true that - perhaps with the exception of the concept of "universal exposition" (7.7) - we do not employ any of Badiou's categorial formalism without significant changes to it, but in the particular case of the theory of transcendentals, we have taken our cue from the Japanese thinker Kojin Karatani, and his alternative "transcendental analysis of world history" [12] in terms of

[69] We must mention here Otto Neurath, who intervened on the "socialist planning" debate from the standpoint of a multi-dimensional theory of measures - see [31]. In current debates around climate change and the ecological constraints on economic growth, similar issues are being addressed, see for example [21] or [9]

the concatenation of different modes of exchange. Badiou himself describes the logics of worlds in very abstract and general terms, restraining himself to complete Heyting Algebras and Topos theory - but there is no suggestion in his work of how to approach a model for worlds that have multiple and relatively autonomous transcendental standpoints. And so, partially influenced by the available literature on multi-layered and multi-scalar social networks[70], we sought to introduce Karatani's theory of multiple modes of exchange in a quasi-formal way, developing it just enough for the purposes of the *Primer*, but conscious that a lot of further work needs to be done for it to properly consist with the rest of the formalism.

Nevertheless, the idea of situating the logic of commodities as the "maximally reaching" sub-transcendental structure in an otherwise broader social formation (8.1) is already helpful. Not only because it prepares the ground for us to enrich our description of capitalism, in a future text, with social phenomena and dynamics that are precisely *not* visible from the standpoint of commodities, but which still play an essential part in the maintenance of a capitalist structure, but also because we are not interested in a theory of organization that intervenes exclusively on the world of commodities[71]. Ultimately, we want a broader framework capable of designing and experimenting with organizational forms that can both benefit from other modes of exchange as well as possibly constitute a new "maximally reaching" transcendental synthesis of some new social formation.

It is from the vantage point of this highly ambitious goal that we can best evaluate our reconstruction of "valorization diagrams" (9.5) "commodity- producing operations" (10.1), the "labour-valuation structure" (11.5), "value- preserving operations" (11.2) and operads (12. - 14.). Readers familiar with Badiou's *Logics of Worlds* might recognize that we adapted his theory of the "four forms of change" and his theory of "decision points", originally developed to account for what radical transformation of a system can mean, into a weaker theory which describes change *within* a capitalist world.

Our definition of capital as an object K that is composed of functions between consistent exchange spaces, such that the statement "money becomes more money" holds true, is in fact partially connected to Badiou's theory of "points" - in technical terms, a surjective homomorphism between different

[70] Most of the literature we assessed approaches the construction of multilayered networks from the standpoint of matrices and weights, focusing on causal emergence in a composite probability space - which is not exactly what we were after, since we are interested, first and foremost, on the different "qualities" of relations, not only their quantitative assessments. Still, we were very much influenced by [5], [13], [4]

[71] We are currently developing, together with Tiago Guidi, a reconstruction of key findings of structural anthropology of Levi-Strauss and Eduardo Viveiros de Castro using this same transcendental approach

transcendental structures that assigns a new, higher, value of existence to parts of an already existing world. Since the philosopher's concern is showing that is rational to believe that the set of these points might constitute a topological space with new properties, i.e., that a new consistent world can be constructed piece by piece, he does not explore the theory of ”weak points”, a theory that would describe real transformations to a world that nevertheless remain constrained by some basic properties of its already existing transcendental, changing ”what” exists inside of it without changing ”how” things exist.

His discussion of such ”weak singularities” [72] is too condensed for our purposes. For us, this weakened notion of points and sets of points proved to be a perfect way to talk about valorization in general (9.2.6) and industrial capital in particular (9.5). On the one hand, it clarifies in which *precise* sense industrial capital really does allow for a ”constant revolution of the production process”, a huge organizational machinery that truly extracts previously unknown capacities from people and materials, as long as these can be ”localized” on the atomic structure of the commodity world. On the other, it is a theory of transformation and novelty whose limits are well-defined enough to prevent us from taking such transformations for the model of radical transformations in general - a distinction which, once more, treats capitalist dynamics as a regional case of a larger space of possible organizational strategies.

A logical theory of labour-power

An important and related issue is the way that, considering all of this, we decided to introduce labour into our reconstruction. Surely many readers of the *Primer* will find it strange that practically no mention to labour, labour-time, abstract and concrete labour is made in our long presentation of the money-form - while, in *Capital*, the ”two-fold status of labour embodied in commodities” is introduced already in the first chapter, presenting several of these concepts as a direct consequence of the ”two factors of the commodity” - use and value.

As we already said, we believe that Marx was working with a conceptual language that cannot so clearly spot certain sudden theoretical jumps, since these are often indistinguishable from ”dialectical movements”. In the case of the distinction between concrete and abstract labour in particular, it would have been possible for us to immediately use our initial distinction between use and exchange sets to define two types of compositional operations that can mobilize the use set of labour: those which compose it in general - with no regards to the value of the product - and those which compose it while at least preserving the value of the commodities being composed together. And this is the way we define them in proposition 11 - but why wait so long to introduce

[72] See book V in [2]

the ”commodity-production point of view”?

It is important to constantly remind ourselves that, when we adopt the method of objective phenomenology, we are obliged to distinguish between what is intelligible to us and what is intelligible from the standpoint of the closed operational space that we are interested in understanding. In actuality, we all see that a worker or a waterfall are quite different from man-made products - but this is not enough to maintain that, for commodities themselves - and for capital, particularly - the labour-commodity has some special distinguishing property.

In objective phenomenology, *you are only allowed to construct an ”observable” if you can also construct an ”observing” object* - a condition that finds resonance, in Marxist literature, with the critique of ”transhistorical” or ”ontological” views of labour, reproaches that highlight the fact that labour only became intelligible as this special object once the conditions for capitalism were already in place. In *Capital* itself there is a precedence for our preferred strategy, because Marx introduces concrete/abstract labour and socially necessary labour time in chapter 1, but then *reintroduces* labour once again in chapter 6, when discussing the ”buying and selling of labour-power”. At this point, the formula of industrial capital has already been introduced and the conditions of the problem are inverted:

> In order to be able to extract value from the consumption of a commodity, our friend, Moneybags, must be so lucky as to find, within the sphere of circulation, in the market, a commodity, whose use-value possesses the peculiar property of being a source of value, whose actual consumption, therefore, is itself an embodiment of labour, and, consequently, a creation of value.

In other words, it is *from the perspective of capital* that commodities come to be distinguished between those that can only transfer their own value to the product and those which can transfer more value then they are worth. This is why labour-power only enters into the picture in our reconstruction once we have the formal means to express its singularity in terms that are intrinsic to the commodity world itself. For us, the following preliminary steps were therefore necessary: to define a consistent exchange space (3.-6.) with a money-commodity (7.), to define a commuting diagram for maps across such exchange spaces (9.) and to define a particular type of diagram for valorization functions (9.5) which do not rely on alternative transcendental indexes (8.). Under these conditions, which define productive capital, we can ask: considering all trajectories which can go from money to a set of commodities that are then composed together, are there relevant differences? Or even: can we distinguish commodities in terms of different correlations between exchange and use maps? And it turns out that labour can be *logically* singled out, rather than ontologically so, from other commodities - not due to some god-given privilege, but because of how, in the world of consistent exchange spaces, different ”productive consumables” (10.2 - 10.3.1) are structured in terms of compositional maps. It follows from this that any creature - organic or otherwise - which sold its useful time for money to be employed in a production process, while remaining responsible for purchasing their means of subsistence with their own payment would be indistinguishable from a labourer.

Once the labour-commodity had good reason to be logically singled out, we then defined two different restrictions on the transformations that compose its use set - we call ”concrete labour” (11.1) all the maps that go from the labour-power use set to some product, no matter its value, while ”abstract labour” (11.2) *restricts* these maps to only those uses which at least preserve the value of the commodities being composed, labour included. The first benefit of this definition is that, once more, we offer an alternative reading of what ”abstractions” mean in capitalist mode of production: rather than implying some ideological or mental abstraction from reality, or an indifference to use-values, abstract labour is characterized as real activities that are *more* restricted and *more* differentiated than the loosened conditions which single out ”concrete” uses of labour - those which merely preserve commodity-form, but not necessarily value.

In this way, just as we were able to distinguish between the ontological substrate of use sets and the logical constraint of use under commodity logic, so too we can now distinguish between concrete labour as ”commodity-preserving” but not ”value-preserving” compositions and the broader, undefined, ontological substrate of *actions* in general, which are not constrained to preserving any properties of the commodity world. Another aspect that is worth mentioning in passing is that, while it is true that value, in capitalism, appears as money and that, therefore, ”value-preserving” transformations are tightly connected with preserving and increasing the monetary value of commodities, it is at least imaginable, in the framework that we have developed here, that other measures of value would induce new types of abstract labour: for example, in a world where value is connected to increased complexity [73] of products, ”abstract labour” could be defined the set of ”complexity increasing” compositional transformations that involve the use of labour.

Deriving the concept of socially necessary labour time

[73] A proposal in this direction is presented in [1] - a text that greatly influenced our work

We come now to what is probably the most polemic aspect of our characterization of labour in the *Primer*, which is that we did not only introduce labour later on in our reconstruction of *Capital*, but also altered the way "socially necessary labour time" (11.5) is defined, taking advantage of our expanded theory of social measures.

From our description of abstract labour use-maps above, it follows that the process of "abstraction by restriction", which takes from concrete to abstract labour, should not be confused with the more usual understanding of abstraction as the singling out of a common property in the substance or essence of things. The former selects maps that enter into compositions such that some property is preserved *in the output* - the exchange sets of productive consumables - while the latter conception would lead us to look for intrinsic common properties *within the inputs themselves* - that is, to search for that which many labours would have in common. The difference, to return to our discussion of method, is that in objective phenomenology, "concepts" or "abstractions" are determined *perspectives* on some elements from the standpoint of others such that the restrictions of the latter singles out specific information about the former - one cannot "abstract" information if there is no abstracting point of view internal to the space. Being aware of this distinction allows us to say that the expression "abstract labour time" is actually meaningless in itself. Nothing guarantees that "time" is the property that all value-preserving uses of labour have in common - not only because they might share many other relevant properties, but also because we have not constructed an object capable of "seeing" time as a commodifiable predicate (5.1). This is why we propose an alternative derivation of the concept of socially necessary labour *use*, as a broader theory of possible measures of labour, one that tries to take advantage of our previous characterization of the logic of value-form as a particular case of a theory of social intelligibility.

The issue that interests us is to actually arrive at a meaningful use of "time" as a measure of labour. How is it that "labour-time" becomes a phenomenologically relevant and intelligible thing - how does it come to *appear* in the commodity world? If we do not assume that abstract labour - those maps that present a "*productive* expenditure of human brains, nerves, and muscles" - already single out chronological time as a relevant category, how else could we introduce it?

The benefit of having developed our predicative value-form theory (5.) is that we have a procedure for constructing new measure-structures without the recourse to an external criteria. In the case of the money-form, this required us to show that changes of quantity became part of the commodity world when value relations found assignable correlations with parts of one single commodity, such that these parts varied in cardinality while preserving the qualitative attributes - a property we called a ”money-compatible resolution”.

More generally, this process allows us to describe the conditions for a ”ruler” to be constructed: for example, we could compare the height of things two by two (accidental comparisons), say, a bottle and a tree, or a bottle and a branch; or we can fix one of the two objects and say that the bottle is ”tree-like” if it is as big as a tree, or ”branch-like” if it is smaller (expanded relative comparison), finally, we can also express what it means to be big like a tree or small like a branch in terms of a single predicate - trees are ”less bottle-like” than branches (general equivalent comparison). This, in itself, would not allow us to interpret the ”more or less” in quantitative terms, or at least not to fully order these comparisons, if we do not choose as our general equivalent an object that can serve as a means to interpret the ”more or less” in mereological terms - more or less parts of itself. In other words, a good ruler for extensions will require an additional property, a ”formal use value”, as Marx calls it, which allows it to internally embody the relations that we seek to describe: a homogeneous piece of wood, which can be divided into parts of the same quality, such that they relate to one another in terms of extensive differences, allows us to define a logical ”atomic unit” that we can use to ground the height in terms of quantities of this unit.

With this in mind, we are able to provide a partial answer as to why labour *time* appears as a relevant property of the use of labour: chronological time is totally ordered, and it is ”homomeric” or self-similar to some degree - a slice of a longer duration is a duration, just as the sum of these returns a longer duration. If we were to measure the expenditure of labour in a consistent way, chronological time could function as a possible ruler. The question that remains, however, is how we move from separate productive processes to a common single ”atomic unit”.

Here we believe to have broken new ground by applying the same measure-constructing procedure that is at stake in the value-form logic to the construction of socially necessary labour time.

Given a certain product such that the value of the productive consumables was at least preserved in it, we can use this constraint as an abstracting perspective that can intrinsically compare different workers involved in its production. What this preliminary condition allows us to do is to correlate the *total* use of labour to a fixed quantity of money - since it was already established that the value was in fact transferred to the product in question. Having defined that, considered as a unit, the labour-power employed was used ”abstractly”, we can now inquire into possible distinctions and comparisons between individual labours - say, the work of ”Arthur” and ”Brian” - from this particular standpoint.

This would be, at first, an accidental comparison (11.5.a): though all the workers employed, taken together, produce the same commodity in the same value-preserving way, it might be the case that some contributed to this transfer more than others, and so we can compare them in this regard: we might say that Arthur did ″more″ than Brian, or we might say they did ″the same″ - but we *cannot* say, at this point, that Arthur's and Brian's labour are the same ″time-wise″ because there is no specific semantic content to labour time. Nevertheless, we can now expand our comparisons: so we compare Arthur not only to Brian, but to Carl and Dora as well. We might conclude that his work is ″more like Brian's″ - because their style of work is similar - and ″less Carl-like″ - in that Carl is faster, but sloppier - or ″less Dora-like″ - because she was slower, but more efficient. Still, these new predicates do not single out any one property that makes Arthur more like Brian at work than like Carl or Dora.

It is only by first finding something about the use set of Arthur's labour - or any other worker we might have fixated our comparison on - that is capable of decomposing the ″proper name predicates″ we already have into one single predicate that is common to all of them that we can then talk about Arthur, Brian, Carl and Dora in terms of presenting ″more or less″ of some common attribute, an attribute that measures all of them in terms of their part in the ″total value-transferring labour″ employed. Such a predicate, projecting the totality of labour back onto the labourers, would truly be an ″averaging″ predicate.

Now, at this point, we add a step that is missing in Marx's theory - certainly because labour use and use in general are not treated as part of the logic of commodity-operations - which is to define as *socially necessary labour use*, or SNLU, any predicate that decomposes the the labours of Arthur, Brian, Carl and Dora into one common ″atomic″ attribute that is capable of ″seeing″ them in a common way. There are an infinity of such attributes and the fact that chronological time is one of them is not enough to single it out. Workers can be compared to one another via example, through determinations of skill or intensity, etc - the move from socially necessary labour use to labour time requires the additional consideration of other constrains on the labour process.

At this point, we should not forget that this particular use-resolution we are after is also constrained by the fact we are comparing labour-uses that have all transferred a certain amount of value to the product - in other words, we are looking at *abstract* labour-uses from the perspective of the *value* of the commodities produced. This additional restriction means that we are looking for an aspect of labour-use that is ″money-compatible″, so that we can assign to it changes in value in the same way that we assign different values to different *quantities* of money. These are the conditions of the problem which make *time* into an appropriate predicate to compare *all* abstract use-maps in terms of how they transfer value into the product. This is *social* because it is a predicate for a comparison across labour uses, not for one given worker, it is *necessary* because it establishes the common denominator between labour and value-transfer, and it is measured in terms of *time* because that is the aspect of labour use that presents the adequate properties to express the measurement of value-transfer.

All of this allows us to say that the social average of abstract labour-use that goes into the production of a certain commodity finds an adequate means of expression in the totally ordered parts of labour-time just as the average value of commodities find their expression in a totally ordered set of parts of a money-commodity. But Marx does not want socially necessary labour-time to be a ”possible measure” of value, he goes as far as stating that ”what *exclusively determines* the magnitude of the value of any article is therefore the labour-time socially necessary for its production”. This seems to imply an important precedence of social labour time over the magnitude of value - and if we disregarded the word ”social”, which already entangles this determination with abstract labour and therefore value-constrains, we could even conclude that it means that value of a product is totally determined by productive labour alone. But even if we avoid this reductive reading, we still need to come to terms with the fact that socially necessary labour time, no matter how much its *form* is dependent on the mode of production and on value, has some logical precedence over the specific magnitude of value of its products.

In our framework, causal relations can be expressed in terms of degrees of logical dependence: we say ”A causes B” if B is true only inasmuch A is true as well, the presence of the former being a condition for the latter. In this sense, we can interpret Marx's formulation to mean that ”the value of a product is of magnitude x only if the labour use y that goes into its production is maximally equivalent to the social labour-time required for its production” - a predicate we already extracted out of the comparison of several labour uses with regards to the value of their production. Magnitude ”x” therefore depends on labour time ”y” - even though ”social labour time” could only have become an adequate measure in a production system constrained by value-preserving operations [74].

Finally, we can benefit from our further definition of a ”rewrite rule” for the commodity space that redescribes the commodity-world in terms of a ”labour-value point of view” (10.4) to consider one final property that confirms the precedence of labour-time determinations over the magnitude of value without the need for any reference to a special ontological nature of labour, namely, the

[74] This is a good example of how we might move past the debates about ”what comes before: production or circulation?”: it is possible for production to include information that only becomes intelligible once further developments allow for new structures to capture additional aspects of what nevertheless conditioned them

ubiquity of labour-power. Marx is right to say that socially necessary labour time ”exclusively determines” the magnitude of value because all other factors can be ”projected” back to a perspective that *only* sees labour uses. If that is the case, then the magnitude of value can be said to be an ”effect”, since there is a perspective which does not feature it amongst the ”causes”.

If we return for a second to our example of an extensive ruler, we could say that while a wooden ruler can measure the size of a tree, dendrology, studying a tree’s intake of nutrients and water from the ground and how much of that becomes useful energy, might develop a measure that correlates with the magnitude of a tree’s height and that can project heights in terms of energy conversion, thus offering a ”rewrite rule” for heights that expresses them in alternative terms. In a sense, we could say ”energy conversion is the cause of the tree’s size”, but logically we are correlating two types of measure - energy conversion and size - and calling the first a cause because we can formally define a space where the latter is translated into the former. But this is deceiving, since a plant’s capacity for absorption of nutrients is conditioned by its size, the area of its roots, not to say further environmental factors - like how much it rains. So the possibility of giving precedence to a measure - social necessary labour-time - over another - magnitudes of value - suggests a relation of dependence that can be true, but does not imply that relevant factors were not left out, or that the form of the measurements were not codependent in a way.

In fact, our derivation of socially necessary labour time was purposefully done within the confines of a particular productive unit, but different ”atoms” of labour-time can be constructed by altering the initial constraint: total value-preserving labour with regards to one production process, or with regards to a type of commodity, or with regards to production in general - and other conditions can be loosened as well, such as a calculation directly on value, or on other forms of price. All we need to produce a whole class of measures, that project different ”average units” onto labour use, is to be able to construct objects that embody the relevant constraints [75].

Production as an organizational site

One of the reasons for painstakingly avoiding an ontological definition of labour was in fact already presented in our introduction. There, we mentioned that we share the ”tektological view” proposed by Alexander Bogdanov, who defended that that concepts of ”labour” and ”production” should not be considered primitive terms in Marxist thinking.

[75] A similar hypothesis is explored by Fred Moseley in his (still unpublished) essay *Socially Necessary Labor-time and Equilibrium Prices: A Critique of the Value Form Interpretation of Marx’s Theory and a Suggestion for Possible Consensus*, where he tries to conciliate SNLTs based on equilibrium and non-equilibrium price-structures

Bogdanov proposed his ”Tektology” ([6]) as a general science of organizations - a framework that would be capable of navigating the specificities of physics, biology, sociology, political economy *and* political action through the adoption of a common set of concepts. Profoundly inspired by Marx in this regard, he was nevertheless cautious of the fact that if we ground concepts mobilized in the critique of political economy directly on some ontological basis, therefore making them the backbone of this new ”science”, this might prevent us from further understanding how these different disciplines and contexts are intrinsically related. For example, if labour in capitalism is taken to mean ”work” in the thermodynamic or informational sense, we are bound to overdetermine the relation between science and politics, either importing too much from statistical mechanics into our social theory or deforming our scientific understanding of entropy and energy to fit our political purposes. The other option is to methodologically require the *compossibility* of concepts while giving them *logical* definitions that rely on how they operate within each specific conceptual space. This allows us to say that there is something about labour-power and labour-use in capitalism - and as a matter of fact, about all ”fictive commodities” (10.2) - that appears there as ”black boxes”, meaning that their internal determinations are not expressible within the commodity-theoretical ”topos”, so that a different ”world” might reveal them under a totally new light.

For Bogdanov, ”the concept of *organizing action* is hidden in the term ’production’” [76], and in order to follow this intuition we looked for a formal approach to the capitalist production process in which, once again, a *restriction* would be needed to arrive at the specificity of labour use and productive work in capitalism - a set of constraints that are logically informed by the internal coherence of the commodity world. By now, the reasoning should be clear: different forms of organization - human or otherwise - appear *as* productive features from the *standpoint* of some specific object previously constructed within the commodity space. Were we to expand our world - as Bogdanov did, by trying to ask about how actions and processes appear ”to organizations in general” - then we might be able to recast what previously appeared as labour in other terms, only visible from a new perspective - in fact, from the perspective that the soviet thinker called ”the organizational point of view”.

This underlying commitment to possible descriptive expansions - or to possible embeddings of the commodity world within richer environments - led us to a sub-field of applied category theory called *resource theory* and, from there, to the theories of *operads* and ”symmetric monoidal categories”. Restricted as it seems, this formalism is in fact very rich, as operads combine the study of partial ordered sets and a compositional algebra into a graphic language of ”wire diagrams” that very naturally describes process-based structures of

[76] See page 3 in [6]

varying degrees of complexity. For these reasons, a lot of research goes today into showing its multiple applications in formalizing aspects of fundamental physics, chemistry, engineering and even manufacture processes. In fact, the authors of our main bibliographical references, *The operad of temporal wiring diagrams*, claim that, with the exception of certain rules that guarantee their underlying generality, operads ”roughly conform with the IDEF0 standard set by the National Institute of Standards and Technology”[36], which defines the current standards for modeling labour-processes in the USA [77].

Besides the fact that they offer us the means to arrive at a definition of the production process that abides to our method of restrictions, a particularly important aspect of the formalism that interests us is that, in privileging maps over elements, or logic over ontological descriptions, we can use operads to define what counts as a ”material base” or a ”material atom” without assuming that this is an absolute ”ground”, or given, coming from outside of the capitalist world [78]. Recall that we introduced, early on in the *Primer*, the notion of a ”use-resolution” for use sets. This means that, given only an object's ontological substrate, we cannot yet tell what counts as its possible uses and what aspects of it are not directly relevant in a compositional process. It might be the case that some type of steel is employed in a production process due to its ”isotropic” macro-scale behaviour - the fact that, in large chunks, it responds similarly to forces applied in any orientation - while another process employs the same steel due to its ”anisotropic” micro-scale behaviour - the fact that, once a certain scale threshold has been crossed, the lattice structure of the material becomes more sensible to different force orientations [79] It might be equally possible that an artist finds new uses for the same steel that are ”invaluable” because they cannot be re-purposed for any valorization venture.

A similar change in ”atomic constitution” can take place with labour use itself since, in a manufacture or factory environment, uses of labour are subjected to different constraints. On the one hand, the crucial condition that labour-power was bought in the production process as a commodity that was sold by individual property owners (10.3.1) prevents labour from ever losing the form of an individual capacity - thus making it impossible to intrinsically see what we called the ”collective worker” (14.4), the generative power of combined labour, as an object within the commodity space. On the other, different production processes - which employ different forms of co-operation of constant and variable capital - can explore and truly capture previously unheard of forms of labour use, ”probing” into its use set for new use-resolutions which only appear as possible forms of action because they become caught up in the

[77] A brief overview of these standards can be found in `https://en.wikipedia.org/wiki/IDEF0`

[78] There are interesting resonances between this resolution-sensitive approach to materials and Gilbert Simondon's theory of individuation in [37] - formal developments that go in this similar direction can be found in [10], [32] and [15]

[79] We take this example from the great essay ”The Greediness of Scales” in [44]

logical space of the production operad. If we think of experiments as attempts to extract new information from a ”black box” such that this new information is logically consistent with a broader conceptual space, then we can affirm that capitalist production, insofar as it seeks to capture non-priced parts of labour use into the value form, is an organizational *site* for social experiments, restricted by underlying commitments to the commodity-form and the capitalist valorization structure. Nothing prevents artists from experimenting with new uses of materials that do not conform to these formal conditions, while militants might engage in new uses of collective cooperation that are only intelligible from other transcendental perspectives.

But this limitation of the ”capitalist laboratory” is not built into the operad structure - in fact, operads allow for infinite nesting of processes: nothing, in principle, prevents us from further unpacking ”atomic” processes into smaller sets of compositional actions, thereby giving form to otherwise ”inert” aspects of a use set, or from treating previously separate processes as composing the inside of a larger box, giving form to new social complexes. The limitations here come from the logical relevance of these alternative expositions, which means that the stage is set for us to redescribe a capitalist production process from a richer ”organizational point of view”, on condition we can construct the operational space that defines the rules of this broader framework, allowing for alternative synthesis of these local constructions.

A final word on the theory of surplus extraction. One of the consequences of integrating the ”operadic” and the diagrammatic languages into our categorial formalism is that we enrich our capacity to describe the way productive processes are affected by changes to organic composition of capital: though, ultimately, the ratio e_k between necessary and surplus labour time measures, for an individual or a set of workers, the degree of exploitation in terms that are compatible with the money-form, this totally-ordered measure should not be confused with the actual structure of production, where capital exploits possible compositions and combinations of labour and means of production. The passage between productive compositions and monetary valorization - between the upper and lower maps in the commuting diagram of section 9 - helps to clarify that the process of extraction of surplus, as a sort of ”mining” for non-priced parts of labour use sets, requires that production open up to a lattice-like structure, where parts are combined in non-trivial ways, which is later covered or ”filtered” by priceable partitions, translating a complex composition of labour and materials into segments of a totally ordered measure of value. In this sense, *surplus labour time* is an already fetichized form of appearance of a mode fundamental category of surplus, *surplus cooperation*, if we understand this as the set of parts of labour-use, which we previously called the ”collective worker”, and which is not equivalent to the trivial set of individual labour-powers employed. Surplus labour time is rather a monotonic mapping of this complex composite onto the value-form, with varying degrees of loss of information - surplus must be *given form* by the productive process, it does not always emerge already conformed to the value logic.

Many debates on contemporary capitalism - from the current discourse on "accelerationism" to the different theories of the "crisis of value" - rely on an understanding of how the centrality of relative surplus extraction leads to a self-sundering of the labour-value relation, affecting the texture of a capitalist world. We believe our objective phenomenological approach can introduce useful new concepts into the discussion. For example, even though relative surplus does imply a dynamic change to production that is not expressible in labour-time in the same way that absolute surplus extraction is, we cannot assume, thereby, that a "crisis of measure" automatically follows when changes to the organic composition rely more and more on technological improvements, etc - something which is often mobilized to state that "labour-time" or "value" are no longer a consistent objects in capitalism, or that value has become "subjective", or even to state that the production process can no longer be discerned from "social life in general". From our perspective, *every crisis of measure is situatable*: the fact that there is relevant information which is not intelligible from the standpoint of some specific universal exposition does not allow us to assume it cannot be seen otherwise. Therefore, before we conclude that no possible intrinsic measure of value exists in the complex world of financial capital and multi-lateral international geopolitics, we should investigate if there aren't new objects in the commodity world which have the appropriate structure to "see" this relevant information.

A similar argument might also help shed a new light on the so-called "transformation problem", which denounces the supposed inconsistency in the different concepts of price introduced between volume 1 and 3 of *Capital*. A possible hypothesis [80], left for a future exercise, is to evaluate if "simple prices", "market prices" and "production prices" cannot be distinguished in terms of the different internal points of view which see the production process from diverging scales, going from the immediate anticipated capital all the way to the "average rate of profit" that is constructed for competing capitals in volume 3.

Appearance, representation, mystification

Even though the *Primer* does not address the different concepts of price, the theory of profit rates and the "trinitarian formula" from Volume 3 ([28]), we do introduce in section 15, on "Wages and the representation of labour, a strategy for the reconstruction of the wage-form which anticipates how we might derive different price-structures and representations of the capitalist dy-

[80] First presented to us by Raquel de Azevedo

namic within our framework. The key result in this section is to demonstrate, only through the reference to arrows and compositional structure of objects, that even though the capitalist only pays for the value of labour-power that corresponds to the "equivalent labour-use" $\mathsf{LP}_{\mathsf{Eq}} = \mathsf{LP}_{\mathsf{MS}}$, the labourer sells the full labour-time utilized in the production process $\mathsf{LP}_{\mathsf{A}} > \mathsf{LP}_{\mathsf{Eq}}$. In other words, the distinction that is visible from the standpoint of variable capital between equivalent and surplus labour is rendered invisible from the standpoint of wages.

Rather than introduce an external cause for this deformation in how the commodity world appears - for example, ideology conceived as a mystifying representation of reality, which would reintroduce an extrinsic dualism in our construction - our formalism allows us to derive this distortion immanently, by considering the different relations that constrain and situate, respectively, m_v and m_w (see 15.2.1). Every concept constructed in the *Primer* is, first of all, defined by the arrows that map onto and out of it and, through this, by the parts of its support set that get mobilized and by these multiple maps [81]. From this theoretical perspective, variable capital is not only money that buys labour, but *money that buys labour whose further composition outputs a product*, while wages are not only money received for the selling of labour, but *money that buys means of subsistence whose further composition outputs the same labour-power*. The same object - a share of anticipated capital m_1 - is "seen" differently depending on the arrows that render it meaningful. And, through this categorical semantics, we can talk about poorer or distorted representations of capitalist reality without recourse to cognition or social discursivity: the loss of information with regards to labour-power is part of the very structure of the wage-form, a consequence of the circuit binding wages, labour and social reproduction.

This co-variance between arrows and object-compositions - which allows not only for different *parts* of a system to "sense" diverging aspects of some object, but for alternative constitutions of the object at different *scales* of the structure - sheds a useful light on how we might reconstruct the categories such as prices of production, equalization dynamics and the rent-form, all of which require us to consider that *not all syntheses of a world converge to to the same structure*. The way value relations allow us to parse the world into "priceable" parts might be different from the way this same world gets decomposed into "valorizing" parts from the perspective of an individual capital, and different from the way this world looks from the standpoint of competing organic *compositions* in a given capitalist sector. As we have constantly emphasized in these concluding remarks, new objects can function as new observing standpoints in

[81] This second condition, which ties together the logical approach to a set-theoretic one, implies that we work with "localic toposes" rather than broader structures - this is not a deficiency, but an enrichment, of our formalism, as it allows us to always have in mind the *material* conditions for logical consistency. This is why Alain Badiou calls this enrichment the "postulate of materialism" in [2]

a system, and new points of view are correlative to *new resolutions* which define the relevant informational content of the parts of the world it composes into a coherent territory.

Scale-sensitivity in capitalist phenomenology

Though the concept of delta-resolutions and resolution boxes play a restricted part in the *Primer*, they should be taken as an initial attempt to make explicit the scale-dependent nature of our framework as a whole.

In a previous publication ([40]), we proposed the rather cheeky distinction between three stages of socialist or communist thinking in modernity. The first one, identified in the *Communist Manifesto* as that of "utopian socialism", developed together with early capitalist social forms, focused on a *spatialized* grammar: capitalism was made intelligible by its spatial forms - by looking at the modern city and its decadence or by noticing the difference between peasant and urban worlds, for example - and emancipation was therefore a battle for free *space* - in short, politics should strive to move us *out* of capitalist society, and certain forms of social organization, like Fourier's phalansteries, sought to construct these social experiments. The second form, which Engels famously called "scientific socialism", emerged out of the recognition that the logic of capital has no necessary spatial limits - it could potentially encompass the whole planet and therefore we cannot rely on identifying an "outside" to find places for political intervention - and this led to a critical shift from a spatial to a *temporal* grammar: not only was capitalist logic analysed in terms of its production and reproduction cycles, historical modes of production, and so on, but, with this critical shift, political emancipation also took a new form - communist politics should strive to lead us to what comes *after* the capitalist world, in a search for free *time*.

For us, the multi-dimensional logic of contemporary "crisis capitalism", which emptied out the future promises of social and technological progress leading to exploding contradictions and structural transformations of society, and the gravity of climate change, which fills the future with a sense of dread and impotence, impose today the need to reinvent the very grammar of political critique and political strategy. This is why, not without considerable hubris, we propose that a new fundamental transformation is needed in communist thinking, one which adds, to the spatial and temporal coordinates of our inherited framework, a *scalar* dimension as well. Rather that conceive of communist strategy as being essentially a problem of moving from the inside to the outside, or from the present to the future, we expand our grammar to also include, as concrete strategic concerns which require their own solutions, the problems of connecting the local and the global, the fine-grained and the coarse-grained, the combination of different synthetic perspectives of a same object, the cognitive mapping of complex social systems and the effort of composing a (new) totality out of its (old) parts.

Even if this is an ambitious hypothesis, it seeks to render intelligible a real and historically-specific process. After all, together with the latest transformations to the capitalist world, a series of new forms of popular protest and political experimentation have emerged - novelties which, seen from our current theoretical and political paradigm, often appear to lack a sense of historical direction and are therefore dismissed due to a supposedly programmatic deficit. What must be included into the picture, however, is that our era of popular revolts and planetary concerns has also brought about several new theoretical developments - coming from disparate fields of knowledge - that help us to reconsider the place and function of scalar variables in highly sophisticated ways, challenging the commonsense understanding that, unlike time and space, the scales and resolutions of a system are a purely subjective effect with no objective constraints. It is no coincidence that the intellectual effort required to conceive of measurement resolutions as being part of the "scale-space" of a coordinate system is quite similar to the effort required to engage with Alain Badiou's objective phenomenology, which displaces the standpoint that senses the relevant differential network of a logical space from our cognition to the structure of the space itself: these two ideas are part of the same paradigm shift. It is, finally, quite similar to the costly shift in perspective which would allow us to affirm that *only certain forms of social organization can intervene at social phenomena of the same form* - a strategic, rather than purely theoretical, principle.

In the *Primer*, this scale-sensitive approach is not only present in determinate concepts which helped us unpack the "black box" of use-composition in production, but is rather built into the formalism as such. Sections 16 and 17, which deal with the reproduction of individual capitals and introduce the point of view of total social capital $\mathcal{K}$ and reserve army of labour $\mathcal{L}$, help to demonstrate this in some detail. As we have previously suggested, the compositional point of view rigorously formalized by category theory allows us to define in purely logical terms what counts as a "part" and what counts as a "whole" in a given system, from a given perspective. This is the principle behind our theory of the cooperative set made up of parts of individual labour-use sets, for example: depending on the object that "sees" it, parts of individual labour-power can compose together more complex and information-rich sets than the trivial bundle of "the set of labourers employed". New local "atoms" are defined as new syntheses become possible, just as new totalities can be constructed once new ways of decomposing previously existing objects emerge. The transcendental logical framework has primacy here over the way parts and wholes will be defined - these are only set in place from the perspective of concrete and materially-supported "instruments" capable of sensing some differences rather than others. And this is precisely the way that a large-scale analysis of capitalist accumulation is constructed in our approach.

Section 16 introduces, first, the diagram for simple reproduction of individual capitals and then adds the key condition that a greater part of surplus value be directed back to the next productive cycle in order to define the expanded reproduction diagram. This small shift, which requires that some new set of commodities "respond" to an increased amount of commodity-capital available, implies that the arrow connecting "more accumulated capital" and "more commodities for consumption" be *seen by other capitalist endeavours* - it is a difference that only makes a difference if other conditions are met, namely, the increased productive output of other capitalist ventures, the increased availability of labour-power or the growing enclosure of natural means of production. In other words, it is a difference that cannot be made to count by a productive endeavour alone - it requires us to consider collective growth rate of individual capitals. However, the moment that we introduce this new object $\mathcal{K}$ we also start looking at single productive units from a new perspective, as new differences in production start to make a bigger difference than before. This is why, early on in section 17, we returned to our operadic description of productive units to re-describe it as a "minimal dynamic unit" (17.1.1) of capitalist accumulation: rather than appear as an unconstrained site where capitalist valorization can explore the non-priced parts of labour-use anyway it wants, now the implicit connection with other similar productive units makes the specific organic composition of capital a crucial feature in the overall satisfaction of conditions for total capital growth - and, *consequentially*, the partition of total growth amongst individual productive units. We therefore began by introducing a local difference (expanded reproduction) which required a new global perspective (total social capital) which then partitions the capitalist space into new parts: individual capitals competing for shares of total social growth.

At the same time, the variations to organic composition allow us to propose a similar reconstruction of individual labour-commodities: first conceived as essentially "productive consumables" which *also* have the property of going through a separate reproductive cycle, the labouring class emerges, from the perspective of capitalist accumulation, as being, first of all, an *unemployed* class - that is, a class determined by its survival - which is only partially, and dynamically, absorbed by the productive sphere.

If we quickly venture into a rushed translation of this complete process in explicitly "Badiouian" terms, we could say that the productive process forms a *site* in which non-priced parts of labour and means of production - parts whose resolutions are not immediately compatible with the capitalist *atomic logic* - can be exploited, as if in a laboratory, and conformed to the value-form through different strategies of social measurement, forming weak *singularities*, small perturbations to regular value relations. The assignment of a higher value of existence to some previous capital, that is, the successful valorization of money, is a weak *point*, and the collection of such points describes a topological interior called an *organ* - or a certain standard of capital's organic composition - which combines, through competition, concentrating, dividing and centralizing capitals. The real *envelope* of some organs - the combination of several competing capital structures under a common financial circuit - forms a monopolistic *body* which supports a determinate form of capitalist accumulation via the internal competitive coherence of its parts. It can be formed by one or multiple corporations whose existence is maximal in the capitalist world - such that they come to immanently substitute the appearance of value logic itself, distorting the regularities of the world from the standpoint of their singular compositions. As the reproduction of this capitalist body gains visible consistency, all the cooperative ties between workers are substituted for the propagation of social information through competition and monopoly, which induce a market logic on the basic parts of social reality. The existence of this monstrous collective body is logically held together, then, at the expense of the increased obsolescence of cooperative connections between people.

We offer this simplified overview of the "greater logic" of capitalist accumulation as a way to highlight two crucial aspects of it. First, that even though there are important spatial and dynamic determinations to this model, it also includes a constant movement between analysis and synthesis, between the exploration of "sub-atomic" resolutions of productive sites and the "gluing together" of these non-priced parts in priceable ways. Capitalism is, therefore, not only an expanding and unfolding process, but also a sort of *lens* that constantly seeks to extract differential potentials across divergent material scales - this is, after all, the way we defined the innovation of industrial capital: capital that also finds differences in value through immanent decomposition and re-composition of parts. However, there is another crucial consequence of this perspective that we must consider, namely, that capital can only extract from cooperative organizations of people and things *those parts that can be indexed back onto value*. This is why we qualify the theory of capitalist change as being one of "weak singularities" and "weak points" - ultimately, even if capitalism can *see everything* it cannot see *everything in every way*, as only those novelties which can be conformed to commodity logic can be "sensed" by this complex system. This means that a scale-sensitive reconstruction of capitalism is also one that is capable of expressing a limit to capital that does not require us to envision some promised land outside of its scope or put our hopes on some teleological end point. It also places a renewed emphasis on social cooperation not only as a tactical and strategic instrument of struggle, but as a tool for decomposing and recomposing social parts in non-capitalist ways.

Cooperative surplus and collective organization as experiment

There are many different recipes for "how to read *Capital*": some give extreme priority to the first chapter, focusing almost exclusively on the logic of value or the theory of fetishism, while others, like Louis Althusser's, suggest that one should only read the first chapter *after* the rest of the first volume in its entirety. If we were to describe our reconstruction in similar terms, we could say that, in our case, we should perform a strange topological "gluing" of the book, creating an impossible continuity between chapter 1, on the commodity-form, and chapter 13, on cooperation, and deforming the reading of the three volumes in such a way that, anamorphically, new shapes might become intelligible. As crazy as this description sounds, it does help us to highlight that, in our reconstruction, the logic of value-form sets up the basic logical constraints for how large-scale collective organization of society appears *to itself* in capitalist societies. Money, prices, wages, profit, social division of labour, social departments of production, production prices and profit rates - all of these concepts can be understood as the complex internal machinery that restricts how the commodity world represents capitalist social organization to itself. This is what Brazilian Marxist thinker Jorge Grespan calls the capitalist "mode of representation" [8] and which we might call "the transcendental conditions of appearance" within the capitalist social formation.

We highlight the chapter on cooperation because it is there - and not with the introduction of surplus value - that we find the emergence of a truly invisible force, something which cannot be directly "seen" by the commodity world that nevertheless mobilizes it [82]. While capital, unlike money, can be understood as a "dynamic ruler" that is capable of measuring increases in value, the objectivity of the "collective worker" - of the forms of organization of men, women, other natural forces and ideas - is only possible within the capitalist social constraints through its indexing onto value, that is, through its objectification into new means of production that already interact in "cooperative" ways, or through their expression in terms of new standards of individual wages, etc. No wonder that other sub-dominant transcendental spaces - the communitarian and legal standpoints - are constantly fighting to make internal determinations of social cooperation count for capitalist production, proposing legal reforms, minimum wages, fixating the collective existence of workers in trade unions, fighting for human and animal rights, organizing protests and strikes, or representing this surplus-coordination power in terms of social groups - "the working class", "the exploited", etc. Even though the support of T_B and T_A is an essential part of any consistent social form, the essential communist question is not how to conform this "cooperative surplus" onto other transcendental layers - giving it legal or communitarian existence - but rather how to use the "organizational point of view" to explore what forms of social

[82] A great proponent of a similar reading, that highlights the chapter on cooperation and its connection to forms of representation in capitalism, is Fredric Jameson - see [11].

composition could inform a new global consistency, irreducible to either of the existing layers of the capitalist world.

It is true that Marx never got to write the (in)famous chapter on class in book 3, but the scant notes that we have of what would have been "chapter 52" suggest that there was some vacillation on his part of how to characterize it. Here is the complete passage:

> The owners merely of labour-power, owners of capital, and land-owners, whose respective sources of income are wages, profit and ground-rent, in other words, wage-labourers, capitalists and land-owners, constitute then three big classes of modern society based upon the capitalist mode of production.
>
> In England, modern society is indisputably most highly and classically developed in economic structure. Nevertheless, even here the stratification of classes does not appear in its pure form. Middle and intermediate strata even here obliterate lines of demarcation everywhere (although incomparably less in rural districts than in the cities). However, this is immaterial for our analysis. We have seen that the continual tendency and law of development of the capitalist mode of production is more and more to divorce the means of production from labour, and more and more to concentrate the scattered means of production into large groups, thereby transforming labour into wage-labour and the means of production into capital. And to this tendency, on the other hand, corresponds the independent separation of landed property from capital and labour, or the transformation of all landed property into the form of landed property corresponding to the capitalist mode of production.
>
> The first question to be answered is this: What constitutes a class? — and the reply to this follows naturally from the reply to another question, namely: What makes wage-labourers, capitalists and landlords constitute the three great social classes?
>
> At first glance — the identity of revenues and sources of revenue. There are three great social groups whose members, the individuals forming them, live on wages, profit and ground-rent respectively, on the realisation of their labour-power, their capital, and their landed property.
>
> However, from this standpoint, physicians and officials, e.g., would also constitute two classes, for they belong to two distinct social groups, the members of each of these groups receiving their revenue from one and the same source. The same would also be true of the infinite fragmentation of interest and rank into which the division of social labour splits labourers as

well as capitalists and landlords-the latter, e.g., into owners of vineyards, farm owners, owners of forests, mine owners and owners of fisheries.

[Here the manuscript breaks off.] [83]

Considering all the conceptual tools we already developed, we can see that Marx first defines ″three big classes of modern society based upon the capitalist mode of production″ in terms that exceed the immanent point of view of the commodity world, to include matters of property ownership and even an empirical example taken from the social composition of British society. Hence ″wage-labourers, capitalists and landlords″ are characterized as owners of labour, money and land, respectively. Now, this is not a purely commodity-theoretical distinction, but a more general one, that considers all transcendental layers of a capitalist social formation - commodities, property rights and group structures - and which therefore seems muddled to him, as other factors ″obliterate lines of demarcation everywhere″.

On the other hand, when he poses the question directly - ″what *constitutes* a class?″ - and tries to reach an answer from the standpoint of the dominant transcendental alone, that is, from the standpoint of the ″identity of revenues and sources of revenue″ which characterize purely commodified paths in capitalist space, he is also not satisfied with the answer, as ″from this standpoint, physicians and officials, e.g., would also constitute two classes, for they belong to two distinct social groups, the members of each of these groups receiving their revenue from one and the same source″. The criteria allows for more partitions of the space than those that define the ″three main classes″ of the capitalist world.

In other words, Marx is unhappy with simply describing ″intra-layer″ class distinctions, since this does not help him single out the specificity of what these three ″big classes″ have in common, requiring the inclusion of other social groups as well, but he is also unhappy with reaching an answer that relies on the ″multilayered″ transcendental as a whole, as these other layers were not fully presented in *Capital* and thus this particular answer becomes a petition of principle rather than a conceptual construction.

There is however one particular aspect that all these three classes have in common and which we now have the means to define, namely, all three mobilize a certain ″organizational surplus″ that place their owners in special positions in the process of decomposition and recomposition of the parts of the capitalist world. Labour-power and land are both ″fictive commodities″ forced into commodity-form via ″layer-conformance″ operations, which implies not

[83] See chapter 52 in [28]

all of their parts are indexed on commodity logic, while money, as we have repeatedly argued, is not only a commodity, but one which plays a crucial role in the immanent representation of social coordination within a commodity-world - a role which it acquires ″for free″.

From a ″tektological″ perspective, what Marx might have been trying to discern is the fact that these three classes are not definable solely by the commodities they own, but rather by the *organizational commons* that haunt the atomic logic of each of them. Labour, land and money can all be composed and decomposed in scale-dependent ways so as to generate new ″logical atoms″ in the form of new uses of labour cooperation and actions, of energy extraction and new material properties, or in the form of new measures of social intelligibility - and this property singles them out without recourse to additional layer-dependences. Such a statement, however, cannot be made from *inside* the capitalist world, which only sees this surplus once it has been localized on a ″priceable part″. And hence the poetic value of the manuscript's interruption, which, to us, only reaffirms the project of reconstructing the critique of political economy within a theoretical framework that can further decompose these ″commons″ into determinate organizational spaces, with their respective constraints and possibilities, and experiment with their capacity to form consistent logical spaces of their own.

To conclude, we invite the reader to help us explore all the different conceptual venues opened up by the *Primer*. As we mentioned in our introductory remarks, this essay is a first condensed exposition of some of the partial results of much broader political project, an attempt of putting to the test of (hopefully generous) interlocutors several connected ideas that still rely on quite shaky foundations.

The two main structural risks we face lie, evidently, in our employment of mathematical formalisms - constantly confronting us with the limits of our self-taught understanding of these topics - and in possible misreadings of Marx's concepts. Still, even if it turns out that there is little to correct in these two fronts, we are keenly aware that several aspects of the *Primer* remain greatly underdeveloped: for example, the theory of a ″multilayered transcendental″ for capitalism not only requires further investigation of useful formal resources, but also demands additional and extensive engagement with anthropology and social theory, just as further architectural features could - and should - be added to our ″organizational point of view″ coming from feminist social reproduction theory, critical ecology and critical race studies.

We anticipate, however, that our next step in this research is to investigate what such an organizational perspective could look like within this new theoretical framework. Since all our efforts in the *Primer* were motivated by the challenge to subsume the current conceptual gap separating our understanding of political economy and of political organization, we must see what happens when we approach worker cooperatives, mutual aid communities, party politics, social movements and other forms of collective organization using these same method and tools. So that's where we are heading.

Notational index

C - commodity (2.)
C_U - use set of commodity (2.1)
C_E - exchange set of commodity (2.1)
$\mathcal{C}$ - the set of commodities, also the category of commodities when equipped with *commodity-preserving operations* (2.)
$\mathcal{O}(A; B)$ - set of commodity preserving operations between A and B (2.3)
$\mathcal{C}_U$ - functor to use sets (2.2.3)
$\mathcal{C}_E$ - functor to exchange sets (2.2.3)
$\psi : \mathcal{C}^N \to \mathcal{C}$ - commodity preserving operations (2.2.2)
Δ-resolution - use-resolution (2.5)
$C_{\Delta U}$ - use-resolution of commodity (2.5)
$C_{\Delta^A U}$ - abstract use resolution (14.5.1.2)
$e(A, B) = p$ - exchangeability function (3.1)
$e(A, A) = M$ - commodity existence (3.4)
$p, q, \mu, M \in T$ - degrees of a transcendental T (3.3)
q^* - completing the exchange (3.2.2)
q_* - slicing the exchange (3.2.3)
q^r - partially completing the exchange (3.2.5)
q_r - partially slicing the exchange (3.2.5)
$p(x)$ - price function (7.3)
E_A - exchange set of commodity A (5.)
$A(x) = p$ - A-component function (5.4)
$a \lceil b = p$ - degree of localization (6.2)
$a \ddagger b$ - degree of compatibility (6.2)
M - money-commodity (7.1)
m_i, m_j - amounts of money-commodity M (7.1)
${T^i}_{jk}$ - multilayered transcendental (8.1)
${T^C}_{AB}$ - transcendental of capitalist formations (8.2)
$0 \to (C)_{AB}$ - layer-conformance (8.2.2)
T^i/T_{jk} - bracketing operation (8.2.3.1)
$Id_A(x, y), T_A$ - gift function, community transcendental (8.2.1)
$Id_B(x, y), T_B$ - contract function, State transcendental (8.2.1)
$(C \to M \to C)_{AB}$ - market (8.3)
$M \to (M)_{AB}$ - currency (8.3.1)
K_ϕ - Capital (9.1)

$K_{\phi I}$ - Interest capital (9.3)
$K_{\phi M}$ - Merchant capital (9.4)
$K_{\phi P}$ - Industrial capital (9.5)
$\nu : m_i \to (LP \otimes MP) \to C \to m_j > m_i$ - valorization process (9.5.1)
$(C_i \otimes C_j) \to P$ - commodity-production (2.3.5 and 10.)
$C_1 \otimes C_2$ - the (symmetric) monoidal product of two commodities (10.1)
P - product (10.2)
MP^P - private means of production (10.3.2)
MP^N - natural means of production (10.3.2)
MP^M - machine (14.5.2)
LP - labour-power or labour-commodity (10.3.2)
C_{LP_A} - the set of abstract labours embodied in private means of production (11.4)
LP^n - n-many labour-commodities employed (11.5)
$SNLU(x)$ - the socially necessary labour use predicate (11.5)
$SNLT_C$ - the socially necessary labour time embodied in a commodity (11.5)
$SNLT(x)$ - the measure of socially necessary labour time, evaluating contributions of different labour uses (11.5)
LP^w - set of parts of employed labour-use (14.4.2) $C \to 1$ - simple consumption (10.3)
$CL : (LP_U \otimes MP_U) \to P_U$ - concrete labor (11.1)
$AL : (LP_U \otimes MP_U) \to P_U \wedge P_E \geqslant LP_E \cup MP_E$ - abstract labour (11.2)
LP_A - labour time-measure (11.3.2)
$P(k)$ - an operad, where $k \in \mathbb{N}$ (12.1)
$W(p_1, p_2, ...p_n; q)$ - operad composition (12.1.1)
$In(x)$ - box inputs (12.1.2)
$Out(x)$ - box outputs (12.1.2)
$V(x)$ - wire evaluation (12.1.4)
$\delta_n : In(x) \to Out(x)$ - value propagator (12.6)
$*$ - delay token (12.6)
dV - average value transfer rate of a production process (12.6.1)
$\vec{\delta_n}$ - a vector of unit requirements associated to each production process (12.6.2)
$K_c : m_c \to m_c$ - constant capital (13.2.1)
$K_\nu : m_\nu \to m_\nu + m_s$ - variable capital (13.2.1)
$AL^{Eq} \subseteq AL$ - equivalent labour (13.3)
$AL_S, AL = AL_{Eq} \cup AL_S$ - surplus labour (13.3.1)
LP_{Eq} - equivalent labour time (13.3)
LP_S - surplus labour time (13.3.1)
P^{Eq} - equivalent product (13.3.1)
P^S - surplus product (13.3.1)
m_s/m_ν - surplus production rate (13.3.2)
LP_S/LP_{Eq} - exploitation rate (13.3.2)
$\delta AL^A = \delta LP^w{}_S/LP^w{}_{Eq}$ - absolute surplus extraction (14.1.1)
$\delta AL^R = \delta LP^w{}_S/\delta LP^w{}_{Eq}$ - relative surplus extraction (14.1.1)

δ^p - a partial resolution (14.5.1)
m_w - total subsistence wage (15.1)
$p(LP_{MS})/u$ - time or piece wages (15.1.1 and 15.1.2)
k as (ϕ_P, rv_i, rv_p) - simple reproduction cycle of capital (16.2)
k^+ as (ϕ_P, rv_i, rv_p) - expanded reproduction cycle of capital (16.6)
$\mathcal{K}$ - total social capital (16.7.1)
a_k - the accumulation rate for a given capital K (17.3)
A_k - the accumulation rate for $\mathcal{K}$, the total social capital (17.5.1)
$\mathcal{K}^{\circlearrowleft}$ - all self-maps of total social capital, representing all possible partitions (17.6.1)
$\mathcal{K}^{\circlearrowleft^{\mathbb{N}}}$ - all ordered histories of partitions of total social capital (17.6.1)
$\mathcal{L}$ - labouring class (17.7.3)
$\mathcal{L}_{\mathcal{A}}$ - active labour force (17.2)
$\mathcal{L}_{\mathcal{U}}$ - unemployed labour force (17.7.1)
$\mathcal{L}_{\mathcal{P}}$ - pauperized labour force (17.7.2)

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